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Mathematics of finance

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PREFACE

IN ADDITION TO the topics that are ordinarily treated in a course on mathematics of finance, this book contains the necessary algebraic background for those topics. The entire book includes enough material for two courses of three semester hours each. The part on mathematics of finance furnishes somewhat more material than would ordinarily be given in a three-semester-hour course; hence, the instructor can fit the course to suit the needs of his class. The algebraic material in this book or a semester of college algebra can be used as a background for the work on mathematics of finance.

The algebraic part makes up about thirty per cent of the book and includes twenty exercises and some 600 problems, whereas the mathematics of finance contains 48 exercises and about 860 problems. We have made a concentrated effort to see that the exercises are a lesson apart; furthermore, the number of exercises in the material likely to be taken in a course is slightly less than the number of non-quiz class periods in it. This enables the instructor to cover the course without having to make any assignment from two or more exercises. The problems are in groups of four that are similar in type; hence, a full coverage can be obtained by assigning every fourth problem and it is not necessary for the instructor to study through the problem list to make a good assignment.

The first chapter on annuities treats only the case in which the interest period and payment interval coincide. The general annuity is treated just before the work on insurance.

The examples are worked out in enough detail that the student should not encounter any real difficulty in following them. They are so chosen that they illustrate the text material that precedes them and act as a guide in working the problems that follow.

Answers are given in the text to three-fourths of the problems. There is a chapter summary and review exercise at the end of each chapter in the finance part of the book.

EDWIN D. MOUZON, JR.
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CONTENTS

7 FUNCTIONS AND GRAPHS

8 PROGRESSIONS, THE BINOMIAL THEOREM

9 STATISTICS

10 SIMPLE INTEREST

11 COMPOUND INTEREST

12 SIMPLE ANNUITIES

13 AMORTIZATION AND SINKING FUNDS

14 DEPRECIATION AND CAPITALIZED COST

15 BONDS

16 GENERAL ANNUITIES

17 LIFE ANNUITIES AND LIFE INSURANCE

TABLES

Mathematics of finance

1

The four fundamental algebraic operations

1-1 INTRODUCTION

To compete successfully in business, one must be familiar with a number of terms and must have a considerable facility with the elementary operations of algebra. The terms and algebraic operations that are needed in a course in mathematics of finance will be introduced, explained, and used at such a rate as to keep the student supplied with something to do at all times without giving him enough at any time to cause a serious case of mental indigestion. The four fundamental operations are addition,

subtraction, multiplication, and division. They will be discussed in the following sections.

1-2 THE FOUR OPERATIONS WITH NUMBERS

We shall need to use the *size* of a number in some of our work; hence, we offer the following definition: The size or numerical value of a number without regard to its sign is called the *absolute value* of the number. Thus, the absolute value of $+7$ is 7 and that of -7 is also 7.

We shall now recall the rules for use in addition, subtraction, multiplication, and division of numbers.

(i) *To add two numbers with like signs, find the sum of their absolute values and give it their common sign. To add two numbers with unlike signs, find their difference and give it the sign of the one with the larger absolute value.*

Example 1. $5 + 3 = 8$.

Example 2. $5 + (-3) = 2$, since the difference between 5 and 3 is 2 and the one with the larger absolute value is positive.

(ii) *To subtract one number from another, we change the sign of the one that is being subtracted and add.*

Example 3. $9 - 4 = 9 + (-4) = 5$.

Example 4. $9 - (-4) = 9 + (+4) = 13$.

(iii) *To multiply one number by another, multiply their absolute values and then prefix a plus or minus sign to this product according as the numbers have the same sign or opposite signs.*

Example 5. $(6)(3) = +18$.

Example 6. $(6)(-3) = -18$.

Example 7. $(-6)(-3) = +18$.

(iv) *To divide one number by another, divide their absolute values and then prefix a plus or minus sign according as the numbers have the same sign or opposite signs.*

Example 8. $\frac{12}{4} = +3.$

Example 9. $\frac{12}{-4} = -3.$

Example 10. $\frac{-12}{-4} = +3.$

1-3 ADDITION AND SUBTRACTION OF ALGEBRAIC EXPRESSIONS

We shall need several new terms in working on algebraic expressions with the four fundamental operations and shall now define some of them. Two or more numbers or symbols that represent numbers are called *algebraic expressions* if they are connected or combined by addition, subtraction, multiplication, or division signs. Parts separated by plus or minus signs are called *terms*. If an expression consists of one term it is called a *monomial;* if it contains two terms, it is a *binomial;* if it consists of more than two terms, it is called a *polynomial.*

Thus, $2a$, ab, and $2a/3b$ are monomial algebraic expressions, whereas $3a + 2b$ is a binomial with $3a$ and $2b$ as terms. If the product of two or more numbers is indicated, each number is a *factor* and each is called the *coefficient* of the other. Thus, a is the coefficient of b and b is the coefficient of a in ab; furthermore, a and b are factors. If the coefficient is a number, as the 2 in $2a$, it is called a *numerical coefficient.* If the coefficient is a letter, as the a in ab, it is called a *literal coefficient.* Two terms are called *like terms* if they have the same literal part; thus, $3a$ and $5a$ are like terms.

To add two or more algebraic expressions, we collect coefficients of like terms. This can be done readily if we write each expression on a separate line and, at the same time, put like terms under one another.

Example 1. Find the sum of $3x + 2y - 4w + 3t$, $2x + 5y + 8w$, and $x - 4y + 6w + 2t$.

Solution. To add more readily, we shall arrange the expressions as suggested above. Thus

$$\begin{array}{r}
3x + 2y - 4w + 3t \\
2x + 5y + 8w \phantom{{}+3t} \\
x - 4y + 6w + 2t \\
\hline
6x + 3y + 10w + 5t
\end{array}$$

To subtract one algebraic expression from another, we subtract the coefficient of each term in one from the coefficient of the like term in the other.

Example 2. Subtract $3x - 2y + 7t$ from $5x + 3y + 6w + 2t$.

Solution.

$$\begin{array}{r} 5x + 3y + 6w + 2t \\ 3x - 2y \qquad\;\; + 7t \\ \hline 2x + 5y + 6w - 5t \end{array}$$

When one quantity is subtracted from another, the one that is subtracted is called the *subtrahend,* the one subtracted from is the *minuend,* and the quantity obtained as a result is the *remainder.* Thus, in Example 2, the minuend is $5x + 3y + 6w + 2t$, the subtrahend is $3x - 2y + 7t$, and the remainder is $2x + 5y + 6w - 5t$.

1-4 SYMBOLS OF GROUPING

The symbol ( ) is called *parentheses* and is one of several symbols used to group together terms that are to be considered as a single term. Brackets [ ] and braces { } are two other symbols used for similar purposes.

If several terms are enclosed in parentheses and if there is a coefficient of the parentheses, then each term in the parentheses must be multiplied by that coefficient when the parentheses are removed. Thus

$$\begin{aligned} 2(3x - 4y) &= 2(3x) - 2(4y) \\ &= 6x - 8y \end{aligned}$$

since 2 times $3x$ is $6x$ and 2 times $-4y$ is $-8y$. Brackets and braces are removed in the same manner.

If two or more symbols of grouping are used in the same problem, it is customary to remove the innermost one first, the one that is then innermost next, and so on until all symbols are removed.

Example. Remove the signs of grouping from

$$2\{5x + 8y - 3[-2x + 4y + 3(x - y)]\}$$

and collect like terms.

Solution. $2\{5x + 8y - 3[-2x + 4y + 3(x - y)]\}$
$= 2\{5x + 8y - 3[-2x + 4y + 3x - 3y]\}$,
removing parentheses,
$= 2\{5x + 8y - 3[x + y]\}$, collecting in the brackets,
$= 2\{5x + 8y - 3x - 3y\}$, removing brackets,
$= 2\{2x + 5y\}$, collecting in braces,
$= 4x + 10y$

Exercise 1-1

Perform the indicated operations in Problems 1 through 4.

1. $4 + 2$, $4 - 2$, 4×2, $4 \div 2$.
2. $5 - 2$, $5 - (-2)$, $5 \times (-2)$, $5 \div (-2)$.
3. $-6 + 3$, $-6 - (-3)$, -6×3, $-6 \div 3$.
4. $-12 - 4$, $-12 - (-4)$, $-12 \times (-4)$, $-12 \div (-4)$.

Find the sum of the algebraic expressions given in each of Problems 5 through 8.

5. $2x + 3y + 5w$, $3x + y + 2w$.
6. $3a + 2b + 4c$, $2a + 3b + c$.
7. $3b + 5c - 7w$, $2b - 4c + 3w$.
8. $3x - 4y + 2z$, $-ax + 2y - 2w$.

9. Subtract $2x + 4y + w$ from $5x + 7y + 2w$.
10. Subtract $2a + 3b + cw$ from $7a - 2b + 4w$.
11. Subtract $3c - 4d - 2x$ from $5c - 5d + ax$.
12. Subtract $-x + 2ay + 5w$ from $2x + by + w$.
13. Subtract the sum of $2x + 3y - 4w$ and $3x - 2y + 3w$ from $5x - y + 2w$.
14. Subtract the sum of $3a - 5b + 6c$ and $2a + 6b - 4c$ from $7a + 2b + 3c$.
15. Subtract $7x + 3y - 4w$ from the sum of $3x + y + 6w$ and $5x + 2y - 2w$.
16. Subtract $ax + 3y + 5w$ from the sum of $2x - 8y - w$ and $3ax + 10y - 4w$.

Remove the symbols of grouping and collect like terms in the following problems.

17. $2(3a - 2b) + 3(a + 3b)$.
18. $3(2x - 3y) - 5(x - 2y)$.
19. $2(5a - 2b) - 3(3a - b)$.
20. $-a(x + 2y) + x(2a + 3)$.
21. $2[3a - 3(2a - 3b) + 2b]$.
22. $4[12x - 3(3x - 5y) - 12y]$.

23. $-3[5x + 2(-3x + 4y) - 6y]$.
24. $-5[-2x + 3(-4x + 2y) - 5y]$.
25. $2\{10a + 3[b - 4(a + 2b)] + 3b\}$.
26. $-3\{9a + 2[b - (a + 3b) + a]\}$.
27. $-2\{3x + 2y - 3[2x - y + 3(-x + y) - y] - 7x\}$.
28. $-4\{2a - 3b - 2[2a - 3b - 2(2a - 3b) + 2a - 3b]\}$.

1-5 EXPONENTS IN MULTIPLICATION

There are times and conditions under which we need to use a factor more than once. We indicate this by writing the number of times the factor is to be used above and to the right of the factor and calling it an *exponent*. The factor is called the *base*. Thus, a^2 means that the factor a is to be used twice; hence, $a^2 = (a)(a)$. Similarly, $(x - 2y)^3$ indicates that $x - 2y$ is to be used three times as a factor.

To understand what to do with exponents of like bases in multiplication, we shall consider

$$\begin{aligned} a^m a^n &= (aa \ldots \text{to } m \text{ factors})\ (aa \ldots \text{to } n \text{ factors}) \\ &= aa \ldots \text{to } (m + n) \text{ factors} \\ &= a^{m+n} \end{aligned}$$

Consequently, we see that

$$a^m a^n = a^{m+n}$$

and put it in words as: *Exponents of like bases are added in multiplication.*

The product of two or more monomials is the product of their numerical parts times the product of their literal parts.

Example 1. $x^3 x^4 = x^{3+4} = x^7$.

Example 2. $(3x^2)(5x^4) = (3)(5)x^{2+4} = 15x^6$.

Example 3. $(2a^2b)(3a^3b^4) = 6a^{2+3}b^{1+4} = 6a^5b^5$.

1-6 MULTIPLICATION OF ALGEBRAIC EXPRESSIONS

The product of two polynomials is obtained by multiplying each term in one by each term in the other and adding the results.

Example 1. Multiply $3x^3 - 5x^2 + 2x - 1$ by $2 + 3x^2 - 4x$.

Solution. We shall arrange each factor in descending powers of x before multiplying and put like terms of the product in columns. Thus

$$\begin{array}{rrrrrrl}
 & & 3x^3 & -\ 5x^2 & +\ 2x & -\ 1 & \\
 & & 3x^2 & -\ 4x & +\ 2 & & \\
\hline
9x^5 & -\ 15x^4 & +\ 6x^3 & -\ 3x^2 & & & \text{multiplying by } 3x^2 \\
 & -\ 12x^4 & +\ 20x^3 & -\ 8x^2 & +\ 4x & & \text{multiplying by } -4x \\
 & & 6x^3 & -\ 10x^2 & +\ 4x & -\ 2 & \text{multiplying by } 2 \\
\hline
9x^5 & -\ 27x^4 & +\ 32x^3 & -\ 21x^2 & +\ 8x & -\ 2 & \text{collecting terms}
\end{array}$$

Example 2. Find the product of $x^2 - xy + y^2$ and $x + y$.

Solution.

$$\begin{array}{rrrrl}
x^2 & -\ xy & +\ y^2 & & \\
x & +\ y & & & \\
\hline
x^3 & -\ x^2y & +\ xy^2 & & \text{multiplying by } x \\
 & x^2y & -\ xy^2 & +\ y^3 & \text{multiplying by } y \\
\hline
x^3 & & & +\ y^3 & \text{collecting terms}
\end{array}$$

1-7 SOME SPECIAL PRODUCTS

Several types of products occur relatively often and the reader should become familiar with them so that he can perform the multiplications mentally. The products given below can be verified by multiplication and should be learned since they are the ones that occur frequently.

(1) $$k(x + y) = kx + ky$$

(2) $$(x + y)^2 = x^2 + 2xy + y^2$$

(3) $$(x - y)^2 = x^2 - 2xy + y^2$$

(4) $$(x + y)(x - y) = x^2 - y^2$$

(5) $$(ax + b)(cx + d) = acx^2 + (ad + bc)x + bd$$

Example 1. $3(2x - b) = 6x - 3b$.

Example 2. $(2a + 3b)^2 = (2a)^2 + 2(2a)(3b) + (3b)^2$
$= 4a^2 + 12ab + 9b^2$.

Example 3. $(3x - 4y)^2 = (3x)^2 - 2(3x)(4y) + (4y)^2$
$= 9x^2 - 24xy + 16y^2.$

Example 4. $(3a + 5b)(3a - 5b) = (3a)^2 - (5b)^2$
$= 9a^2 - 25b^2.$

Example 5. $(2x + 3)(4x - 5) = (2)(4)x^2 + [2(-5) + 3(4)]x + 3(-5)$
$= 8x^2 + [-10 + 12]x - 15$
$= 8x^2 + 2x - 15.$

Exercise 1-2

Multiply the first expression by the second in each of Problems 1 through 24.

1. a^2, a^5.
2. a^3, a^4.
3. $2x^3, 3x^2$.
4. $3x^2, 4x^3$.
5. $3x^5, -7x^2$.
6. $-2y^5, 4y^4$.
7. $-7b^3, -2b^2$.
8. $-9w^4, -3w$.
9. $2ab, 3a^2b^3$.
10. $4x^2y, -6x^3y^2$.
11. $-3c^3d^2, 5c^2d$.
12. $-6x^4y^3, -7x^2y^5$.
13. $2x + 3, x^2 + 2x - 3$.
14. $3x - 2, 2x^2 - x + 4$.
15. $5x - 2, 3x^2 + 2x - 1$.
16. $4x + 3, x^2 - 3x + 5$.
17. $3x + 1, 2x^3 - 4x^2 - 3x + 5$.
18. $2x - 5, 3x^3 - 2x^2 + 4x + 2$.
19. $5x - 3, 2x^3 - 3x^2 - x + 3$.
20. $2x + 3, 4x^3 + 2x^2 - 3x + 1$.
21. $x^2 - x + 2, 2x^3 + 3x^2 - 4x + 2$.
22. $3x^2 - 2x - 1, 4x^3 - 3x^2 - 5x + 3$.
23. $2x^2 - 3x + 4, x^3 - 2x^2 + 3x + 4$.
24. $2x^2 - 5x + 3, 3x^3 + 4x^2 - 2x + 5$.

Using the special products (1) to (5) of Section 1-7, write the values of the following products.

25. $4(3x - 2y)$.
26. $-2(2a - 5b)$.
27. $-3(2x - 5y)$.
28. $5(-2y + 7b)$.
29. $(2a + b)^2$.
30. $(3a - 2b)^2$.
31. $(3x + 4y)^2$.
32. $(4x - y)^2$.
33. $(2a + 3b)(2a - 3b)$.
34. $(3a + 5b)(3a - 5b)$.
35. $(x^2 + 2y)(x^2 - 2y)$.
36. $(3x^2 + y^3)(3x^2 - y^3)$.
37. $(2x - 3y)(3x + 2y)$.
38. $(5a + 3b)(2a - 5b)$.
39. $(3x + 5y)(2x - 7y)$.
40. $(5x - 4y)(2x + 3y)$.

1-8 EXPONENTS IN DIVISION

It was pointed out in Section 1-5 that $a^n = aa \ldots$ to n factors. We shall now use this definition and see what to do to exponents of like bases in division. By use of the definition of a positive integral exponent,

$$\begin{aligned}\frac{a^m}{a^n} &= \frac{aa \ldots \text{ to } m \text{ factors}}{aa \ldots \text{ to } n \text{ factors}} \\ &= aa \ldots \text{ to } (m-n) \text{ factors for } m > n^* \\ &= a^{m-n}\end{aligned}$$

Consequently, we can write

$$\frac{a^m}{a^n} = a^{m-n}$$

and put it in words as: *In dividing one power of a quantity by another power of the same quantity we subtract the exponent of the divisor from that of the dividend.*

The quotient of two monomials is the quotient of their numerical parts times the quotient of their literal parts.

Example 1. $\dfrac{a^{10}}{a^3} = a^{10-3} = a^7.$

Example 2. $\dfrac{12x^7}{3x^2} = \dfrac{12}{3}x^{7-2} = 4x^5.$

1-9 DIVISION OF ALGEBRAIC EXPRESSIONS

It is desirable to put each algebraic expression in terms of ascending or descending powers of the letter in dividing one algebraic expression by another. The procedure will be illustrated by an example.

Example. Divide $6x^3 - 9x + 13x^2 - 10$ by $3x + 2x^2 - 5$.

Solution. The terms of the dividend and divisor are arranged in descending powers and the divisor put on the left in line 2. Then the first term of the divisor is divided into the first term of the dividend and the first term, $3x$, of the quotient is obtained. It is put in line 1 and multiplied by the divisor. The product is put in line 3 and subtracted from the divi-

*The symbols $m > n$ are read "m greater than n."

dend. The remainder is put on line 4 and the first term of it is divided by the first term of the divisor. Thus we obtain 2, which is put in the quotient along with $3x$. The 2 is then multiplied by the divisor and the product put in line 5. Now subtracting, we get zero; hence, the quotient is $3x + 2$ and the remainder is zero.

$$
\begin{array}{lr|l}
(1) & & \quad\; 3x + 2 \\
\cline{3-3}
(2) & 2x^2 + 3x - 5 & 6x^3 + 13x^2 - \;\; 9x - 10 \\
(3) & & \underline{6x^3 + \;\; 9x^2 - 15x} \\
(4) & & \qquad\quad\;\; 4x^2 + \;\; 6x - 10 \\
(5) & & \qquad\quad\;\; \underline{4x^2 + \;\; 6x - 10}
\end{array}
$$

1-10 FACTORING

If the same factor can be and is taken from each term of an expression, we say that a *common factor* has been removed. This should be the first step in any factoring.

Example 1. The factor $2x$ can be removed from each term of $6x^2 + 8xy + 4xy^2$. If this is done, we have

$$6x^2 + 8xy + 4xy^2 = 2x(3x + 4y + 2y^2)$$

We should also be able to factor an expression of the type $Ax^2 + Bx + C$. In order to do this, we must find two quantities whose product is $Ax^2 + Bx + C$. It would be desirable to know that two such quantities exist before attempting to find them. We shall now state, without proof, a test for determining whether such factors exist. It is: *The expression $Ax^2 + Bx + C$ can be factored into two factors that do not contain radicals if $B^2 - 4AC$ is a perfect square.*

Example 2. In $2x^2 - 5x - 3$, we have $A = 2$, $B = -5$, and $C = -3$; hence, $B^2 - 4AC = (-5)^2 - 4(2)(-3) = 49 = 7^2$. Therefore, the given expression can be broken into factors without radicals.

In order to find the factors after we know they exist, we make use of (5) of Section 1-7. If the right and left members are interchanged, it becomes

$$acx^2 + (ad + bc)x + bd = (ax + b)(cx + d)$$

Consequently, *the factors of an expression of the type $Ax^2 + Bx + C$ must be such that:*

(1) *the product of the coefficients of x in the factors is the coefficient of x^2,*
(2) *the product of the second terms in the factors is the constant term, and*
(3) *the sum of the cross products* is the middle term of the expression that is to be factored.*

Example 3. Find the factors of $4x^2 + 11x - 3$.

Solution. Since $A = 4$, $B = 11$, and $C = -3$ it follows that $B^2 - 4AC = 11^2 - 4(4)(-3) = 169 = 13^2$ and the expression can be factored. The coefficients of x in the factors can be 4 and 1 or 2 and 2 since $4(1) = 2(2) = 4$; the constant terms can be 3 and -1 or -3 and 1 since the product of each pair is -3. We must now select a combination of these possibilities that gives the correct sum for the cross products. If we try $4x - 3$ and $x + 1$, the sum of the cross products is $4x + (-3x) = x$; hence, we must try another pair of the possibilities. If we use $4x - 1$ and $x + 3$, the sum of the cross products is $12x - x = 11x$; consequently, these are the factors and we write

$$4x^2 + 11x - 3 = (4x - 1)(x + 3)$$

In order to factor with facility, one should be able to recognize and know the factors of a perfect square and of the difference of two squares. They can be symbolized by

$$x^2 + 2xy + y^2 = (x + y)^2$$
$$x^2 - 2xy + y^2 = (x - y)^2$$

and

$$x^2 - y^2 = (x + y)(x - y)$$

Example 4. The expression $4x^2 - 12x + 9$ may be a perfect square since $4x^2 = (2x)^2$ and $9 = 3^2$ are perfect squares; furthermore, the middle term is minus twice the product of the two terms whose squares occur. Therefore, the given expression is a perfect square and we write

$$4x^2 - 12x + 9 = (2x - 3)^2$$

Example 5. The expression $x^2 + 6x + 9$ is a perfect square since the first and third terms are perfect squares and the second term is twice the product of the two whose squares occur. We can now write

$$x^2 + 6x + 9 = (x + 3)^2$$

*The cross products in $(ax + b)(cx + d)$ are adx and bcx.

Example 6. Since $9x^2 = (3x)^2$ and $25 = 5^2$ are perfect squares it follows that $9x^2 - 25$ is the difference of two squares. It can be factored into $3x + 5$ and $3x - 5$; hence

$$9x^2 - 25 = (3x + 5)(3x - 5)$$

Exercise 1-3

Perform the indicated divisions in Problems 1 through 12.

1. $\dfrac{a^7}{a^3}$
2. $\dfrac{b^{12}}{b^5}$
3. $\dfrac{c^{11}}{c^6}$
4. $\dfrac{d^9}{d^7}$
5. $\dfrac{6x^5}{2x^2}$
6. $\dfrac{15x^7}{3x^2}$
7. $\dfrac{12y^8}{2y^6}$
8. $\dfrac{6y^6}{2y^2}$
9. $\dfrac{4x^3 - 12x^2 + 13x - 6}{2x^2 - 3x + 2}$
10. $\dfrac{4x^3 - 21x^2 + 7x - 10}{4x^2 - x + 2}$
11. $\dfrac{6x^3 - 19x^2 - 11x + 14}{3x - 2}$
12. $\dfrac{6x^3 - 13x^2 + 14x - 12}{2x - 3}$

Factor each expression in Problems 13 through 36.

13. $3x - 12$
14. $5x + 15$
15. $7x + 28y$
16. $3x^2 + 6x + 12$
17. $x^2 + 2x - 8$
18. $x^2 + x - 2$
19. $x^2 + 8x + 15$
20. $x^2 + 4x - 12$
21. $2x^2 + x - 1$
22. $3x^2 + x - 2$
23. $6x^2 - 17x + 12$
24. $6x^2 - 11x - 10$
25. $9x^2 - 24x + 16$
26. $25x^2 + 30x + 9$
27. $16x^2 + 40x + 25$
28. $49x^2 - 28x + 4$
29. $x^2 - 16$
30. $4x^2 - 49$
31. $25a^2 - 4b^2$
32. $9b^2 - 16a^2$
33. $6x^3 + 5x^2 + x$
34. $2x^3 + 7x^2 + 3x$
35. $12x^2 + 2x - 4$
36. $4x^2 + 10x + 6$

2

Linear equations in one unknown

2-1 INTRODUCTION

We are all accustomed to stating that two things are equal or hearing other people do so. A statement that two expressions are equal is called an *equation*. The two expressions are called *members* of the equation. The equation is called an *identity* if it is true for all values of the letter or letters in it. We shall be primarily interested in equations that are true for some values of the unknown and not true for others. Such equations are called conditional equations or simply equations. We shall study a type

of conditional equation known as a *linear equation in one unknown*, which is an equation that contains only one unknown letter and it to the first and no other power. Any value of the unknown for which the equation is a true statement is called a *root* or *solution* of the equation.

Example. The statement that $3x + 2 = 5x - 6$ is an equation; furthermore, $x = 4$ is a solution of it since the two members are equal if x is replaced by 4. The truth of this statement is clear since $3(4) + 2 = 14$ and $5(4) - 6 = 14$. Furthermore, $x = 2$ is not a solution since the two members are not equal for $x = 2$ because the left member is $3(2) + 2 = 8$ and the right member is $5(2) - 6 = 4$.

2-2 SOLUTION OF A LINEAR EQUATION

We pointed out in the last section that any value of the unknown which makes the equation a true statement is a solution; we shall now find out how to determine the solution. Two equations are said to be *equivalent* if they have the same solutions. We shall give two axioms concerning equivalent equations which enable us to find the roots.

Axiom 1. *If the same quantity is added to or subtracted from each member of an equation, the new equation is equivalent to the original.*

Example 1. If we add $b - cx$ to each member of

(1) $$ax - b = cx + d,$$

we see that it and

(2) $$ax - cx = b + d$$

are equivalent equations.

An examination and comparison of these two equivalent equations shows that (2) could have been obtained from (1) by moving $-b$ and cx from one member of (1) to the other and changing the sign of each term that is moved to another member. This procedure is justified by use of Axiom 1 and is known as *transposing*.

One step in solving a linear equation consists of collecting coefficients of the unknown on one side and collecting constants on the other side.

This can be done by transposing or by use of Axiom 1. We then have an equation of the form $Ax = B$. To solve this equation, we shall state a second axiom.

Axiom 2. *If each member of an equation is multiplied or divided by the same nonzero constant, the new equation is equivalent to the original.*

Example 2. Solve $5x - 2 = 3x + 4$.

Solution. If we add $2 - 3x$ to each member, we have

$$5x - 2 + 2 - 3x = 3x + 4 + 2 - 3x$$

$$2x = 6 \quad \text{collecting}$$

Now, dividing by 2 as justified by Axiom 2, we see that

$$x = 3$$

is the solution of the equation. This can be checked by replacing x by 3 in the given equation. Thus, the left member is $5(3) - 2 = 13$ and the right member is $3(3) + 4 = 13$; consequently, $x = 3$ checks as the solution.

Example 3. Solve the following equation for x and check.

$$3(x - 1) - 2 = 19 - 2(x + 2)$$

Solution.

$$\begin{aligned} 3(x-1) - 2 &= 19 - 2(x+2) && \text{given equation} \\ 3x - 3 - 2 &= 19 - 2x - 4 && \text{removing parentheses} \\ 3x + 2x &= 19 - 4 + 3 + 2 && \text{transposing} \\ 5x &= 20 && \text{collecting} \\ x &= 4 && \text{dividing by 5} \end{aligned}$$

Now, we must check. Substituting 4 for x in the left member gives $3(4 - 1) - 2 = 7$; furthermore, for $x = 4$, the right member is $19 - 2(4 + 2) = 7$. Consequently, the solution checks.

Exercise 2-1

Solve equations 1 through 20 for x and check.

1. $5x = 15$.

2. $-3x = 12$.

3. $2x = -10$.

4. $-2x = -8$.

5. $3x - 5 = 2x - 3$.

6. $7x - 2 = 5x + 4$.

7. $3x + 5 = 5x + 7$.
8. $4x + 9 = 3x + 7$.
9. $x + 7 = 3x - 3$.
10. $2x + 4 = 3x + 7$.
11. $4x + 11 = 1 - x$.
12. $2 - 5x = -8 - 7x$.
13. $2(x + 1) - 3(2x - 3) = 3$.
14. $5(2x + 7) - (x + 2) = 6$.
15. $2(x - 3) + 3(2x + 4) = 6$.
16. $5(2x + 3) - 4(x + 2) = 1$.
17. $3(2x - 4) - 2(x + 1) = 3(2 - 2x)$.
18. $7(2x + 3) - 5(3x + 2) = 3(1 - 3x)$.
19. $5(x - 4) - 4(x - 6) = 2(x + 2)$.
20. $3(2x - 5) - 2(x - 1) = 3(x - 3)$.
21. Solve $2mx + 3 = 5x$ for x.
22. Solve $5(x - c) = 3a$ for x.
23. Solve $bx + ay = ab$ for y.
24. Solve $3(y + c) = a(y - 2)$ for y.
25. Solve $I = Prt$ for P.
26. Solve $S = P(1 + rt)$ for P.
27. Solve $S = P(1 + i)^n$ for P.
28. Solve $(1 - r)S = a - rl$ for r.

2-3 WORD PROBLEMS

Situations often arise in which we want the value of a quantity and do not have an equation containing the unknown but do have the information that is needed to make up the equation. Under such circumstances, we represent the unknown quantity by a letter, get two expressions for the same thing, equate them and solve the resulting equation. The big stumbling block in this procedure is in getting two expressions for the same thing. This can be done more readily if the reader knows the facts given in the problem before he attempts to use them.

Example 1. The sum of the monthly incomes of a father and son is \$1100. Find each if the son makes \$250 less than the father.

Solution. If we decide to represent the son's income by s, then we must use $s + \$250$ for the father's, since he makes \$250 more than the son. Hence, we are led to the statement that $s + s + \$250$ is the total monthly income. We are told in the statement of the problem that this is \$1100. Therefore, we have two expressions for the same thing and can write the equation

$$\begin{aligned} s + s + \$250 &= \$1100 \\ 2s &= \$\ 850 && \text{collecting and transposing} \\ s &= \$\ 425 && \text{the son's income} \\ s + \$250 &= \$\ 675 && \text{the father's income} \end{aligned}$$

Example 2. How many pounds of coffee that is worth \$.70 per pound should be mixed with 40 pounds of coffee that sells for \$1.05 per pound in order to obtain a blend to sell for \$.90 per pound?

Solution. If we let x represent the number of pounds of \$.70 coffee, then the total number of pounds in the blend is $40 + x$; consequently

$$\begin{aligned}
\$.70x &= \text{the value of the lower-priced coffee} \\
40(\$1.05) &= \text{the value of the } \$1.05 \text{ coffee} \\
(40 + x)\,(\$.90) &= \text{the value of the blend}
\end{aligned}$$

Since the value of the blend and the value of the types of coffee blended are equal, we have

$$\begin{aligned}
.70x + 40(1.05) &= (40 + x)\,(.90) \\
.70x + 42.00 &= 36.00 + .90x \qquad \text{removing parentheses} \\
-.20x &= -6.00 \qquad \text{transposing and collecting} \\
x &= 30
\end{aligned}$$

This result can be checked by comparing the value of the blend and the components of the blend. The value of the blend is $(40 + 30)\,(\$.90) = \$63$ and the value of the components is $(40)\,(\$1.05) + 30(\$.70) = \$42 + \$21 = \$63$.

Example 3. If each side of a rectangle is increased by 2 inches the area is increased by 30 square inches. Find the original dimensions if the length is 1 inch more than the width.

Solution. If we represent the width by w, then $w + 1$ represents the length, since it is 1 inch more than the width. Furthermore, $w(w + 1)$ is the original area and $(w + 2)\,(w + 3)$ is the area after its dimensions are increased by 2. Consequently, since the area was increased by 30,

$$\begin{aligned}
(w + 2)\,(w + 3) - w(w + 1) &= 30 \\
w^2 + 5w + 6 - w^2 - w &= 30 \qquad \text{expanding} \\
4w &= 24 \qquad \text{transposing and collecting}
\end{aligned}$$

Hence, $w = 6$ and $w + 1 = 7$.

Exercise 2-2

1. The sum of two numbers is 19. Find each if they differ by 7.
2. The sum of two numbers is 15. The sum of three times one of them and four times the other is 53. What are the numbers?

3. The difference of two numbers is 9. What are the numbers if one of them is 2 more than twice the other?

4. The difference of two numbers is 4. What are they if three times one of them is 3 more than twice the other?

5. Two brothers made a total of \$160 during a vacation period. How much did each make if one made \$20 less than the other?

6. A dealer sold two cars for a total of \$4900. The prices differed by \$500. What was each price?

7. A man is seven times as old as his son. Find the age of each now if in 5 years the father will be four times as old as the son.

8. A farmer sold two calves and received \$160. How much did each bring if the prices differed by \$10?

9. Find the side of a square if increasing it by 3 increases the area by 75.

10. What is the side of a square if increasing one pair of opposite sides by 2 and the other pair by 4 changes the area by 68?

11. If the width of a rectangle is increased by 2 and the length by 3, the area is increased by 33. Find the dimensions if the length is 6 more than the width.

12. The area of a rectangle is decreased by 26 if the length is decreased by 4 and the width increased by 2. Find the original length and width if the length is 3 more than the width.

13. Tom Smith paid a total of \$73.80 in city, county, and state taxes. The city tax was \$12.80 more than the county and the state tax was one-half as much as the county. How much was each?

14. A family spent \$190 one month for food, rent, and clothes. The rent was \$10 less than the food and the clothes were \$30 less than the rent. Find the amount spent for each.

15. A dealer sold three cars one day for \$8400. The first was \$400 less than the second and the third was \$700 more than the second. What was the price of each?

16. A soda clerk sold 210 drinks for \$16.10. Find the number of each if some were \$.05 and the others were \$.10.

17. How many pounds of coffee worth \$.80 per pound should be mixed with 35 pounds worth \$1.04 per pound to get a mixture that should sell at \$.94 per pound?

18. A farmer has two adjacent parcels of land. One is worth \$120 per acre and the other, \$75. How many acres of the \$75 land should he sell along with 40 acres of the better land if he gets \$93 per acre for the combination?

19. How many pounds of hard candy at \$.30 per pound should be mixed with 15 pounds at \$.42 per pound in order to get a mixture worth \$.35 per pound?

20. There were 84 coins worth \$10.50 in a collection plate that contained only nickels, dimes, and quarters. How many of each were there if the number of quarters was twice the number of dimes?

3

Fractions and fractional equations

3-1 MULTIPLICATION AND REDUCTION OF FRACTIONS

The indicated division of one quantity by another is called a *fraction*. If the fraction is $\frac{a}{b} = a \div b$, then a is called the *numerator* and b, the *denominator* of the fraction; furthermore, the numerator and denominator are called the *members* or *terms* of the fraction.

The *product* of two fractions is a fraction whose numerator is the product of the numerators of the separate fractions and whose denominator is the product of the denominators. This can be put in symbolic form as

(1) $$\left(\frac{a}{b}\right)\left(\frac{c}{d}\right)=\frac{ac}{bd}$$

Example 1. $$\left(\frac{3}{5}\right)\left(\frac{7}{13}\right)=\frac{(3)\,(7)}{(5)\,(13)}=\frac{21}{65}$$

Example 2. $$\left(\frac{x+2}{2x-3}\right)\left(\frac{3x-1}{x+3}\right)=\frac{(x+2)\quad(3x-1)}{(2x-3)\quad(x+3)}$$
$$=\frac{3x^2+5x-2}{2x^2+3x-9}$$

We shall now consider a special case of (1) and, from it, obtain a general principle for use in connection with fractions. If $d = c$ in (1), it becomes

(2) $$\frac{a}{b}\,\frac{c}{c}=\frac{ac}{bc}$$

However, $\frac{c}{c} = 1$ and we have

(3) $$\frac{a}{b}\,\frac{c}{c}=\left(\frac{a}{b}\right)(1)=\frac{a}{b}$$

The left members of (2) and (3) are equal; consequently, the right members are also equal and we have

(4) $$\frac{ac}{bc}=\frac{a}{b}$$

Now, by reading (4) from left to right and from right to left, we see that: *The value of a fraction is not changed if both numerator and denominator are divided or multiplied by the same non-zero quantity.*

This fact enables us to change the form of a fraction by removing any factor that occurs in both the numerator and denominator. If all factors that are common to the numerator and denominator of a fraction are removed, we say that the fraction has been reduced to *lowest terms*. The work of reducing a fraction to lowest terms is most readily performed if both numerator and denominator are factored since factors that occur in both members then stand out.

Example 3. $$\frac{15}{24}=\frac{(3)\,(5)}{(8)\,(3)}\qquad\text{factoring}$$
$$=\frac{5}{8}\qquad\text{removing the common factor}$$

Example 4. $$\frac{3x^2 + 11x - 4}{2x^2 + 11x + 12} = \frac{(3x-1)(x+4)}{(2x+3)(x+4)} \quad \text{factoring}$$
$$= \frac{3x-1}{2x+3} \quad \text{removing the common factor}$$

3-2 DIVISION OF FRACTIONS

We shall now derive a procedure for dividing one fraction by another. We have

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{\frac{a}{b}\frac{d}{c}}{\frac{c}{d}\frac{d}{c}} \quad \text{multiplying numerator and denominator by } \frac{d}{c}$$
$$= \frac{\frac{a}{b}\frac{d}{c}}{1} \quad \text{since } \frac{c}{d}\frac{d}{c} = 1$$
$$= \frac{a}{b}\frac{d}{c}$$

Consequently, we can now say that: *To divide one fraction by another, invert the one by which we are dividing and multiply.*

Example 1. If we invert the denominator and multiply, we have
$$\frac{5}{7} \div \frac{3}{2} = \left(\frac{5}{7}\right)\left(\frac{2}{3}\right) = \frac{10}{21}$$

Example 2. $$\frac{10}{21} \div \frac{2}{7} = \left(\frac{10}{21}\right)\left(\frac{7}{2}\right) \quad \text{inverting and multiplying}$$
$$= \frac{(5)}{(7)}\frac{(2)}{(3)}\left(\frac{7}{2}\right) \quad \text{factoring}$$
$$= \frac{5}{3} \quad \text{removing common factors}$$

Example 3. $$\frac{2x^2 - x - 3}{3x^2 - 4x + 1} \div \frac{2x-3}{x-1} = \frac{2x^2 - x - 3}{3x^2 - 4x + 1} \cdot \frac{x-1}{2x-3} \quad \text{inverting}$$
$$= \frac{(2x-3)(x+1)}{(3x-1)(x-1)} \cdot \frac{x-1}{2x-3} \quad \text{factoring}$$
$$= \frac{x+1}{3x-1} \quad \text{removing common factors}$$

Exercise 3-1

Perform the indicated operations in Problems 1 through 16.

1. $\left(\frac{2}{3}\right)\left(\frac{5}{9}\right)$
2. $\left(\frac{3}{7}\right)\left(\frac{5}{2}\right)$
3. $\left(\frac{6}{11}\right)\left(\frac{5}{7}\right)$
4. $\left(\frac{5}{8}\right)\left(\frac{3}{7}\right)$
5. $\frac{3}{4} \div \frac{2}{5}$
6. $\frac{6}{11} \div \frac{7}{5}$
7. $\frac{4}{9} \div \frac{5}{2}$
8. $\frac{3}{5} \div \frac{5}{2}$
9. $\left(\frac{x}{x+1}\right)\left(\frac{2x+5}{3x-2}\right)$
10. $\left(\frac{5x+3}{x-2}\right)\left(\frac{2x-1}{3x-5}\right)$
11. $\left(\frac{2x-1}{x-2}\right)\left(\frac{3x+4}{4x-3}\right)$
12. $\left(\frac{3x-1}{x+2}\right)\left(\frac{5x+2}{2x+3}\right)$
13. $\frac{x-1}{2x-5} \div \frac{5x+2}{x+3}$
14. $\frac{2x-7}{x+4} \div \frac{5x-3}{x+2}$
15. $\frac{x-1}{x+1} \div \frac{2x-1}{2x+1}$
16. $\frac{3x-4}{4x+3} \div \frac{3x+4}{4x-3}$

Reduce each fraction in Problems 17 through 24 to lowest terms.

17. $\frac{6}{15}$
18. $\frac{16}{20}$
19. $\frac{15}{25}$
20. $\frac{12}{18}$
21. $\frac{2x^2-5x+3}{2x^2+x-6}$
22. $\frac{6x^2+x-1}{6x^2+7x-3}$
23. $\frac{6x^2+5x-6}{6x^2-13x+6}$
24. $\frac{15x^2+26x+8}{15x^2+14x-8}$

Perform the indicated operations and reduce to lowest terms in each of Problems 25 through 40.

25. $\left(\frac{3}{4}\right)\left(\frac{2}{12}\right)$
26. $\left(\frac{2}{21}\right)\left(\frac{3}{8}\right)$
27. $\left(\frac{6}{35}\right)\left(\frac{10}{9}\right)$
28. $\left(\frac{21}{6}\right)\left(\frac{4}{14}\right)$
29. $\frac{3}{7} \div \frac{6}{21}$
30. $\frac{5}{9} \div \frac{10}{27}$
31. $\frac{30}{7} \div \frac{18}{14}$
32. $\frac{35}{12} \div \frac{10}{6}$
33. $\frac{2x^2+x-1}{x^2+3x+2} \cdot \frac{x+2}{2x-1}$
34. $\frac{3x^2+11x+6}{x^2-x-12} \cdot \frac{x-4}{x+3}$
35. $\frac{6x^2+x-2}{3x^2-x-2} \cdot \frac{x-1}{2x+1}$
36. $\frac{6x^2+11x+3}{6x^2+7x-3} \cdot \frac{3x-1}{3x+2}$
37. $\frac{2x^2+x-1}{x^2-x-2} \div \frac{x+2}{2x-1}$
38. $\frac{x^2-x-12}{2x^2+9x+9} \div \frac{x-4}{3x+2}$
39. $\frac{3x^2-x-2}{6x^2+7x+2} \div \frac{x-1}{2x-1}$
40. $\frac{6x^2+7x-3}{9x^2+3x-2} \div \frac{2x+3}{3x+1}$

3-3 LOWEST COMMON MULTIPLE

Any expression that has each of several others as a factor is called a *multiple* of them. If the expressions are arithmetic, the smallest number that contains each of them as a factor is called their *least common multiple.* If the expressions are algebraic, or algebraic and arithmetic, the expression of lowest degree that has each of them as a factor is called their *lowest common multiple.* Lowest common multiple is abbreviated as L.C.M.

Example 1. It is clear that 24 is a multiple of 4 and 6 since $(4)(6) = 24$; however, it is not their least common multiple since 4 and 6 are both factors of 12.

Example 2. Since $(x - 1)(2x + 3) = 2x^2 + x - 3$ it follows that $2x^2 + x - 3$ is a common multiple of $x - 1$ and $2x + 3$; furthermore, it is their lowest common multiple since no expression of lower degree has both $x - 1$ and $2x + 3$ as factors.

In order to get the lowest common multiple of several expressions, we should form the product of all factors that occur in any expression and use each factor to the highest power that it enters in any expression. This can be done most readily if each expression is factored.

Example 3. If the expressions in factored form are $(x + 2)(2x - 1)^3$, $(x + 2)^2(3x - 4)$, and $(2x - 1)(3x - 2)$, then the lowest common multiple is

$$(x + 2)^2 (2x - 1)^3 (3x - 4)(3x - 2)$$

since that is the product of all factors that are in any one of the expressions and each factor enters to the highest power that it occurs in any expression.

3-4 ADDITION AND SUBTRACTION OF FRACTIONS

To add or subtract two fractions with the same denominator, we add or subtract the numerators and put the result over their common denominator.

Example 1. $$\frac{5}{11} + \frac{2}{11} = \frac{5 + 2}{11} = \frac{7}{11}$$

Example 2. $$\frac{3a}{a - b} - \frac{2a - b}{a - b} = \frac{3a - (2a - b)}{a - b} = \frac{a + b}{a - b}$$

To add or subtract two fractions that have different denominators, we first change each of them to an equivalent fraction which has a denominator that is a common multiple of the denominators. This can be done by multiplying numerator and denominator of each fraction by the factor or factors in the common multiple which do not appear in the denominator of that fraction. We then have fractions with the same denominator and proceed accordingly.

Example 3. Perform the indicated operations.

$$\frac{2x-1}{(x+2)(2x+1)} - \frac{3x+4}{(x+2)(x-1)} + \frac{4x+5}{(x-1)(2x+1)}$$

Solution. The common denominator is $(x+2)(2x+1)(x-1)$; hence, the first denominator must be multiplied by $x-1$ to obtain the common denominator, the second must be multiplied by $2x+1$ and the third by $x+2$; consequently each numerator must be multiplied by the same factor. Thus, we get

$$\begin{aligned}
&\frac{2x-1}{(x+2)(2x+1)} - \frac{3x+4}{(x+2)(x-1)} + \frac{4x+5}{(x-1)(2x+1)} \\
&= \frac{2x-1}{(x+2)(2x+1)}\,\frac{x-1}{x-1} - \frac{3x+4}{(x+2)(x-1)}\,\frac{2x+1}{2x+1} \\
&\quad + \frac{4x+5}{(x-1)(2x+1)}\,\frac{x+2}{x+2} \\
&= \frac{(2x-1)(x-1) - (3x+4)(2x+1) + (4x+5)(x+2)}{(x+2)(2x+1)(x-1)} \\
&= \frac{2x^2 - 3x + 1 - (6x^2 + 11x + 4) + 4x^2 + 13x + 10}{(x+2)(2x+1)(x-1)} \\
&= \frac{-x+7}{(x+2)(2x+1)(x-1)}
\end{aligned}$$

Exercise 3-2

Find the L.C.M. of the expressions given in each of Problems 1 through 16.

1. 4, 6, 8
2. 6, 10, 15
3. 13, 39, 12
4. 15, 21, 35
5. $2x^2y$, $3xy^2$
6. $3a^2b^3$, $5a^3b$

7. $3ab^2c^3,\ 6a^2bc$

8. $2x^2y^0,\ 8xy^3$

9. $(2x - 1)(3x - 4)^2,\ (2x + 1)(3x - 4)$

10. $(x + 2)(x - 2)^2,\ (x + 2)^2(x - 2)$

11. $(2x + 1)^3(x - 3),\ (x - 3)^2(2x + 3),\ (2x + 3)^2(2x + 1)$

12. $(x + 4)^2(x - 2)^4,\ (x - 2)(2x + 5),\ (x + 4)^3(2x + 5)^2$

13. $4x^2 - 4x + 1,\ 2x^2 - 5x + 2,\ x^2 - 2x$

14. $9x^2 + 12x + 4,\ 9x^2 - 4,\ 3x^2 + 2x$

15. $x^2 - 2x + 1,\ x^2 + 2x - 3,\ x^2 + 6x + 9$

16. $4x^2 + 12x + 9,\ 4x^2 - 4x + 1,\ 4x^2 + 4x - 3$

Perform the indicated operations in each of Problems 17 through 36.

17. $\frac{1}{2} + \frac{1}{3} - \frac{2}{9}$

18. $\frac{2}{3} - \frac{3}{5} + \frac{7}{15}$

19. $\frac{2}{7} - \frac{1}{3} + \frac{5}{21}$

20. $\frac{2}{5} - \frac{3}{7} + \frac{1}{3}$

21. $\frac{2}{a} + \frac{3}{b} - \frac{1}{ab}$

22. $\frac{3}{x} - \frac{2}{y} + \frac{5}{xy}$

23. $\frac{4}{x} - \frac{3}{y} - \frac{2}{x^2y}$

24. $\frac{5}{a} - \frac{2}{b} + \frac{3}{ab^2}$

25. $\frac{2}{x - 1} + \frac{3}{x - 2} + \frac{7}{x^2 - 3x + 2}$

26. $\frac{3}{2x + 1} - \frac{2}{x + 3} + \frac{x}{2x^2 + 7x + 3}$

27. $\frac{1}{x + 2} - \frac{2}{2x + 1} + \frac{3}{2x^2 + 5x + 2}$

28. $\frac{3}{2x + 5} - \frac{1}{x + 4} - \frac{3}{2x^2 + 13x + 20}$

29. $\frac{x + 4}{2x - 1} - \frac{x - 3}{x + 2} + \frac{x^2 - 11x - 1}{2x^2 + 3x - 2}$

30. $\frac{2x + 5}{3x + 2} - \frac{x - 1}{x + 1} + \frac{x^2 - 7x - 6}{3x^2 + 5x + 2}$

31. $\frac{3x - 1}{5x + 3} - \frac{x + 2}{x - 2} + \frac{20x + 12}{5x^2 - 7x - 6}$

32. $\frac{2x - 7}{7x + 2} - \frac{x + 3}{x - 1} + \frac{5x^2 + 34x - 3}{7x^2 - 5x - 2}$

33. $\frac{3}{(5x + 1)^2} - \frac{2}{5x + 1} + \frac{2}{5x - 2}$

34. $\frac{3}{(2x + 1)^2} - \frac{1}{2x + 1} + \frac{1}{2x + 2}$

35. $\frac{1}{(x - 1)^2} + \frac{1}{x - 1} - \frac{1}{x + 2}$

36. $\frac{5}{(2x + 1)^2} + \frac{1}{2x + 1} - \frac{1}{2x - 2}$

3-5 FRACTIONAL EQUATIONS

If an equation contains fractions, it is called a *fractional equation*. The first step in solving such an equation is to eliminate the fractions by multiplying both members of the equation by the L.C.M. of the denominators of the fractions. If these denominators include the unknown, there may be solutions of the equation that is obtained by eliminating the fractions that are not solutions of the given equation. Such numbers are called *extraneous roots* but, in reality, they are not roots. Because of the possibility of introducing extraneous roots if we multiply through by an expression that contains the unknown, we must check each possible solution by substituting it in the original equation.

Example. Solve the following equation for x.

$$1 - \frac{x+1}{x-3} = \frac{4}{x-1}$$

Solution. Multiplying both members by the L.C.M. $(x-3)(x-1)$ of the denominators, we have

$$\begin{aligned}(x-1)(x-3) - (x+1)(x-1) &= 4(x-3) \\ x^2 - 4x + 3 - x^2 + 1 &= 4x - 12 \quad \text{expanding} \\ -8x &= -16 \quad \text{collecting} \\ x &= 2 \quad \text{dividing through by } -8\end{aligned}$$

Therefore, 2 is the only possible root of the given equation. We shall substitute it for x to see if it is a root. Thus, the left member becomes

$$1 - \frac{2+1}{2-3} = 1 - \frac{3}{-1} = 4$$

and the right member is

$$\frac{4}{2-1} = 4$$

Consequently, the two members are equal and 2 is a root.

3-6 WORD PROBLEMS

Many stated problems lead to fractional equations. The equations are formed as in Section 2-3 and solved as in Section 3-5.

Example. The numerator of a fraction is 3 less than the denominator. If the numerator is increased by 5 and the denominator decreased by 4, the new number is 4. Find the original fraction.

Solution. If we represent the original denominator by x, then we must use $x - 3$ for the numerator since it is 3 less than the denominator; consequently

$$\frac{x-3}{x} \quad \text{is the original fraction}$$

Therefore $$\frac{x-3+5}{x-4} = \frac{x+2}{x-4} \quad \text{is the new fraction}$$

Hence $$\frac{x+2}{x-4} = 4 \quad \text{since the new number is 4}$$

Now, clearing of fractions, we have

$$\begin{aligned} x + 2 &= 4(x - 4) \\ &= 4x - 16 \quad \text{removing parentheses} \\ -3x &= -18 \quad \text{collecting} \\ x &= 6 \end{aligned}$$

Therefore, the fraction is $\frac{3}{6}\cdot$ This is readily checked by adding 5 to the numerator and subtracting 4 from the denominator and obtaining 4 as stated in the problem.

Exercise 3-3

Solve the following equations for x.

1. $\dfrac{2x+5}{3} - \dfrac{x+2}{2} = 1$.
2. $\dfrac{2x-1}{5} + \dfrac{x+3}{6} = 2$.
3. $\dfrac{x-1}{3} + \dfrac{x+2}{2} = 4$.
4. $\dfrac{2x+1}{3} + \dfrac{2-x}{2} = 1$.
5. $\dfrac{x+1}{x-2} - \dfrac{3x-1}{3x-4} = 0$.
6. $\dfrac{2x+3}{3x-5} = \dfrac{6x+2}{9x-16}$.
7. $\dfrac{x+1}{2x-2} = \dfrac{2x-1}{4x-7}$.
8. $\dfrac{3x+1}{2x-2} = \dfrac{3x-5}{2x-5}$.
9. $\dfrac{2x+3}{2x-3} - \dfrac{2x+4}{x-1} = -1$.
10. $\dfrac{3x-5}{x-1} - \dfrac{2x-5}{x-2} = 1$.

11. $\dfrac{5x+2}{x} - \dfrac{3x+7}{x+1} = 2\,.$

12. $\dfrac{7x+13}{x+1} - \dfrac{3x+3}{x+3} = 4\,.$

13. $\dfrac{3}{2x-1} - \dfrac{2}{x+1} = \dfrac{2}{(2x-1)(x+1)}\,.$

14. $\dfrac{5}{x+3} + \dfrac{4}{x+2} = \dfrac{20}{(x+3)(x+2)}\,.$

15. $\dfrac{3}{2x+3} + \dfrac{2}{x+4} = \dfrac{4}{(2x+3)(x+4)}$

16. $\dfrac{2}{x-3} + \dfrac{10}{2x-5} = \dfrac{30}{(x-3)(2x-5)}\,.$

17. Solve $l = a + (n-1)d$ for d and for n.

18. Solve $S = \dfrac{a - rl}{1 - r}$ for l and for r.

19. Solve $S = P(1 + rt)$ for t and for r.

20. Solve $S = P(1 - dt)$ for t and for d.

21. Mr. Jones left an estate of \$18,000 that was divided between his wife and three children. The wife received $\frac{1}{3}$ of the total, the younger son $\frac{1}{3}$ of the amount left after his mother got her share, and the daughter \$1000 more than her older brother. How much did each get?

22. Tom has $\frac{3}{4}$ as many marbles as Robert. After Robert found 4 and Tom lost 3, Tom had $\frac{1}{2}$ as many as Robert. How many did each have at first?

23. The numerator of a fraction is 1 less than the denominator. If 5 is added to the numerator and 2 to the denominator, the new fraction is 1.5. Find the value of the original fraction.

24. The denominator of a fraction is 1 more than the numerator. If 10 is added to the numerator and 3 to the denominator, the new fraction is 2. Find the original fraction.

25. A man can do a paint job in 6 days and his son in 8 days. How long will it take if both work on the job?

26. A grocer thought he had enough bread to last 6 hours but it lasted only 4 hours since he sold 8 more loaves per hour than he had expected to. How many loaves did he have?

27. Mr. Smith sometimes walks to his office at 4 miles per hour and at other times rides at 20 miles per hour. How far is it to his office if he gets there in 24 minutes less time by riding than by walking?

28. A student rode from the campus to his home in a truck at 30 miles per hour and returned in a car at 45 miles per hour. How far does he live from the campus if one trip required 1.5 hours more than the other?

4

Percentage

4-1 PER CENT

The term *per cent* comes from the Latin phrase *per centum* and means "by the hundred." The symbol % is used to indicate per cent. Thus, 7% is seven hundredths and, of course, may be written as .07. Since the number of hundredths in a decimal number is obtained by moving the decimal point two places to the right it follows that to find the number of per cent in a number we move the decimal two places to the right also. Consequently, to change a decimal number to per cent, we move the decimal

point two places to the right and affix the per cent symbol; furthermore, to change from per cent to a decimal number we drop the % sign and move the decimal point two places to the left.

Example 1. $.376 = 37.6\%$.

Example 2. $.0396 = 3.96\%$.

Example 3. $4.2\% = .042$.

Example 4. $259\% = 2.59$.

A common fraction can be changed to per cent by first changing to a decimal fraction and then to a per cent.

Example 5. $\frac{3}{4} = .75 = 75\%$.

Example 6. $\frac{5}{7} = .7143 = 71.43\%$.

4-2 BASE, RATE, PERCENTAGE

There are three quantities involved in many situations in which per cent is used. The number of which a per cent is to be taken is called the *base*. The per cent of the base that is to be taken is called the *rate*. The result obtained by multiplying the base times the rate is called the *percentage*.

The relation

$$\textbf{base} \times \textbf{rate} = \textbf{percentage} \tag{1}$$

is used in many problems and should be learned. It should be considered as an equation in the three quantities, base, rate, and percentage. If any two of them are known, the equation can be solved for the third one.

Example 1. Find 23.4% of \$1680.

Solution. If we represent 23.4% of \$1680 by P, we have

$$\begin{aligned} P &= 23.4\% \times \$1680 \\ &= (.234)\ \$1680 \\ &= \$393.12 \end{aligned}$$

Example 2. 16.2 is what per cent of 497.04?

Solution. If we represent the rate by r and use (1), we get

$$497.04\,r = 16.2$$

since the base is 497.04 and the percentage or part of the base is 16.2. Now, solving for r by dividing through by 497.04 gives

$$r = \frac{16.2}{497.04}$$
$$= .0326$$

Therefore, moving the decimal two places to the right, we see that $r =$ 3.26%.

Example 3. Find a number such that 23.6% of it is 17.2.

Solution. The base is to be determined since we want a number and know a part of it. Use of (1) with the base represented by B gives

$$.236\text{B} = 17.2$$

since 23.6% = .236. Now dividing by .236 leads to

$$B = 72.9$$

Exercise 4-1

In the following problems, express each of the numbers as a per cent and each per cent as a decimal.

1. .235	**2.** .3025	**3.** .571	**4.** .8762
5. .025	**6.** 3.052	**7.** 32.714	**8.** .0027
9. 2.3%	**10.** 5.83%	**11.** 62.3%	**12.** 41.5%
13. 38.6%	**14.** .1%	**15.** .63%	**16.** .025%

Find the base, rate, or percentage that is missing in each of Problems 17 through 28.

17. base = 327, rate = .26.

18. base = 27.3, rate = .76.

19. base = 2.98, rate = 3.7%.

20. base = 929, rate 43.8%.

21. base = 32.9, percentage = 16.2.

22. base = 2.38, percentage = 1.47.

23. base = 863, percentage = 905.

24. base = 7.64, percentage = 8.41.

25. rate = 62%, percentage = 5.3.

26. rate = 54%, percentage = .49.

27. rate = .713, percentage = 71.3.

28. rate = .7%, percentage = 6.3.

29. If 37.5% of a class made "C" on an examination, how many made that grade if there were 32 people in the class?

30. A farmer's income in 1959 was 106% of that in 1958. What was his income in 1959 if he took in \$5728.50 in 1958?

31. The marked price of a suit was \$65 but a discount of 20% was allowed. What was the cost?

32. A company asked \$2732 for a car but allowed a discount of 18.3% for cash and no trade-in. What did the car cost?

33. A teacher's salary increased 8% and he then got \$3456. What had been his salary before the increase?

34. The owner of a pickle factory expected to put up a building in 1956 but decided to wait until 1959. The price of building went up 11% during that time and the building cost \$68,200. What would it have cost in 1956?

35. If a dealer sells a refrigerator for \$308 and makes a profit of 40% on what it cost him, find his cost.

36. Smith made a profit of 32% on his cost by selling a lot for \$4752. What did it cost him?

4-3 INCOME AND PROPERTY TAXES

The United States and some state governments levy a tax on all income beyond a specified sum. The rate of the tax depends on the amount of taxable income and increases as the income does. In many states, the state income tax is paid on the taxable income that remains after the federal tax is subtracted.

Example 1. The federal income tax on a taxable income between \$2000 and \$4000 is \$400 plus 22% of the excess over \$2000 for a single person, and 20% of the entire taxable income for a married person filing a joint return. Find the federal tax that must be paid by a person with a taxable income of \$3673.84 if he is single and if he is married and files a joint return. Assume that the state tax is 3% of the sum left after subtracting the federal tax from the taxable income.

Solution. For a single taxpayer, the federal income tax is

$$\begin{aligned}\$400 + .22\,(\$3673.84 - \$2000) &= \$400 + .22\,(\$1673.84)\\ &= \$768.24\end{aligned}$$

The state income tax is 3% of (\$3673.84 − \$768.24); hence, is \$87.17.
For a married taxpayer, the federal income tax is

$$.20(\$3673.84) = \$734.77$$

Hence, the state income tax is 3% of (\$3673.84 − 734.77), or \$88.17.

There is a tax on real estate and many other types of property. The amount of the tax is determined by the assessed value of the property and the tax rate. The *assessed value* is a fixed per cent of a reasonable sale price. This fixed per cent may vary from one political subdivision to another but is the same for all property in any one subdivision. The unit of tax is called the *mil* and it is one-thousandth of the assessed value.

Example 2. Find the tax on a \$17,500 house if it is assessed at 35% of its value and if the rate is 37.3 mils.

Solution. The assessed value is .35(\$17,500) = \$6125 and the tax is .0373 of that amount since the rate is 37.3 mils. The tax is, therefore

$$.0373(\$6125) = \$228.46$$

Exercise 4-2

The following is an excerpt from a tax computation table printed by the United States government for use in determining the income tax for a recent year.

TAXABLE INCOME		TAX COMPUTATION
Over	*Not over*	*Single person*
\$ 2,000	\$ 4,000	\$400 plus 22% of the excess over \$2,000
\$ 4,000	\$ 6,000	\$840 plus 26% of the excess over \$4,000
\$ 6,000	\$ 8,000	\$1,360 plus 30% of the excess over \$6,000
\$ 8,000	\$ 10,000	\$1,960 plus 34% of the excess over \$8,000
\$ 10,000	\$ 12,000	\$2,640 plus 38% of the excess over \$10,000
\$150,000	\$200,000	\$111,820 plus 90% of the excess over \$150,000

Find the federal income tax that must be paid by a single person on the given taxable income in keeping with the table printed above. Also find the state income tax if one is involved and if it is at the stated rate of the difference between taxable income and federal income tax.

1. \$3825.74
2. \$2742.18
3. \$5205.19
4. \$5838.78
5. \$7344.62
6. \$6677.88
7. \$10,768.07
8. \$178,289.39
9. \$5934.43, state tax of 3%.
10. \$7799.26, state tax of 3.6%
11. \$9636.00, state tax of 4.1%.
12. \$8302.12, state tax of 3.7%.

Find the state, county, and municipal tax in each of Problems 13 through 20 if the value of the property, the per cent of the value that is used as assessed value, and tax rates are as given. The state rate is given first, followed by the county and then the municipal.

13. \$9800, 35%, 5 mils, 18.7 mils, 4.2 mils.

14. \$11,500, 33%, 4.7 mils, 19.8 mils, 3.5 mils.

15. \$37,800, 40%, 3.9 mils, 26.3 mils, 4.6 mils.

16. \$13,750, 45%, 2.7 mils, 30.6 mils, 3.4 mils.

17. \$9,200, 37.5%, 3.4 mils. 27.3 mils, 4.1 mils.

18. \$7,500, 42.5%, 2.9 mils, 28.4 mils. 3.2 mils.

19. \$26,300, 32.5%, 3.5 mils, 20.6 mils, 4.7 mils.

20. \$8,750, 47.5%, 2.8 mils, 29.3 mils, 4.5 mils.

What is the tax rate in mils if the assessed value and taxes are as stated in Problems 21 through 24?

21. Assessed value, \$4,500; taxes, \$139.50.

22. Assessed value, \$7,600; taxes, \$205.20.

23. Assessed value, \$3,800; taxes, \$119.32.

24. Assessed value, \$17,400; taxes, \$563.76.

What is the assessed value if the rate and amount of taxes are as given in Problems 25 through 28?

25. Rate, 27 mils; taxes, \$194.40.

26. Rate, 31 mils; taxes, \$418.50.

27. Rate, 29 mils; taxes, \$95.70.

28. Rate, 32 mils; taxes, \$1331.20.

How much tax must one pay under the conditions given in Problems 29 through 32? Use the table at beginning of this exercise to determine the income tax and use the assessed value and rate to find the property tax.

	Taxable income	*Assessed value*	*Rate*
29.	\$ 7248	\$4950	23.7 mils
30.	\$ 2359	\$2550	31.2 mils
31.	\$ 5934	\$6430	28.5 mils
32.	\$11,543	\$7562	30.6 mils

4-4 TRADE DISCOUNT

Many dealers find it advantageous to publish a catalogue in which they give the price of each article they have for sale. The price given in the catalogue is called the *list price* and it is often higher than the price the

dealer expects to receive. In order to determine the price that is to be paid, the buyer is allowed one or more discounts that are called *trade discounts*. Each trade discount is expressed as a per cent. The first one is based on the list price and the price after the first discount is called the *first net price*. The second and later trade discounts are based on the price that remains after all earlier discounts are subtracted. If there is more than one discount, they are called a *discount series* or *chain of discounts*. The price to be paid after all discounts is called the *final net price*.

Example. If the list price of an article is \$120, find the net price after discounts of $16\frac{2}{3}\%$, 10%, and 12.5%. What is the total trade discount?

Solution.

List price = \$120
First discount = $(16\frac{2}{3}\%)$ (\$120) = \$20
First net price = \$120 − \$20 = \$100
Second discount = (10%) (\$100) = \$10
Second net price = \$100 − \$10 = \$90
Third discount = (12.5%) (\$90) = \$11.25
Final net price = \$90 − \$11.25 = \$78.75

The total trade discount is D_t = \$120 − \$78.75 = \$41.25 and could have been obtained by adding the three discounts.

4-5 SINGLE DISCOUNT EQUIVALENT TO A DISCOUNT SERIES

We shall now see how to find a single discount that is equivalent to a chain of discounts; furthermore, we shall see that the order in which the discounts of a series are taken is immaterial. In order to find the single discount that is equivalent to a chain of discounts, we shall let

L = the list price
r_1, r_2, r_3, r_4 = the 1st, 2nd, 3rd, 4th discounts
N_1, N_2, N_3, N_4 = the 1st, 2nd, 3rd, 4th net prices

Then

$$N_1 = L - Lr_1 = L(1 - r_1)$$
$$N_2 = N_1 - N_1r_2 = N_1(1 - r_2) = L(1 - r_1)(1 - r_2)$$
$$N_3 = N_2 - N_2r_3 = N_2(1 - r_3) = L(1 - r_1)(1 - r_2)(1 - r_3)$$
$$N_4 = N_3 - N_3r_4 = N_3(1 - r_4) = L(1 - r_1)(1 - r_2)(1 - r_3)(1 - r_4)$$

If there are only four discounts in the series, the single discount that is equivalent to them can be obtained by subtracting N_4 from L and then dividing this difference by L. This is a practical procedure but we shall proceed differently in case there are n discounts. If we continue the procedure followed above, we find that the net price after n discounts is

$$N = L(1 - r_1)(1 - r_2)(1 - r_3)\dots(1 - r_n)$$

Furthermore, if r is the equivalent single discount rate then

$$N = L(1 - r)$$

Consequently, equating these two expressions for N, taking out the common factor L, and solving for r, we see that

$$\textbf{(1)} \qquad \boldsymbol{r = 1 - (1 - r_1)(1 - r_2)(1 - r_3)\dots(1 - r_n)}$$

Since the product of the factors that are subtracted from 1 is the same regardless of the order in which they are taken, it follows that the order in which the discounts of a series are taken is immaterial.

Example 1. By use of (1), find the single discount rate that is equivalent to the series $16\frac{2}{3}\%$, 10%, and $12\frac{1}{2}\%$.

Solution. Using the given data, we have $r_1 = 16\frac{2}{3}\% = \frac{1}{6}$, $r_2 = 10\% = \frac{1}{10}$ and $r_3 = 12\frac{1}{2}\% = \frac{1}{8}$.
Consequently

$$\begin{aligned} r &= 1 - (1 - \tfrac{1}{6})(1 - \tfrac{1}{10})(1 - \tfrac{1}{8}) \\ &= 1 - \tfrac{5}{6}\,\tfrac{9}{10}\,\tfrac{7}{8} = 1 - \tfrac{21}{32} = \tfrac{11}{32} = 34.4\% \end{aligned}$$

Example 2. Find the trade discount and net price on a refrigerator if it is listed at \$400 and discounts of 20% and 6.25% are allowed.

Solution. In order to find the discount, we shall first find the single rate by use of (1). Thus, since $r_1 = 20\% = .20$ and $r_2 = 6.25\% = .0625$, we have

$$\begin{aligned} r &= 1 - (1 - .20)(1 - .0625) \\ &= 1 - (.80)(.9375) = 1 - .75 = 25\% \end{aligned}$$

Hence, the discount is

$$D_t = \$400(.25) = \$100$$

and the net price is

$$N = \$400 - \$100 = \$300$$

Example 3. What was the list price of an article that sold for \$162 after discounts of 10% and 10%?

Solution. The single discount that is equivalent to the given discount series is

$$r = 1 - (1 - .1)(1 - .1) = 1 - (.9)(.9) = 19\%$$

If we put $r = .19$ and $N = \$162$ in the relation

$$N = L(1 - r)$$

it becomes

$$\$162 = L(1 - .19) = .81L$$

Hence, solving for L gives

$$L = \frac{\$162}{.81} = \$200$$

as the list price.

4-6 CASH DISCOUNT

In presenting his bill or invoice the manufacturer or distributor gives the list price and the trade discounts that are to be deducted and figures the net price. Along with the bill, he tells when it must be paid. To encourage prompt payment, the seller sometimes offers an additional discount if the bill is paid in cash and a smaller one if paid within a certain period of time. A discount of this type is called a *cash discount*. The cash discount and the time the bill is to be paid are called the *terms*. If the terms are 4, $2/15$, $n/30$, a discount of 4% is allowed if the bill is paid at once, a discount of 2% is allowed if the bill is paid within 15 days, and the net price must be paid in 30 days.

Example. The invoice for a bill of goods shows a list price of \$800 with trade discounts of 12.5% and 8%. The terms are 5, $3/20$, $n/30$. Find the cost if the buyer pays (a) immediately, (b) in 19 days, (c) in 30 days.

Solution.

List price = \$800
First discount = (\$800) (.125) = \$100
First net price = \$800 − \$100 = \$700
Second discount = \$700 (.08) = \$56
Final net price = \$700 − \$56 = \$644

We shall use this and the terms as the basis for determining the cost.

(a) If the buyer pays cash, the discount is \$644(.05) = \$32.20; hence, the cost is \$611.80.

(b) If the buyer pays in 19 days, he gets a cash discount of 3%; hence, he must pay 97% of the final net price. Therefore, the cost is \$644(.97) = \$624.68.

(c) If the payment is made in 30 days, the net price of \$644 must be paid.

Exercise 4-3

Find the final net price in each of Problems 1 through 4 if the list price and discount series are as given. Do not use (1) of Section 4-5.

1. List price, \$150; discount of 20%, 15%, 8%.

2. List price, \$260; discount of 10%, 10%, 10%.

3. List price, \$720; discounts of 12.5%, 10%, 7%.

4. List price \$500; discounts of 8%, 10%, 6%.

Find the single discount that is equivalent to the series in each of Problems 5 through 8 by use of (1) of Section 4-5.

5. 20%, 10%, 12.5%.

6. $33\frac{1}{3}$%, 50%, 50%.

7. 25%, $16\frac{2}{3}$%, 25%.

8. 10%, 10%, 6%.

Use (1) of Section 4-5 to find the final net price if the list price and discount series are as given in Problems 9 through 12.

9. List price, \$200; discounts of 25%, 10%, 6%.

10. List price, \$300; discounts of $16\frac{2}{3}$%, 20%, 7%.

11. List price, \$468; discounts of $11\frac{1}{9}$%, 25%, 9%.

12. List price, \$540; discounts of 15%, 10%, 6%.

What was the list price if the final net price and discount series are as given in Problems 13 through 16?

13. Final net price, \$174.60; discounts of 10% and 3%.

14. Final net price, \$144.90; discounts of 10% and 8%.

15. Final net price, \$504.99; discounts of 25% and 7%.

16. Final net price, \$1065.96; discounts of 10%, $33\frac{1}{3}$% and 6%.

17. The net price of a bill of goods is \$652 and the terms are 6, 4/20, *n*/30. What is the cost (a) if cash is paid, (b) if the bill is paid in 3 days, (c) if the bill is paid in 28 days?

18. The net price of a purchase is \$287 and the terms are 5, 3/15, $n/25$. What is the cost if the bill is paid (a) immediately, (b) in 3 days, (c) in 20 days?

19. What must be paid in 10 days to pay off a bill of \$238.74 if the terms are 4, 2/12, $n/15$?

20. What is the cost of paying off a bill of \$829.37 in 8 days if the terms are 5, 3/10, $n/15$?

Find the cost 9 days after delivery if the list price, trade discounts, and terms are as given in Problems 21 through 24.

	List price	*Discounts*	*Terms*
21.	\$ 750	20%, 7%	5, 3/10, $n/20$
22.	\$ 640	10%, 3%	6, 4/15, $n/30$
23.	\$1720	10%, 4%	7, 5/12, $n/20$
24.	\$ 825	8%, 5%	3, 2/8, $n/15$

25. A store received a bill for 37 suits at \$41.20 per suit. If discounts of 10% and 10% were allowed and if the terms were 5, 3/15, $n/20$, find the cost provided the bill was paid in 12 days.

26. A grocer received a bill for 410 cases of canned goods at \$2.87 per case less discounts of 10% and 6%. The terms were 3, 2/5, $n/10$. Find the cost on the fourth day.

27. A store for men received a bill for 132 shirts at \$1.80 each less discounts of 12.5% and 3%. Find the cost on the seventh day if the terms were 5, 4/10, $n/15$.

28. A filling station bought 72 tires at \$18.84 each less discounts of 12.5% and 5% and with terms of 4, 3/12, $n/20$. Find the cost if the bill was paid on the tenth day.

4-7 MARK-UP AND MARK-DOWN

Quite often a merchant or one of his representatives needs to be able to find what per cent of the cost an article must be marked up to make a desired per cent of profit on the selling price after allowing a customer a specified per cent of the marked price as a discount. The proper data to begin with is the cost to the merchant. We shall represent the cost to the merchant by C and the per cent of mark-up by m; hence, the marked price is $C + mC = C(1 + m)$. If we now represent the per cent discount allowed on the marked price by d, the discount is $dC(1 + m)$; hence, subtracting this from the marked price, we see that the selling price is $C(1 + m) - dC(1 + m) = C(1 + m)(1 - d)$. Consequently, subtracting the cost, we find that the profit is

(1) $$P = C(1 + m)(1 - d) - C$$

We want this to be a specified per cent, say p, of the selling price; thus, we want

(2) $$P = pC(1 + m)(1 - d)$$

We now have two expressions for the profit and shall equate them and simplify. Doing this gives

$$C(1 + m)(1 - d) - C = pC(1 + m)(1 - d)$$

$$C(1 + m)(1 - d)(1 - p) = C \quad \text{transposing and collecting}$$

$$(1 + m)(1 - d)(1 - p) = 1 \quad \text{dividing by C}$$

Hence, *the per cent of mark-up is determined by*

$$\boldsymbol{(1 + m)(1 - d)(1 - p) = 1}$$

where m, d, and p are the per cents of mark-up, discount, and profit. It is a simple matter to determine the marked price if the cost and per cent of mark-up are known.

Example 1. A refrigerator cost a merchant \$200. What should the marked price be if he wants to make a profit of $16\frac{2}{3}\%$ of the selling price after allowing a discount of 25% of the marked price?

Solution. In this problem, we have $d = 25\%$ and $p = 16\frac{2}{3}\%$. Consequently, $(1 + m)(1 - d)(1 - p) = 1$ becomes

$$(1 + m)(1 - \tfrac{1}{4})(1 - \tfrac{1}{6}) = 1$$

$$(1 + m)(\tfrac{3}{4})(\tfrac{5}{6}) = 1$$

$$(1 + m)(\tfrac{5}{8}) = 1$$

$$1 + m = \tfrac{8}{5}$$

$$m = \tfrac{3}{5} = 60\%$$

Therefore, the marked price should be $\$200 + .60(\$200) = \$320$.

Exercise 4-4

Find the marked price in each of the following problems if the cost, profit, and discount allowed by the retailer are as given. The profit is given as a per cent of the selling price and the discount is given as a per cent of the marked price.

	Cost	*Profit*	*Discount by retailer*
1.	\$ 120	$14\frac{2}{7}\%$	12.5%
2.	\$ 153	25%	20%

	Cost	*Profit*	*Discount by retailer*
3.	$ 400	20%	$16\frac{2}{3}\%$
4.	$ 200	20%	$16\frac{2}{3}\%$
5.	$ 100.98	25%	$11\frac{1}{9}\%$
6.	$ 209	50%	$33\frac{1}{3}\%$
7.	$ 500	$16\frac{2}{3}\%$	$14\frac{2}{7}\%$
8.	$ 700	12.5%	20%
9.	$ 384	$11\frac{1}{9}\%$	25%
10.	$ 552	20%	20%
11.	$1980	$22\frac{2}{9}\%$	10%
12.	$2160	25%	20%

5

Exponents and radicals

5-1 USES FOR EXPONENTS

Compound interest, annuities, bonds, and insurance are studied in courses in mathematics of finance and each of them deals with problems that would be quite long and difficult if only ordinary arithmetic processes were used. Much of the work is simplified if one can handle exponential expressions readily; additional simplification is afforded by use of logarithms. A familiarity with exponents is needed in learning how to use logarithms. Consequently we shall present the basic laws of exponents.

5-2 LAWS OF EXPONENTS

The definition of a positive integral exponent was given in Chapter 1, as were laws for use in multiplication and division of exponential expressions. These will be reviewed here and others will be added.

In a^n, the letter a is called the *base* and n is its *exponent* which tells how many times to use the base as a factor. Thus, $7^3 = (7)(7)(7)$.

By use of the definition, we have

$$a^m = aa \ldots \text{ to } m \text{ factors}$$

and

$$a^n = aa \ldots \text{ to } n \text{ factors}$$

Hence

$$\begin{aligned} a^m a^n &= aa \ldots \text{ to } (m+n) \text{ factors} \\ &= a^{m+n} \end{aligned}$$

and

$$\begin{aligned} \frac{a^m}{a^n} &= aa \ldots \text{ to } (m-n) \text{ factors} \\ &= a^{m-n} \qquad m > n \end{aligned}$$

We can now state again that

(1) $$\mathbf{a^m a^n = a^{m+n}}$$

and

(2) $$\mathbf{\frac{a^m}{a^n} = a^{m-n} \qquad m > n}$$

If stated in words, (1) and (2) become: *In multiplying two powers of the same base, we add the exponents. In dividing one power of a base by another power of it, we subtract the exponent of the denominator from that of the numerator.*

Example 1. $$x^5 x^3 = x^{5+3} = x^8$$

Example 2. $$\frac{x^7}{x^2} = x^{7-2} = x^5$$

If we make use of the definition of an exponent, we have

$$\begin{aligned} (ab)^n &= (ab)(ab) \ldots \text{ to } n \text{ factors} \\ &= (aa \ldots \text{ to } n \text{ factors})(bb \ldots \text{ to } n \text{ factors}) \\ &= a^n b^n \end{aligned}$$

Hence, we know that

(3) $$(ab)^n = a^n b^n$$

This, in words, is: *In order to raise the product of two factors to a power, we raise each factor to that power.*

Example 3. $(3x)^4 = 3^4 x^4 = 81x^4$

Using the definition again gives

$$\begin{aligned}\left(\frac{a}{b}\right)^n &= \frac{a}{b}\,\frac{a}{b} \ldots \text{to } n \text{ factors}\\ &= \frac{a\ a \ldots \text{to } n \text{ factors}}{b\ b \ldots \text{to } n \text{ factors}}\\ &= \frac{a^n}{b^n}\end{aligned}$$

Therefore, we know that

(4) $$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

and can say: *In order to raise a fraction to a power, we raise the numerator and denominator to that power.*

Example 4. $$\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4} = \frac{16}{81}$$

We shall now see how to raise a power to a power.

By definition

$$\begin{aligned}(a^m)^n &= a^m a^m \ldots \text{to } n \text{ factors}\\ &= a^{mn}\end{aligned}$$

Hence, we see that

(5) $$(a^m)^n = a^{mn}$$

and can say: *The exponent of a power of a power of a number is the product of the exponents of the two powers.*

Example 5. $(y^3)^2 = y^6$ since $(3)\ (2) = 6$

Example 6. $(2a^4)^3 = 2^3(a^4)^3 = 8a^{12}$

Example 7. $(x^3 y^4)^2 = (x^3)^2 (y^4)^2 = x^6 y^8$

Exercise 5-1

Perform the indicated operations.

1. 2^3	2. 3^2	3. 5^4	4. 7^3
5. $2^2\, 2^3$	6. $3^2\, 3^3$	7. $5^1\, 5^2$	8. $7^2\, 7^2$
9. $\frac{2^7}{2^3}$	10. $\frac{9^7}{9^5}$	11. $\frac{7^6}{7^3}$	12. $\frac{5^8}{5^5}$
13. $[(2)\,(3)]^3$	14. $[(5)\,(3)]^2$	15. $[(2)\,(5)]^4$	16. $[(2)\,(7)]^3$
17. $\left(\frac{3}{2}\right)^4$	18. $\left(\frac{5}{7}\right)^3$	19. $\left(\frac{6}{7}\right)^2$	20. $\left(\frac{5}{3}\right)^5$
21. $(2^2)^3$	22. $(3^3)^2$	23. $(2^3)^3$	24. $(3^4)^2$
25. $m^2\, m^3$	26. $a^3\, a^4$	27. $b^m\, b^p$	28. $c^t\, c^s$
29. $\frac{32a^5\, b^3}{8a^2\, b^2}$	30. $\frac{45a^8\, b^9}{15a^5\, b^3}$	31. $\frac{72r^{10}s^7}{18r^7\, s^6}$	32. $\frac{81m^8\, p^5}{27m^5\, p^2}$
33. $\frac{(1+i)^6}{(1+i)^2}$	34. $\frac{(1+i)^9}{(1+i)^7}$	35. $\frac{1.02^3}{1.02}$	36. $\frac{1.05^9}{1.05^4}$
37. $x^{5n}\, x^{n-1}\, x^{2-n}$		38. $(a^2x)^2\,(ax^2)^3$	
39. $b^{n+1}\, b^{2n-3}\, b^{2-n}$		40. $(a^3x^2)^3 \div (a^2x)^4$	

5-3 ZERO AND NEGATIVE EXPONENTS

The meaning of zero as an exponent and of negative exponents will be such that we can continue using the laws that have been used in connection with positive integral exponents.

To decide on the meaning to assign to a^0, we shall use (2) of Section 5-2 with $m = n$. Thus, we get

$$\frac{a^n}{a^n} = a^{n-n} = a^0,\ a \neq 0$$

However, $\frac{a^n}{a^n} = 1$, since it is a quantity divided by itself.

Consequently, we say that

$$a^0 = 1 \tag{1}$$

To obtain a meaning for a^{-n}, we shall multiply and divide it by a^n. This is equivalent to multiplying by 1. Thus

$$a^{-n} = a^{-n}\,\frac{a^n}{a^n}$$

$$= \frac{a^0}{a^n} \qquad \text{by use of (1) of Section 5-2}$$

$$= \frac{1}{a^n}$$

Consequently, in order to be able to continue to use (1) of Section 5-2 in multiplication, we say that

$$a^{-n} = \frac{1}{a^n} \tag{2}$$

Example 1. $5^0 = 1$

Example 2. $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

Quite often, we have to deal with an expression that contains one or more negative exponents. In such situations, we eliminate each negative exponent as shown in the next example.

Example 3. Express $\frac{2 - x^{-3}}{y^{-2}}$ without negative exponents.

Solution. We can eliminate the x^{-3} by multiplying by x^3 and can offset this by dividing by the same thing. Furthermore, we can get rid of y^{-2} by multiplying the denominator by y^2 and can offset this by multiplying the numerator by y^2. Thus

$$\begin{aligned}\frac{2 - x^{-3}}{y^{-2}} &= \frac{2 - x^{-3}}{y^{-2}} \cdot \frac{x^3}{x^3} \cdot \frac{y^2}{y^2} \\ &= \frac{2x^3y^2 - x^{-3+3}y^2}{y^{-2+2}x^3} \\ &= \frac{2x^3y^2 - y^2}{x^3}\end{aligned}$$

since $x^0 = y^0 = 1$.

5-4 FRACTIONAL EXPONENTS

We defined a positive integral exponent in Section 5-2 and extended the definition to include negative integers in Section 5-3. Now let us examine the meaning and use of fractional exponents. If (1) of Section 5-2 is to hold for fractional exponents, we must have

$$a^{1/q}a^{1/q} \ldots \text{ to } q \text{ factors equal to } a$$

since $\frac{1}{q} + \frac{1}{q} + \ldots$ to q terms is 1.

Thus, $a^{1/q}$ *is one of the q equal factors whose product is a.* Such a quantity is often called a *qth root* of a and is written $\sqrt[q]{a}$. The number q is the *index* and a is the *radicand.*

Now if (5) of Section 5-2 is to hold for fractional exponents, we must have

$$(a^{1/q})^p = a^{p/q}$$

and

$$(a^p)^{1/q} = a^{p/q}$$

Hence, in terms of radicals, we write

$$a^{p/q} = (\sqrt[q]{a}\ )^p = \sqrt[q]{a^p} \qquad a > 0$$

If there is a positive *qth* root of a, it should be used; however, if there is a negative one but no positive one, the negative one is used.

Example 1. $8^{2/3} = (\sqrt[3]{8})^2 = 2^2 = 4$

Example 2. $81^{3/4} = (\sqrt[4]{81})^3 = 3^3 = 27$

Exercise 5-2

Find the value of each of the following expressions.

1. 2^{-3} **2.** 3^{-1} **3.** 4^{-2} **4.** 5^{-4}
5. $8^{1/3}$ **6.** $4^{1/2}$ **7.** $16^{1/4}$ **8.** $64^{1/6}$
9. $4^{3/2}$ **10.** $27^{2/3}$ **11.** $32^{3/5}$ **12.** $81^{3/4}$

Express each of the following without negative exponents.

13. x^{-3} **14.** y^{-5} **15.** a^{-1} **16.** b^{-2}
17. $(x + y)^{-1}$ **18.** $(1 + i)^{-3}$ **19.** $(1 + i)^{-5}$
20. $(2x - y)^{-4}$ **21.** $\dfrac{1}{x^{-2} - y^{-1}}$ **22.** $\dfrac{x^{-1}}{x^{-2} - y^{-3}}$
23. $\dfrac{x^{-2} + y^{-1}}{x^{-3}}$ **24.** $\dfrac{x^{-3} - y^{-1}}{y^{-2}}$ **25.** $\dfrac{1 - 1.03^{-2}}{1.03^{-2}}$
26. $\dfrac{1 - 1.04^{-3}}{1.04^{-3}}$ **27.** $\dfrac{1 - 1.02^{-7}}{1.02^{-7}}$ **28.** $\dfrac{1 - (1 + i)^{-n}}{(1 + i)^{-n}}$

Put each of the following in a form that does not contain fractional exponents.

29. $(a + x)^{2/3}$ **30.** $1.03^{1/2}$ **31.** $1.02^{3/4}$ **32.** $a^{-2/3}$

Put each of the following in a form that does not contain radicals or negative exponents.

33. $\sqrt{1.03}$ **34.** $\sqrt[3]{1.02}$ **35.** $\sqrt[4]{1.01}$
36. $\sqrt[6]{1.04}$ **37.** $\sqrt[3]{(1 + i)^{-2}}$ **38.** $\sqrt[4]{(1 + i)^3}$
39. $\sqrt[4]{(1 + i)^{-5}}$ **40.** $\sqrt[12]{(1 + i)^{-7}}$

5-5 PRODUCTS AND QUOTIENTS OF RADICALS

In order to see how to get the product or quotient of two radicals with the same index, we shall express the radical in terms of a fractional exponent, do the necessary work, and then change the result back in terms of radicals. Thus

$$\begin{aligned}\sqrt[q]{a}\ \sqrt[q]{b} &= a^{1/q}\ b^{1/q}\\ &= (ab)^{1/q}\\ &= \sqrt[q]{ab}\end{aligned}$$

Consequently, we know that

$$\sqrt[q]{a}\ \sqrt[q]{b} = \sqrt[q]{ab}$$

and can put it in words as: *The product of two radicals with the same index is a radical with that index and with a radicand that is the product of the two radicands.*

Example 1. $\sqrt[6]{2}\ \sqrt[6]{16} = \sqrt[6]{32}$ since $(2)(16) = 32$

We shall now find how to get the quotient of two radicals with the same index.

$$\begin{aligned}\frac{\sqrt[q]{a}}{\sqrt[q]{b}} &= \frac{a^{1/q}}{b^{1/q}} = \left(\frac{a}{b}\right)^{1/q}\\ &= \sqrt[q]{\frac{a}{b}}\end{aligned}$$

Hence, we see that: *The quotient of two radicals with the same index is a radical with that index and with a radicand that is the quotient of the two radicands.*

Example 2. $\dfrac{\sqrt[3]{75}}{\sqrt[3]{5}} = \sqrt[3]{15}$ since $\dfrac{75}{5} = 15$

5-6 RATIONALIZING DENOMINATORS AND SIMPLIFYING RADICALS

It is often desirable to change a fraction with a radical in the denominator to an equivalent fraction that has a denominator free of radicals. This is called *rationalizing the denominator*. We shall discuss two such cases. One of them is a fraction in which the denominator is a single radical or contains a radical as a factor and the other is the situation in which the denominator is the sum or difference of two terms with at least one of them a radical.

If the denominator contains a radical of order q, we change the form of the fraction by multiplying the denominator by whatever factor is required in order to make it a perfect qth power and then offset this by multiplying the numerator by the same thing.

Example 1.

$$\sqrt{\frac{3}{5}} = \sqrt{\frac{3}{5}\,\frac{5}{5}} \qquad \text{multiplying the radicand by } \frac{5}{5}$$

$$= \sqrt{\frac{15}{5^2}} = \frac{\sqrt{15}}{5}$$

Example 2.

$$\sqrt[7]{\frac{x^9}{y^3}} = \sqrt[7]{\frac{x^9}{y^3}\,\frac{y^4}{y^4}} \qquad \text{multiplying by } \frac{y^4}{y^4}$$

$$= \sqrt[7]{\frac{x^9y^4}{y^7}}$$

If the denominator is the sum or difference of two terms and at least one of them is a radical, we rationalize by making use of the fact that the product of the sum and difference of two quantities is the difference of their squares.

Example 3. Rationalize the denominator of

$$\frac{14}{3 - \sqrt{2}}$$

Solution. We shall multiply the denominator by $3 + \sqrt{2}$ since that will rationalize it; hence, we must multiply the numerator by the same thing to keep from changing the value of the fraction. Thus

$$\begin{aligned}\frac{14}{3-\sqrt{2}} &= \frac{14}{3-\sqrt{2}}\;\frac{3+\sqrt{2}}{3+\sqrt{2}}\\ &= \frac{14(3+\sqrt{2}\;)}{7} \qquad \text{since } (3-\sqrt{2}\;)(3+\sqrt{2}\;)=7\\ &= 2(3+\sqrt{2}\;)\end{aligned}$$

by dividing numerator and denominator by 7.

To simplify a fraction that includes radicals, we rationalize the denominator and also take out any qth powers that occur in the radicand of a radical of index q. Consequently, the final form in Example 2 is not the simplified form since $x^9 = x^7\,x^2$ occurs under a radical with 7 as its index.

Example 4. Simplify

$$\frac{\sqrt{5}}{\sqrt{10}-1}$$

Solution.

$$\begin{aligned}\frac{\sqrt{5}}{\sqrt{10}-1} &= \frac{\sqrt{5}}{\sqrt{10}-1}\;\frac{\sqrt{10}+1}{\sqrt{10}+1} \quad \text{rationalizing}\\ &= \frac{\sqrt{50}+\sqrt{5}}{9} \quad \text{since } (\sqrt{10}-1)(\sqrt{10}+1)=9\\ &= \frac{5\sqrt{2}+\sqrt{5}}{9}\end{aligned}$$

since $\sqrt{50} = \sqrt{(25)(2)} = \sqrt{25}\,\sqrt{2} = 5\sqrt{2}$

Exercise 5-3

Perform the indicated operations in Problems 1 through 24 and take out factors from the radicals if possible.

1. $\sqrt{3}\,\sqrt{5}$
2. $\sqrt{2}\,\sqrt{7}$
3. $\sqrt{5}\,\sqrt{11}$
4. $\sqrt{2}\,\sqrt{13}$
5. $\dfrac{\sqrt{10}}{\sqrt{5}}$
6. $\dfrac{\sqrt{6}}{\sqrt{2}}$
7. $\dfrac{\sqrt{15}}{\sqrt{3}}$
8. $\dfrac{\sqrt{21}}{\sqrt{7}}$
9. $\sqrt{3}\,\sqrt{27}$
10. $\sqrt{14}\,\sqrt{56}$
11. $\sqrt{2}\,\sqrt{8}$
12. $\sqrt{5}\,\sqrt{125}$
13. $\sqrt{a^3 b}\,\sqrt{a^5 b^4}$
14. $\sqrt{2x^3 y}\,\sqrt{8x^0 y^4}$
15. $\sqrt{6xy^3}\,\sqrt{15x^2y^2}$
16. $\sqrt{5a^4 b^3}\,\sqrt{15a^0 b^2}$
17. $\dfrac{\sqrt{15ab^3}}{\sqrt{3a^{-1}b}}$
18. $\dfrac{\sqrt{24x^3 y^{-1}}}{\sqrt{6x^0 y^{-3}}}$

19. $\dfrac{\sqrt{45ax^2}}{\sqrt{5a^{-2}x^{-2}}}$ 20. $\dfrac{\sqrt{50a^{-3}b}}{\sqrt{2a^{-5}b^{-2}}}$ 21. $\dfrac{\sqrt{8}\sqrt{6}}{\sqrt{3}}$

22. $\dfrac{\sqrt{27}\sqrt{2}}{\sqrt{6}}$ 23. $\dfrac{\sqrt[3]{6}\sqrt[3]{12}}{\sqrt[3]{9}}$ 24. $\dfrac{\sqrt[3]{20}\sqrt[3]{28}}{\sqrt[3]{70}}$

Rationalize each denominator and then take out factors from the radicand if possible.

25. $\sqrt{\dfrac{x}{2z}}$ 26. $\sqrt{\dfrac{2a}{3c}}$ 27. $\sqrt{\dfrac{3b}{5d}}$ 28. $\sqrt{\dfrac{2w}{7s}}$

29. $\sqrt{\dfrac{14xy^3}{20x^2y}}$ 30. $\sqrt[3]{\dfrac{12x^2y}{14x^5y^0}}$

31. $\sqrt[3]{\dfrac{54a^3b^0}{12ab^5}}$ 32. $\sqrt{\dfrac{7x^2y^5}{2x^{-1}y^{-2}}}$

33. $\dfrac{6}{2 - \sqrt{3}}$ 34. $\dfrac{35}{3 + \sqrt{2}}$

35. $\dfrac{15}{\sqrt{7} - \sqrt{2}}$ 36. $\dfrac{2}{\sqrt{5} + \sqrt{3}}$

6

Logarithms

6-1 INTRODUCTION

As pointed out in Chapter 5, there are situations in mathematics of finance that require a considerable bit of computation. The amount of physical labor can be materially reduced if we make use of the concept of the logarithm of a number to a base. This is closely related to an exponent of a base as seen by the following definition:

The logarithm of a positive number N to a base b is the exponent L that the base must have in order to produce the number.

Some readers may prefer the following symbolic form of the definition:

(1) $$\log_b N = L \text{ if and only if } b^L = N$$

Each form of the definition states that the logarithm L is the exponent that the base b must have to produce the number N.

Example 1. $\log_{64} 64 = 1$ since $64^1 = 64$.

Example 2. $\log_8 64 = 2$ since $8^2 = 64$.

Example 3. $\log_4 64 = 3$ since $4^3 = 64$.

Example 4. $\log_{16} 64 = \frac{3}{2}$ since $16^{3/2} = 64$.

Example 5. Find N, if $\log_5 N = 3$.

Solution. By the definition of a logarithm of a number to a base, we know that $5^3 = N$; hence, $N = 125$.

Example 6. If $\log_b 343 = 3$, find b.

Solution. If we change the given equation to exponential form, we have $b^3 = 343 = 7^3$; hence, $b = 7$.

Exercise 6-1

Put each equation in Problems 1 through 8 in exponential form.

1. $\log_4 16 = 2$. **2.** $\log_2 8 = 3$. **3.** $\log_2 128 = 7$.
4. $\log_6 216 = 3$. **5.** $\log_b 17 = 2$. **6.** $\log_3 N = 2.7$.
7. $\log_3 72 = L$. **8.** $\log_a K = 5$.

Put each equation in Problems 9 through 16 in logarithmic form.

9. $3^4 = 81$. **10.** $2^7 = 128$. **11.** $5^3 = 125$.
12. $6^5 = 7776$. **13.** $b^5 = 183$. **14.** $3^a = 407$.
15. $13^{1.9} = N$. **16.** $x^y = z$.

Find the value of the letter in each of Problems 17 through 36.

17. $\log_4 64 = L$. **18.** $\log_3 81 = L$. **19.** $\log_{25} 5 = L$.
20. $\log_{216} 6 = L$. **21.** $\log_{25} 125 = L$. **22.** $\log_{343} 49 = L$.
23. $\log_3 1/9 = L$. **24.** $\log_2 1/64 = L$. **25.** $\log_3 N = 2$.
26. $\log_4 N = 3$. **27.** $\log_2 N = -2$. **28.** $\log_9 N = -1$.

29. $\log_9 N = \frac{1}{2}$ · **30.** $\log_{16} N = \frac{3}{4}$ · **31.** $\log_8 N = -\frac{2}{3}$ ·

32. $\log_4 N = -\frac{3}{2}$ · **33.** $\log_b 16 = 2$. **34.** $\log_b 8 = \frac{3}{2}$ ·

35. $\log_b 16 = \frac{4}{3}$ · **36.** $\log_b 27 = \frac{3}{2}$ ·

6-2 COMMON OR BRIGGS SYSTEM OF LOGARITHMS

If the base b is used in determining the logarithms of numbers, we say that we have a *system of logarithms to the base b*. Any positive number except one can be used as a base. If 10 is used, we have a *common* or *Briggs* system. We shall assume in the remainder of this chapter that 10 is the base unless another is specified.

6-3 MANTISSA AND CHARACTERISTIC

Since $10^2 = 100$ and $10^3 = 1000$, we know that $\log 100 = 2$ and $\log 1000 = 3$; however, we do not know log 234 since we do not know the power to which 10 must be raised in order to produce 234. It seems reasonable that log 234 should be between 2 and 3 since 234 is between 10^2 and 10^3. There are tables from which an approximation to the value of log 234 can be obtained; to four decimal places it is 2.3692. This information can be put in exponential form as

$$234 = 10^{2.3692}$$

Hence

$$2340 = 234(10) = 10^{2.3692} \times 10^1 = 10^{3.3692}$$

and

$$\log 2340 = 3.3692$$

Furthermore

$$.0234 = 234(10^{-4}) = 10^{2.3692} \times 10^{-4} = 10^{-2+.3692}$$

and

$$\log .0234 = -2 + .3692$$

If the logarithm of a number is expressed as an integer plus a positive fraction, then the integer is called the *characteristic* and the positive fraction is called the *mantissa*.

Example 1. The characteristic of log 234 is 2 and the mantissa is .3692 since log 234 = 2.3692.

The mantissa of the logarithm of a three-digit number can be found by use of Table 1 in the back of this book. The mantissa is found by locating the first two digits of the number in the column headed by N and then looking in line with this and in the column headed by the third digit of the number.

Example 2. The mantissa of log 234, except for the decimal point, is across from 23 and under 4. Thus, supplying the decimal, we see that it is .3692 as used above.

We shall use the following definition in determining the characteristic: *The reference position for the decimal point is immediately to the right of the first non-zero digit in the number.*

Example 3. The decimal point in 2.58 is in reference position.

Example 4. In .0258, the decimal point is two places to the left of reference position.

Example 5. The decimal point in 25.8 is one place to the right of reference position.

If we multiply a number by 10^n, the effect is to move the decimal point n places to the right if n is positive and n places to the left if n is negative; furthermore, if the decimal point is in reference position, the number is between 10^0 and 10^1, its logarithm is zero plus a positive fraction, and the characteristic is zero. Consequently, we have the following method for determining the characteristic of the logarithm of a number:

The characteristic of the logarithm of N is numerically equal to the number of places the decimal point is removed from the reference position. It is positive if the decimal point is to the right of reference position and negative if the decimal is to the left of reference position.

Example 6. The decimal point in 76.2 is one place to the right of reference position; hence, the characteristic of log 76.2 is 1; furthermore, the char-

acteristic of log .00296 is -3 since the decimal is three places to the left of reference position. This should be written as $7 - 10$ in keeping with the customary practice of writing a negative characteristic as the appropriate positive integer minus 10.

6-4 GIVEN log N, TO FIND N

We learned how to find the logarithm of a number in the last section and shall now see how to determine the number if its logarithm is given. In order to do this, we must locate the mantissa in the body of the table and then the sequence of digits in the number is made up of the two digits to the left of and the one above this mantissa. The position of the decimal point is determined by the characteristic.

Example 1. Find N if $\log N = 1.8797$.

Solution. We look for the mantissa in the body of the table and find it across from 75 and under 8; consequently, the sequence of digits in N is 758. The decimal point is one place to the right of reference position since the characteristic is 1; hence, $N = 75.8$.

If the given mantissa is not in the table, we use the one that is in it and closer to the given mantissa than any other entry (unless we want to use *interpolation*, which will be discussed in the next section).

Example 2. If $\log N = 9.6565 - 10$, find N.

Solution. This mantissa is not in the table. The entry nearest it is 6561 and the corresponding sequence of digits is 453. Consequently $N = .453$ since the characteristic is $9 - 10 = -1$.

Exercise 6-2

Find the characteristic of the logarithm of each number in Problems 1 through 12.

1. 3.24	**2.** 78.2	**3.** .569
4. 684	**5.** 3075	**6.** .047

7. 3295.1	**8.** .0301	**9.** .0031
10. 61,723	**11.** .00059	**12.** 6.7205

Find the logarithm of each number in Problems 13 through 24.

13. 5.87	**14.** 2.96	**15.** 39.9
16. 48.7	**17.** 987	**18.** 403
19. 7.23	**20.** 60.2	**21.** .507
22. .0352	**23.** .00518	**24.** .0505

Find the value of N in each of Problems 25 through 36 if log N has the given value.

25. 1.4031	**26.** 0.2923	**27.** 2.3927
28. 0.5856	**29.** 1.5868	**30.** 0.6964
31. 2.9996	**32.** 1.9076	**33.** 9.9304 − 10
34. 7.4860 − 10	**35.** 8.3861 − 10	**36.** 8.7188 − 10

6-5 INTERPOLATION

If we want to find the logarithm of a number that contains more digits than can be read directly from the available table, we resort to a procedure known as *linear interpolation.* It will be explained by means of two examples. Since the expression "the mantissa of the logarithm of N" will be used frequently, we shall abbreviate it by writing ml N.

Example 1. Use linear interpolation to find log 5472.

Solution. We make use of the fact that the mantissa of the logarithm of a number is not affected by the position of the decimal; hence we add a zero to 547 and to 548 so as to have four-digit numbers since the given number has four digits. We know that 5472 is between 5470 and 5480; in fact, it is $\frac{2}{10}$ of the way from 5470 to 5480. We now assume that ml 5472 is .2 of the way from ml 5470 = .7380 to ml 5480 = .7388. The difference between ml 5480 and ml 5470 is .7388 − .7380 = .0008 and we want .2 of it; hence, we want .2(.0008) = .00016 = .0002 to four decimal places. We now go this much from ml 5470 toward ml 5480 and have .7380 + .0002 = .7382 for ml 5472. Therefore, ml 5472 = .7382 and log 5472 = 3.7382 since the characteristic is 3.

This discussion can be shown in the form of a diagram as:

$$\begin{array}{cclcc}
 & & \text{ml } 5470 = .7380 & & \\
 & 2 & & .0002 & \\
 & & \text{ml } 5472 = & & .0008 \\
10 & & & & \\
 & & \text{ml } 5480 = .7388 & &
\end{array}$$

$$\begin{aligned}
\frac{2}{10}(.0008) &= .00016 \\
&= .0002 \text{ to four decimal places}
\end{aligned}$$

Hence

$$\begin{aligned}
\text{ml } 5472 &= .7380 + .0002 \\
&= .7382
\end{aligned}$$

Therefore

$$\log 5472 = 3.7382$$

Example 2. Find N if $\log N = 8.4863 - 10$.

Solution. This mantissa is not in the table; hence, we interpolate between the two entries in the table that are closest to it. These entries are $.4857 = \text{ml } 306 = \text{ml } 3060$ and $.4871 = \text{ml } 307 = \text{ml } 3070$. The zero was added in each case so as to have four-digit numbers to interpolate between. The remainder of the work will be shown in a diagram:

$$\begin{array}{cclcc}
 & & \text{ml } 3060 = .4857 & & \\
 & 4 & & .0006 & \\
 & & \text{ml } N \quad\;\; = .4863 & & \\
10 & & & & .0014 \\
 & & \text{ml } 3070 = .4871 & &
\end{array}$$

$$\frac{.0006}{.0014}(10) = 4$$

Therefore, the sequence of digits in N is 3064. Furthermore, $N = .03064$ since the characteristic of log N is $8 - 10 = -2$.

6-6 ROUNDING OFF

If we are asked to find the logarithm of a number that consists of more than four digits by use of the table in this book, we round off to four digits

and then interpolate. In order to round off to n significant figures,* we shall follow the usual practice:

(1) *Replace all digits beyond the nth by zeros.*
(2) *Consider the decimal fraction made up by placing a decimal point at the left of the replaced digits. If this is less than .5, leave the nth digit as it is. If the fraction is greater than .5, increase the nth digit by 1. If the fraction is .5, leave the nth digit unchanged if it is even and increase it by 1 if it is odd.*

Example 1. 477863 rounded off to four figures is 477900 since $.63 > .5$.

Example 2. 477849 rounded off to four figures is 477800 since $.49 < .5$.

Example 3. 47785 rounded off to four figures is 47780 since the fourth digit is even.

Example 4. 47735 rounded off to four figures is 47740 since the fourth digit is odd.

Exercise 6-3

Find the logarithm of each number in Problems 1 through 8 by interpolation.

1. 257.8	**2.** 32.76	**3.** 1.884	**4.** 90.37
5. 3892	**6.** .05401	**7.** .2979	**8.** 7354

If log N is as given in Problems 9 through 16, find N to four figures.

9. 1.2366	**10.** 2.7843	**11.** 0.2875
12. 3.7184	**13.** 9.1862−10	**14.** 8.4239−10
15. 9.6446−10	**16.** 7.3859−10	

Round off the number in each of Problems 17 through 24 to four figures.

17. 34276	**18.** 50.9723	**19.** 73.285
20. 7.3275	**21.** 67.365	**22.** 67.355
23. 491.874	**24.** 20.3914	

Round off the number in each of Problems 25 through 32 to four figures and then find the logarithm by interpolation.

25. 12.345	**26.** .013579	**27.** 9.7634
28. 79.848	**29.** 36.938	**30.** 872.53
31. .025935	**32.** 43.292	

*All digits except zero are always significant; zero is never significant if on the left, is always significant if between two other digits, and may or may not be significant if on the right.

6-7 COMPUTATION THEOREMS

We shall now make use of the laws of exponents in order to obtain several theorems for use in computation with logarithms.

We shall first see how to determine the logarithm of the product of two factors. If we represent the factors by

(1) $$M = b^m \text{ and } N = b^n$$

then

(2) $$\log_b M = m \text{ and } \log_b N = n$$

Consequently

$$\begin{aligned} \log_b MN &= \log_b b^m b^n \\ &= \log_b b^{m+n} && \text{adding exponents} \\ &= m + n && \text{definition of a logarithm} \end{aligned}$$

Now, using (2), we can write

(3) $$\boldsymbol{\log_b MN = \log_b M + \log_b N}$$

and put it in words as: ***The logarithm of the product of two factors is equal to the sum of the logarithms of the two factors.***

This can be extended to any number of factors. Thus, $\log MNP = \log(MN)P = \log(MN) + \log P = \log M + \log N + \log P$.

Example 1. Use logarithms to find the value of $P = (5.43)\ (27.8)$.

Solution.

$$\begin{aligned} \log P &= \log\ (5.43)\ (27.8) \\ &= \log 5.43 + \log 27.8 && \text{using (3)} \\ &= 0.7348 + 1.4440 && \text{using the table} \\ &= 2.1788 && \text{adding} \end{aligned}$$

Hence

$$P = 151 \quad \text{to three figures}$$

To find a formula for the logarithm of a fraction, we shall use (1) and (2) above and let the fraction be $\frac{M}{N}$, then

$$\begin{aligned} \log_b \frac{M}{N} &= \log_b \frac{b^m}{b^n} && \text{using (1)} \\ &= \log_b b^{m-n} && \text{subtracting exponents} \\ &= m - n && \text{definition of a logarithm} \end{aligned}$$

Now, using (2), we see that

$$\textbf{(4)} \qquad \log_b \frac{M}{N} = \log_b M - \log_b N$$

and can say that: *The logarithm of a fraction is equal to the logarithm of the numerator minus the logarithm of the denominator.*

Example 2. Evaluate $Q = \dfrac{37.6}{1.45}$ by use of logarithms.

Solution.

$$\begin{aligned} \log Q &= \log \frac{37.6}{1.45} & \\ &= \log 37.6 - \log 1.45 & \text{using (4)} \\ &= 1.5752 - 0.1614 & \text{using the table} \\ &= 1.4138 & \text{subtracting} \end{aligned}$$

Hence

$$Q = 25.9 \quad \text{to three figures}$$

As a third computation theorem, we shall find a formula for the logarithm of a power of a number. We shall represent the number by $N = b^n$ and the power by p. Then $N^p = (b^n)^p = b^{pn}$ and

$$\begin{aligned} \log_b N^p &= \log_b b^{pn} \\ &= pn \end{aligned}$$

Hence

$$\textbf{(5)} \qquad \log_b N^p = p \log_b N$$

since $n = \log_b N$. Equation (5) can be put in words as: *The logarithm of a power of a number is equal to the exponent of the power times the logarithm of the number.*

Example 3. Evaluate $R = 3.78^4$ by use of logarithms.

Solution.

$$\begin{aligned} \log R &= \log 3.78^4 & \\ &= 4 \log 3.78 & \text{using (5)} \\ &= 4\ (0.5775) & \text{using the table} \\ &= 2.3100 & \text{multiplying} \end{aligned}$$

Hence

$$R = 204 \quad \text{to three figures}$$

6-8 LOGARITHMIC COMPUTATION

There are some possible difficulties against which the reader must be warned in using the three computation theorems illustrated in the last section.

If the denominator of a fraction is larger than the numerator, then the logarithm of the fraction is negative including a negative fraction; consequently, we cannot find the number by use of the available table since each fraction in it is positive. This situation can be avoided by adding 10 to the logarithm of the numerator and subtracting 10 from it.

Example 1. If we use logarithms to evaluate $Q = \dfrac{7.84}{25.6}$, we have

$$\begin{aligned}\log Q &= \log \frac{7.84}{25.6} \\ &= \log 7.84 - \log 25.6\end{aligned}$$

We shall look up these logarithms and put one under the other and have

$$\begin{array}{l}\log 7.84 \;=\; 0.8943 \\ \underline{\log 25.6 \;=\; 1.4082} \\ \log Q \quad\;\; = \end{array}$$

where $\log Q$ is to be found by subtraction. If this is done, we get a negative value for $\log Q$ but can keep from getting a negative fraction by adding 10 to log 7.84 and subtracting 10 from it. Thus we have

$$\begin{array}{llll}\log 7.84 & = & 10.8943-10 & \text{adding and subtracting 10} \\ \log 25.6 & = & 1.4082 & \\ \hline \log Q & = & 9.4861-10 & \text{subtracting}\end{array}$$

Hence, using the table, we see that

$$Q = .306$$

There is another situation in which we may be led to a negative fraction in computing with logarithms. It comes about when we are using the logarithm of a power of a number in extracting a root, as can be done by making use of the fact that

$$\sqrt[q]{N^p} = N^{p/q}$$

Example 2. If we use logarithms to evaluate $R = \sqrt[3]{.805}$, we have

$$\log R = \log \sqrt[3]{.805} = \log .805^{1/3} = \tfrac{1}{3} \log .805$$
(1) $$= \tfrac{1}{3}(9.9058-10)$$
$$= 3.3019 - 3.3333$$
$$= -.0314$$

This, however, is a negative fraction and cannot be used in connection with our tables. This situation can be avoided by adding and subtracting any number within the parentheses in (1) so that the total subtracted is divisible by 3. Thus, we can add and subtract 20 in that equation and have

$$\log R = \tfrac{1}{3}(29.9058 - 30)$$
$$= 9.9686-10$$

Therefore, use of the tables enables us to say that

$$R = .930$$

Exercise 6-4

Express each logarithm in Problems 1 through 8 in terms of the sum and difference of logarithms.

1. $\log ab$ **2.** $\log 2cd$ **3.** $\log c^2d$

4. $\log \frac{s}{t}$ **5.** $\log \frac{ab}{c}$ **6.** $\log \frac{a}{bc}$

7. $\log \sqrt{ab}$ **8.** $\log \sqrt[3]{\frac{a^2}{b}}$

Use logarithms to find the value, to three digits, of the combination of numbers in each of Problems 9 through 24.

9. $(38.2)(2.17)$ **10.** $(7.84)(5.93)$ **11.** $(22.2)(33.3)$

12. $(.801)(975)$ **13.** $\frac{68.5}{41.2}$ **14.** $\frac{3.84}{4.83}$

15. $\frac{92.7}{81.5}$ **16.** $\frac{51.8}{72.9}$ **17.** $\frac{(25.7)(3.02)}{60.6}$

18. $\frac{(973)(.128)}{76.4}$ **19.** $\frac{30.9}{(27.1)(1.58)}$ **20.** $\frac{.471}{(2.80)(.103)}$

21. $\sqrt{93.7}$ **22.** $\sqrt[3]{.723}$ **23.** $\sqrt[4]{.915}$

24. $\sqrt[7]{1.09}$

SECTION 6-8

Use logarithms and interpolation to find the value, to four figures, of each combination of numbers in Problems 25 through 32.

25. (12.58) (7.634)

26. (79.38) (.2726)

27. (48.37) (61.42)

28. (.8059) (.3857)

29. $\frac{1786}{41.34}$

30. $\frac{28.47}{357.3}$

31. $\sqrt{31.72}$

32. $\frac{\sqrt{5.072}}{\sqrt[3]{8.614}}$

7

Functions and graphs

7-1 CONSTANTS AND VARIABLES

We have had various experiences in which we knew the values of certain quantities and were to find the value or values of others by use of the ones that were known. For example, if we know the radius and height of a right circular cylinder, we can find its volume by use of the equation $V = \pi r^2 h$ where V represents volume, r stands for the radius, and h is the height. If the cylinder has a radius of 4 inches, then $V = 16\pi h$. If we think of the cylinder as being filled by pouring water into it, we

have a situation in which the number π never changes, the quantity r has the value 4 in this discussion, and h may have any value from zero to the height of the cylinder. These types of quantities are called *constants* and *variables* and may be defined as follows:

A constant is a quantity that does not change in value during a discussion and may never change.

A variable is a quantity that may have any value within a given range or may have any value.

Example 1. The value of 2 and of $\sqrt{7}$ never change; hence, they are constants.

Example 2. The value of the circumference of a given circle does not change; hence, it is a constant as long as the discussion is restricted to one circle.

Example 3. The angle of elevation of the sun and the amount of water in a cistern vary and are variables.

7-2 FUNCTIONS

The volume of a right circular cylinder of radius 4 and height h is

$$V = 16\pi h \tag{1}$$

as given in the previous section; hence, the value of V depends on the value of h and is determined if a value is assigned to h. This type of situation is described in mathematical language by saying that "V is a function of h." The concept of a function is important in mathematics and is defined by saying that one variable is a *function* of another if a value is determined for it when a value is assigned to the other. There may be physical or other reasons why the range of values that can be assigned is limited. In the case of the cylinder whose volume is given by (1), the range of values for h is from zero to the height of the cylinder. It is common practice to indicate that one variable is a function of another by putting the second one in parentheses and placing it immediately after the first. Thus, $y(x)$ indicates that y is a function of x. If we want to show just how y depends on x we equate $y(x)$ to a specific combination of x's. Finally, to show that x has been given a particular value, we replace x in each member of the equation by that value.

Example 1. If $y(x) = 3x^2 - 5x + 6$, then
$$y(2) = 3(2)^2 - 5(2) + 6 = 12 - 10 + 6 = 8$$
as obtained by replacing x by 2 and simplifying.

Example 2. If $s(t) = t^3 - 5t + 1$, then
$$s(4) = 4^3 - 5(4) + 1 = 64 - 20 + 1 = 45$$
and
$$s(2x) = (2x)^3 - 5(2x) + 1.$$

One variable may be a function of several others.

Example 3. The symbol $s(n, i)$ indicates that s depends on n and i; furthermore
$$s(n, i) = \frac{(1 + i)^n - 1}{i}$$
shows the combination of n and i that gives s. Finally, if we want to replace i by .03 and n by 2, we write
$$\begin{aligned} s(2, .03) &= \frac{1.03^2 - 1}{.03} \\ &= \frac{1.0609 - 1}{.03} \quad \text{expanding } 1.03^2 \\ &= 2.03 \end{aligned}$$

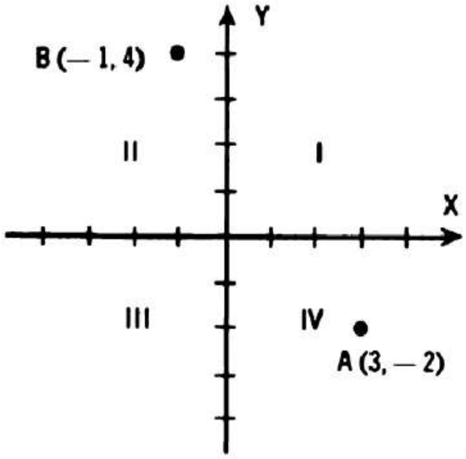
Exercise 7-1

1. If $y(x) = 2x - 3$, find $y(1)$, $y(3)$, $y(\frac{1}{2})$.
2. If $y(x) = 3x + 2$, find $y(0)$, $y(-1)$, $y(-\frac{2}{3})$.
3. If $r(s) = 6s - 5$, find $r(1)$, $r(\frac{1}{2})$, $r(3)$.
4. If $t(u) = 12u - 7$, find $t(2)$, $t(\frac{1}{4})$, $t(0)$.
5. If $y(x) = x^2 - x + 2$, find $y(2)$, $y(1)$, $y(\frac{1}{2})$.
6. If $y(x) = 3x^2 - 6x + 1$, find $y(2)$, $y(-1)$, $y(\frac{1}{3})$.
7. If $g(t) = 16t^2 - 12t + 3$, find $g(2)$, $g(0)$, $g(\frac{1}{2})$.
8. If $F(r) = 2r^2 - 3r + 1$, find $F(3)$, $F(1)$, $F(\frac{1}{2})$.
9. If $F(x, y) = 3x - 4y + 1$, find $F(2, 1)$, $F(0, \frac{1}{2})$.
10. If $F(x, y) = 2x - y - 3$, find $F(1, -1)$, $F(\frac{1}{3}, 1)$.
11. If $F(r, s) = 4r - 6s + 5$, find $F(0, 0)$, $F(\frac{1}{4}, \frac{1}{2})$.
12. If $F(v, s) = 2v + 3s + 2$, find $F(3, -2)$, $F(\frac{1}{2}, 1)$.

13. If $r(s) = s^2 - s + 1$, find $r(x + 1)$, $r(2x - 1)$.
14. If $s(t) = 6t^2 - 10t + 3$, find $s(x + 2)$, $s(x - 1)$.
15. If $g(t) = t^2 + 3t$, find $g(x - 3)$, $g(x + 1)$.
16. If $W(h) = 2h^2 - 5h - 1$, find $W(h - 2)$, $W(2h - 1)$.
17. If $l(a, r, n) = ar^{n-1}$, find $l(2, 2, 5)$, $l(1, 3, 4)$.
18. If $l(a, d, n) = a + (n - 1)d$, find $l(-3, 2, 7)$, $l(2, -1, 6)$.
19. If $S(n, a, l) = \frac{n}{2}(a + l)$, find $S(6, -3, 7)$, $S(4, 1, 10)$.
20. If $S(a, r, l) = \dfrac{a - rl}{1 - r}$, find $S(1, 2, 32)$, $S(3, -2, 48)$.
21. If $S(P, i, n) = P(1 + i)^n$, find $S(200, .04, 3)$, $S(400, .02, 2)$.
22. If $A(S, i, n) = S(1 + i)^{-n}$, find $A(300, .05, 2)$, $A(500, .03, 3)$.
23. If $S(R, i, n) = R\dfrac{(1 + i)^n - 1}{i}$, find $S(50, .03, 2)$, $S(70, .04, 3)$.
24. If $A(R, i, n) = R\dfrac{1 - (1 + i)^{-n}}{i}$, find $A(90, .01, 3)$, $A(130, .03, 4)$.
25. If $F(x) = \dfrac{2x^2 - 5x - 3}{x - 1}$, find $F(3)$, $F(0)$, $F(-1)$.
26. If $g(x) = \dfrac{3x^2 - 2x - 5}{2x - 1}$, find $g(-1)$, $g(0)$, $g(-\frac{1}{2})$.
27. If $s(t) = \dfrac{5t^2 + 9t - 2}{t + 4}$, find $s(-2)$, $s(\frac{1}{5})$, $s(3)$.
28. If $h(u) = \dfrac{6u^2 + u - 2}{2u - 1}$, find $h(2)$, $h(\frac{1}{2})$, $h(1.5)$.

7-3 THE RECTANGULAR COORDINATE SYSTEM

In order to locate a point in a plane, we give its distance and direction from each of two perpendicular lines. These lines are ordinarily taken as a horizontal and a vertical and are called the X and Y *axes*, respectively, as indicated in the accompanying figure. The point of intersection of the axes is called the *origin*. Distances measured to the right or up are considered to be positive whereas those measured to the left or down are called negative. The directed distance of a point from the Y axis is called the *abscissa* and the directed distance from the X axis is called the *ordinate*. The abscissa and ordinate are called the *coordinates* of the point and are written as a pair of numbers separated by a comma and enclosed in parentheses. The abscissa is always written first. The four portions into which the axes divide the plane are called *quadrants* and are numbered I, II, III, and IV as shown in the figure on the next page.



Example 1. The point that is 3 units to the right of the Y axis and 2 units below the X axis is designated by $(3, -2)$ and is shown as A in the figure.

Example 2. The point $(-1, 4)$ is one unit to the left of the Y axis and 4 units above the X axis and is shown as B in the figure.

7-4 GRAPHS AND ZEROS OF FUNCTIONS

To locate a point that is on the graph of a function, we assign a value to the independent variable x, compute the corresponding value of the dependent variable y, and plot the point (x, y). This procedure is repeated until we have located any desired number of points. The range of values assigned to x should be such as to give the entire graph or the desired portion of it. After the coordinates of the points have been determined and the points located, a smooth curve is drawn through them; this is called the ***graph*** of the function. Each value of x for which y is zero is called a ***zero of the function***.

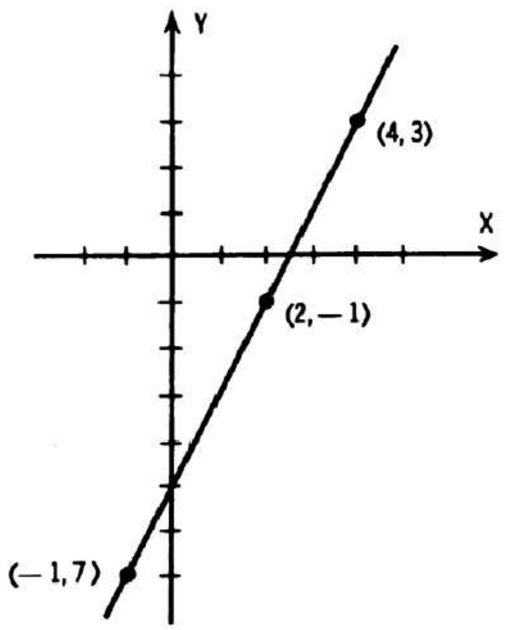
Example 1. Sketch the graph of $y(x) = 2x - 5$.

Solution. We shall assign only three values to x since the graph is a line. If we replace x by -1 in $y(x)$, we get $y(-1) = (2)(-1) - 5 = -7$. This information is listed in the table below along with the coordinates

of other points on the graph. If $x = 2$, then $y(2) = (2)(2) - 5 = -1$; similarly, $y(4) = (2)(4) - 5 = 3$.

x	-1	2	4
y	-7	-1	3

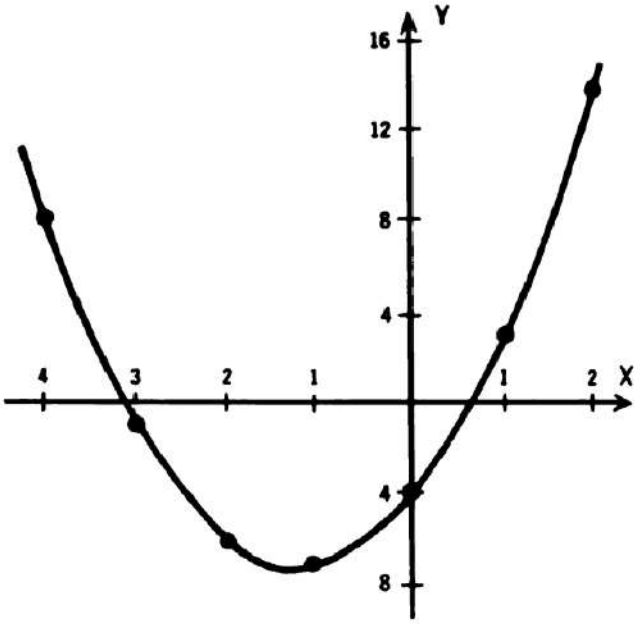
Finally, we get the graph by locating each point (x, y) that is indicated in the table and then drawing a smooth curve through them as shown in the figure below. The graph appears to cross the X axis at $x = 2.5$; hence, we say that $x = 2.5$ is the zero of the function.



Example 2. Sketch the graph and estimate the zeros of $y(x) = 2x^2 + 5x - 4$.

Solution. We shall begin by assigning several values to x and computing each corresponding value of y. Thus, if $x = -4$, we have $y(-4) = 2(-4)^2 + (5)(-4) - 4 = 8$. This and other pairs of values are shown in the table.

x	-4	-3	-2	-1	0	1	2
y	8	-1	-6	-7	-4	3	14



If we locate these points and draw a smooth curve through them, we get the curve shown above. It appears to cross the X axis for $x = -3.2$ and $x = .7$; hence, we say that these are the zeros of the function.

Exercise 7-2

Locate the following points or tell the quadrant in which each lies if k is positive.

1. $(3, 4)$, $(2, -1)$, $(-1, 2)$, (k, k), $(k, -k)$.
2. $(2, 7)$, $(-2, -3)$, $(4, 0)$, $(-k, k)$, $(-k, -k)$.
3. $(5, -1)$, $(0, -2)$, $(0, 0)$, $(k, -1)$, $(-k, 4)$.
4. $(-3, -4)$, $(-6, 1)$, $(2, -3)$, $(3, -k)$, $(-2, -k)$.

Sketch the part of the graph of each of the following functions that corresponds to the given range on x. Estimate the zeros.

5. $y(x) = 3x - 4$, $\quad x = -1$ to $x = 3$.
6. $y(x) = 5x - 9$, $\quad x = -1$ to $x = 3$.
7. $y(x) = 2x + 7$, $\quad x = -4$ to $x = 0$.
8. $y(x) = 3x + 8$, $\quad x = -5$ to $x = 1$.
9. $y(x) = 8x + 5$, $\quad x = -2$ to $x = 1$.

10. $y(x) = 5x + 11, \qquad x = -4$ to $x = 0.$
11. $y(x) = 6x - 7, \qquad x = -1$ to $x = 3.$
12. $y(x) = 2x - 7, \qquad x = 1$ to $x = 5.$
13. $y(x) = x^2 - 3x + 1, \qquad x = -1$ to $x = 4.$
14. $y(x) = x^2 - 5x + 3, \qquad x = -1$ to $x = 5.$
15. $y(x) = 2x^2 + 3x - 1, \qquad x = -3$ to $x = 2.$
16. $y(x) = 2x^2 + x - 2, \qquad x = -3$ to $x = 3.$
17. $y(x) = 3x^2 - 5x - 3, \qquad x = -2$ to $x = 3.$
18. $y(x) = 3x^2 + 8x - 4, \qquad x = -4$ to $x = 2.$
19. $y(x) = 4x^2 + 4x - 5, \qquad x = -4$ to $x = 3.$
20. $y(x) = 4x^2 - 5x - 4, \qquad x = -2$ to $x = 3.$

8

Progressions, the binomial theorem

8-1 ARITHMETIC PROGRESSIONS

Any collection of things is called a *set*. If a set is arranged in a definite order, we call it a *sequence*. If a sequence is such that each term after the first can be obtained from the one immediately preceding by adding a fixed number, the sequence is called an *arithmetic progression* and the fixed number is called the *common difference*.

Example 1. $-2, 1, 4, 7, 10$ is an arithmetic progression since each term, after the first, can be obtained from the one immediately preceding by adding the number 3.

In working with arithmetic progressions, it is customary to let

$a =$ the first term
$d =$ the common difference
$n =$ the number of terms
$l =$ the last or *nth* term
$s =$ the sum of the n terms

If we do this, the progression becomes

$$a, a + d, a + 2d, \ldots, a + (n - 1)d$$

as seen by adding d to the first term a, then adding d again and again until we have reached the *nth* term by adding the common difference $(n - 1)$ times. We then have

$$(1) \qquad \boldsymbol{l = a + (n - 1)d}$$

and can indicate the sum by

$$(2) \qquad s = a + a + d + a + 2d + \ldots + a + (n - 1)d$$

If we start with l and obtain the progression by subtracting d from it, then subtracting again and again until $(n - 1)$ subtractions have been made, we can write

$$(3) \qquad s = l + l - d + l - 2d + \ldots + l - (n - 1)d$$

Now, adding corresponding members of (2) and (3), we see that

$$(4) \qquad \begin{aligned} 2s &= (a + l) + (a + l) + \ldots + (a + l) \text{ to } n \text{ terms} \\ &= n(a + l) \end{aligned}$$

since for each d in (2) $-d$ occurs in (3) and since there are n terms in the progression. If we divide each member of the second form of (4) by 2, we find that

$$(5) \qquad \boldsymbol{s = \frac{n}{2}(a + l)}$$

is a formula for the sum of an arithmetic progression. The form of this equation can be changed by replacing l by its value as given in (1). Thus

$$s = \frac{n}{2}[a + a + (n - 1)d]$$

or, collecting terms,

$$(6) \qquad \boldsymbol{s = \frac{n}{2}[2a + (n - 1)d]}$$

is a variation of the formula for the sum of an arithmetic progression.

Example 2. Find the sum of an arithmetic progression of 11 terms if the first one is -5 and the common difference is 3.

Solution. In finding the sum, we shall use (6) since we know the value of each letter in the right-hand member of it. The values are $n = 11$, $a = -5$, and $d = 3$; hence, (6) becomes

$$\begin{aligned} s &= \frac{11}{2}[2(-5) + (11 - 1)3] \\ &= \frac{11}{2}(-10 + 30) \\ &= 110 \end{aligned}$$

Example 3. A college graduate accepts a position that pays \$310 per month with an agreement that he is to receive a raise of \$10 per month at the beginning of each month for a year. How much does he receive for his work during the first year with the company?

Solution. The monthly sums form an arithmetic progression since each can be obtained from the one immediately before it by adding \$10. The progression is such that $a = \$310$, $d = \$10$, and $n = 12$; hence, (6) becomes

$$\begin{aligned} s &= \frac{12}{2}[2(\$310) + (12 - 1)\$10] \\ &= 6(\$620 + \$110) \\ &= \$4380 \end{aligned}$$

Exercise 8-1

Find the terms of each arithmetic progression described in Problems 1 through 8.

1. $a = 1, d = 2, n = 7.$
2. $a = 8, d = -3, n = 6.$
3. $a = -9, d = 3, n = 8.$
4. $a = -1, d = -2, n = 5.$
5. $a = 3$, 2nd term $= 5, n = 6.$
6. $a = 2$, 2nd term $= 3, n = 8.$
7. $a = -1$, 3rd term $= 3, n = 7.$
8. $a = 4$, 3rd term $= 0, n = 6.$

Find the *nth* term and the sum of the terms in each of Problems 9 through 16.

9. $a = 2, d = 1, n = 11.$
10. $a = 3, d = -1, n = 7.$
11. $a = 7, d = -2, n = 9.$
12. $a = -5, d = 2, n = 8.$
13. $a = -3$, 2nd term $= -1, n = 8.$
14. $a = 6$, 2nd term $= 3, n = 7.$
15. $a = 12$, 3rd term $= 7, n = 9.$
16. $a = 15$, 3rd term $= 8, n = 9.$
17. Find the sum of all odd integers from 1 to 49, inclusive.

18. Find the sum of all even integers between 1 and 51.

19. Find the sum of all integral multiples of 3 between 13 and 103.

20. Find the sum of all positive integral multiples of 7 that are less than 138.

21. If a boy got 10 cents for delivering the first of 15 packages and 1 cent more for each than for the one just before it, how much did he get for delivering all of them?

22. A student is to be hired for 13 days during the Christmas holidays. He has the choice of working for $9.25 per day or of getting $6 the first day and an increase of $.50 per day each day. Which should he choose and how much will he profit by the choice?

23. A van owner was given a choice between taking a customer's household goods from Houston to El Paso for $600 or receiving $6 for the first hundred miles and $20 more for each hundred than for the last. Which should he choose if it is 800 miles from Houston to El Paso? How much would he gain by the choice?

24. If a body falls 16 feet the first second, 48 the second, 80 the third, and continues to increase at that rate, how far will it fall in the 9th second and how far in 9 seconds?

8-2 GEOMETRIC PROGRESSIONS

If a sequence is such that each term after the first can be obtained from the one immediately preceding by multiplying by a fixed number, the sequence is called a *geometric progression* and the fixed number is called the *common ratio*.

Example 1. The sequence $2, -6, 18, -54, 162$ is a geometric progression since each term can be obtained from the immediately preceding one by multiplying by -3.

When working with geometric progressions, it is customary to let

$$\begin{aligned} a &= \text{the first term} \\ s &= \text{the sum of the terms} \\ n &= \text{the number of terms} \\ l &= \text{the last or } nth \text{ term} \\ r &= \text{the common ratio} \end{aligned}$$

If we do this, the progression becomes

$$a, ar, ar^2, \ldots, ar^{n-1}$$

as seen by multiplying the first term a by r, then multiplying by r again and again until we have reached the nth term by multiplying by the common ratio $(n - 1)$ times. We then have

(1) $$l = ar^{n-1}$$

and can indicate the sum by

(2) $$s = a + ar + ar^2 + \ldots + ar^{n-1}$$

Furthermore

(3) $$rs = ar + ar^2 + ar^3 + \ldots + ar^n$$

as seen by multiplying each member of (2) by r. Now, subtracting each member of (3) from the corresponding member of (2) gives

(4) $$s - rs = a - ar^n$$

since all other terms cancel out. Finally, factoring the left member of (4) into $s(1 - r)$ and dividing by $1 - r$, we see that

(5) $$s = \frac{a - ar^n}{1 - r}$$

is a formula for the sum of a geometric progression. The form of this equation can be changed by writing ar^n as rar^{n-1} and then replacing ar^{n-1} by l from (1). Thus, we get

(6) $$s = \frac{a - rl}{1 - r}$$

as another form for the sum of a geometric progression. Whether (5) or (6) is to be used at any time is determined by the things that are known.

Example 2. Find the 10th term and the sum of the terms of a geometric progression of 10 terms if the first term is 3 and the common ratio is -2.

Solution. In this problem, we know that $n = 10$, $a = 3$, and $r = -2$. Substituting these values in (1) gives

$$l = 3(-2)^9 = -1536$$

We can now use either (5) or (6) to find s. If we use (6), we have

$$\begin{aligned} s &= \frac{3-(-2)\,(-1536)}{1 - (-2)} \\ &= -1023 \end{aligned}$$

Exercise 8-2

Find the terms of each geometric progression described in Problems 1 through 8.

1. $a = 1, r = 2, n = 6$.
2. $a = 2, r = 2, n = 7$.
3. $a = 128, r = \frac{1}{2}, n = 8$.
4. $a = 15{,}625, r = .2, n = 8$.
5. $a = \frac{1}{4}$, 2nd term $= \frac{1}{12}$, $n = 6$.
6. $a = \frac{1}{32}$, 2nd term $= \frac{1}{16}$, $n = 9$.
7. $a = 81$, 3rd term $= 9$, $n = 7$.
8. $a = 256$, 3rd term $= 16$, $n = 6$.

Find the *nth* term and the sum of the terms in each of Problems 9 through 16.

9. $a = \frac{1}{8}, r = 2, n = 6$.
10. $a = \frac{1}{125}, r = 5, n = 5$.
11. $a = 243, r = -\frac{1}{3}, n = 7$.
12. $a = 512, r = -\frac{1}{2}, n = 9$.
13. $a = 27$, 2nd term $= -9$, $n = 5$.
14. $a = \frac{81}{16}$, 2nd term $= \frac{27}{8}$, $n = 5$.
15. $a = 64$, 3rd term $= 16$, $n = 7$.
16. $a = 3125$, 3rd term $= 125$, $n = 8$.

17. Find the sum of all positive integral powers of 2 that are less than 1025.
18. Find the sum of all positive integral powers of 3 from 3 to 3^{10}, inclusive.
19. Find the sum of all positive integral powers of $\frac{1}{2}$ from $\frac{1}{2}$ to $(\frac{1}{2})^6$.
20. Find the sum of all positive integral powers of $\frac{2}{3}$ from $\frac{2}{3}$ to $\frac{64}{729}$.

Find an expression for each sum by use of (5) or (6) of Section 8-2.

21. $1 + 1.03 + 1.03^2 + 1.03^3 + 1.03^4 + 1.03^5$.
22. $1 + 1.04 + 1.04^2 + 1.04^3 + 1.04^4$.
23. $1 + 1.01 + 1.01^2 + 1.01^3 + \ldots + 1.01^{17}$.
24. $1 + 1.02 + 1.02^2 + 1.02^3 + \ldots + 1.02^{13}$.
25. $1 + 1.02^{-1} + 1.02^{-2} + 1.02^{-3} + 1.02^{-4}$.
26. $1 + 1.04^{-1} + 1.04^{-2} + 1.04^{-3} + 1.04^{-4} + 1.04^{-5}$.
27. $1 + 1.01^{-1} + 1.01^{-2} + \ldots + 1.01^{-19}$.
28. $1 + 1.03^{-1} + 1.03^{-2} + \ldots + 1.03^{-11}$.

29. A blacksmith received 1 cent for the first nail used in shoeing a horse, 2 cents for the second, 4 cents for the third, and so on for the 16 nails used. How much did he get in all?

30. A boy had a 10-day holiday and was given the opportunity of working for 5 cents the first day and twice as much as on the previous day for each day thereafter. Instead of taking this job, he accepted another at \$5 per day. How much did he lose by this choice?

31. Twelve men are fishing from a pier. The first is worth \$1000, the second \$2000, the third \$4000, etc. How many millionaires are fishing there?

32. If there are no duplicates, find the number of ancestors that a pair of twins have in the six generations immediately preceding them.

8-3 EXPANSION OF A BINOMIAL WITH POSITIVE INTEGRAL EXPONENTS

We can obtain the following expansions by raising each binomial to the indicated power:

$$(x + y)^1 = x + y$$
$$(x + y)^2 = x^2 + 2xy + y^2$$
$$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$
$$(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$
$$(x + y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$$

By referring to the above expansions, we can readily see that the following properties of the expansion of $(x + y)^n$ hold for $n = 1, 2, 3, 4,$ and 5:

(1) The sum of the exponents of x and y in any term is n.

(2) The first term in the expansion is x^n and the exponent of x decreases by 1 from term to term.

(3) The second term in the expansion is $nx^{n-1}y$ and the exponent of y increases by 1 from term to term.

(4) If the coefficient in any term is multiplied by the exponent of x in that term and the product divided by the number of the term, the coefficient of the next term is obtained.

(5) Two terms that are equidistant from the ends of the expansion have the same coefficient.

We have shown these properties to hold only for $n = 1, 2, 3, 4,$ and 5, but they do hold and we shall use them in all cases for n a positive integer.

The theorem that states they are true in all cases is called the *binomial theorem*. If we use it or the above properties to expand $(x + y)^n$, we obtain

$$(x + y)^n = x^n + nx^{n-1}y + \frac{(n)(n-1)x^{n-2}y^2}{(1)(2)} + \frac{n(n-1)(n-2)x^{n-3}y^3}{(1)(2)(3)} + \cdots + nxy^{n-1} + y^n$$

This is called the *binomial formula*.

Example 1. Find the expansion of $(2a + b)^7$.

Solution. We could find the expansion by using the binomial formula but shall find it by making use of the four properties listed above. According to properties (2) and (3), the first two terms in the expansion are

$$(2a)^7 \text{ and } 7(2a)^6b$$

If we indicate the sum of these two terms and find two others by use of property (4), we have

$$(2a)^7 + 7(2a)^6b + \frac{7(6)\,(2a)^5b^2}{2} + \frac{7(6)\,(5)\,(2a)^4b^3}{2(3)}$$
$$= (2a)^7 + 7(2a)^6b + 21(2a)^5b^2 + 35(2a)^4b^3$$

as the first four terms in the expansion. The coefficients of the other four terms are the same as these by use of property (5), since corresponding to each of the first four terms one of the last four is the same distance from an end; hence, the expansion is

$$(2a + b)^7 = (2a)^7 + 7(2a)^6b + 21(2a)^5b^2 + 35(2a)^4b^3 + 35(2a)^3b^4 + 21(2a)^2b^5 + 7(2a)b^6 + b^7$$

Now, expanding each power of $2a$, we have

$$(2a + b)^7 = 128a^7 + 448a^6b + 672a^5b^2 + 560a^4b^3 + 280a^3b^4 + 84a^2b^5 + 14ab^6 + b^7$$

Example 2. Find the value of 1.04^5.

Solution. $1.04^5 = 1^5 + 5(1^4)\,(.04) + 10(1^3)\,(.04)^2 + 10(1^2)\,(.04)^3 + 5(1)\,(.04)^4 + .04^5$

$= 1 + .20 + .016 + .00064 + .0000128 + .0000001024$

$= 1.2166529024$

8-4 THE BINOMIAL THEOREM FOR FRACTIONAL AND NEGATIVE EXPONENTS

If n is a fraction or a negative integer there is no end to the number of terms in the expansion of $(x + y)^n$, since in using property (4) of the previous section we never obtain zero for a coefficient. This is seen to be true since, if we begin with a fraction or a negative number and subtract 1 again and again, we never obtain zero. Consequently, we must be content with any desired or indicated number of terms. We are in no position to prove but should become familiar with the following fact:

The expansion of $(x + y)^n$ for n fractional or negative is valid only if the value of y is between x and $-x$.

Example 1. Find the first four terms in the expansion of $(3 + y)^{1/2}$. For what range on y is the expansion valid?

Solution. We shall use properties (1), (2), (3), and (4) of the previous section. The first term is given by (2), the second term by (3), other terms by use of (4), and a check on the sum of the exponents is obtained by (1). Thus, the first four terms in the expansion of $(3 + y)^{1/2}$ are

$$3^{1/2} + \tfrac{1}{2}(3^{-1/2})y + \frac{(\frac{1}{2})(-\frac{1}{2})(3^{-3/2})y^2}{2} + \frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})(3^{-5/2})y^3}{2(3)}$$

The form of this expression can be changed by simplifying each coefficient and eliminating negative exponents without introducing fractional exponents in the denominators. Thus, we get

$$\begin{aligned}\sqrt{3} + \frac{\sqrt{3}}{6}\ y - \frac{\sqrt{3}}{72}\ y^2 + \frac{\sqrt{3}}{432}\ y^3 \\ = \frac{\sqrt{3}}{432}\ (432 + 72y - 6y^2 + y^3)\end{aligned}$$

The expansion is valid for y between 3 and -3.

Example 2. Find the sum of the first four terms in the expansion of 1.03^{-5}.

Solution. By use of the properties given in the previous section we find that the first four terms are

$$\begin{aligned}&1^{-5} + (-5)(1^{-6})(.03) + \frac{(-5)(-6)(1^{-7})(.03)^2}{2} \\ &\qquad + \frac{(-5)(-6)(-7)(1^{-8})(.03)^3}{2(3)} \\ &= 1 - 5(.03) + 15(.0009) - 35(.000027) \\ &= .862555\end{aligned}$$

Exercise 8-3

Expand the binomial in each of Problems 1 through 16.

1. $(a + b)^7$	2. $(a + 3)^4$	3. $(1 + x)^5$	4. $(2 + x)^6$
5. $(x - y)^6$	6. $(x - 3)^4$	7. $(1 - a)^5$	8. $(2 - b)^7$

9. $(a - 2b)^5$ **10.** $(3x - y)^4$ **11.** $(2x - y)^6$ **12.** $(x - 4y)^3$
13. $(x^2 - 2)^3$ **14.** $(x + y^3)^4$ **15.** $(a^2 + b^3)^5$ **16.** $(2a^3 - b^2)^4$

Find the first four terms in the expansion of the binomial in each of Problems 17 through 24.

17. $(a + b)^{1/2}$ **18.** $(1 - a)^{2/3}$ **19.** $(2 - x)^{3/4}$
20. $(1 + x)^{1/4}$ **21.** $(1 + x)^{-2}$ **22.** $(1 - x)^{-3}$
23. $(2 + x)^{-1}$ **24.** $(2 - x)^{-2}$

Find the sum of the first four terms of each expansion.

25. 1.02^3 **26.** 1.03^5 **27.** 1.01^7 **28.** 1.04^4
29. 1.02^{-3} **30.** 1.03^{-5} **31.** 1.05^{-3} **32.** 1.01^{-5}
33. $1.04^{1/2}$ **34.** $1.02^{1/4}$ **35.** $1.06^{1/6}$ **36.** $1.03^{1/4}$
37. $1.02^{-1/4}$ **38.** $1.05^{-1/2}$ **39.** $1.03^{-1/3}$ **40.** $1.06^{-1/6}$

9

Statistics

9-1 INTRODUCTORY CONCEPTS

Modern statistics often requires the use of advanced mathematics. We will concern ourselves here only with some of the more elementary aspects of the subject. These include the collection and organization of data, drawing inferences from the data, and checking the reliability of those inferences.

Some rearrangement of the data is usually in order to make facts stand out. One rearrangement consists of separating the data into classes and showing the number

of items in each class. This is called combining into a *frequency distribution*. If a frequency distribution is used, the classes should all be of the same size. Furthermore, each item is used as if its value were the middle of the class. Thus, for a class of 17 to 23, use 20 for each item.

Example. The weights in pounds of the 30 boys in an eleventh-grade class were 140, 127, 173, 162, 177, 132, 143, 122, 154, 162, 139, 157, 134, 169, 123, 158, 129, 143, 150, 134, 148, 137, 171, 138, 126, 157, 153, 160, 147, 151.

The least weight was 122 pounds and the heaviest boy weighed 177 pounds; hence, the lowest class must begin at 122 or less and the highest must end at 177 or more. We can choose any desired number of classes so long as the entire data range is included and each class size is the same. We shall arbitrarily choose 7 classes beginning at 121.5; hence, each class size must be 8 to include all data. We shall make a table that shows each class, the tally, and the frequency in it.

Class	*Tally*	*Frequency*
121.5 to 129.5	~~IIII~~	5
129.5 to 137.5	~~IIII~~	4
137.5 to 145.5	~~IIII~~	5
145.5 to 153.5	~~IIII~~	5
153.5 to 161.5	~~IIII~~	5
161.5 to 169.5	III	3
169.5 to 177.5	III	3

The largest and smallest values in a class are called the *class boundaries* and their difference is called the *class width*. Half of the sum of the class boundaries is called the *class mark*. Thus, the class boundaries for the smallest class in the above tally are 121.5 and 129.5, the class mark for that class is 125.5, and the class width for each class is 8.

The totality of items of data is called the *population*. At times, it is not practicable to use the entire population; hence, methods for obtaining a sample have been devised. One of them is called a *random sample* and it is obtained so that each member or item of the population has an equal chance of being selected. Another is called a *selective sample* and it is obtained in such a way that all characteristics of the population are represented in proper proportion. This method is generally used by the institutes of public opinion.

9-2 AVERAGES

The term *average* or *measure of central tendency* is used to describe any term that locates or gives a central tendency or central value of a set of data or frequency distribution. Several such quantities are used and each has its advantages and its limitations. The average that is most desirable depends on the use to which it is to be put.

The average that is obtained by dividing the sum of the items by the number of items is called the *arithmetic mean*. If a frequency distribution is used, the class mark is used as the value of each item.

Example 1. If the heights of the 9 men on a basketball squad are 70, 71, 71, 72, 75, 76, 76, 76, and 79 inches, find the arithmetic mean.

Solution. We shall find the sum of these heights and divide that by 9 since there are 9 items in the population. The sum is $70 + 71 + 71 + 72 + 75 + 76 + 76 + 76 + 79 = 666$; hence, the arithmetic mean is $666 \div 9 = 74$.

The middle item after an odd number of items have been arranged in order of size is another measure of central tendency and is called the *median*.

Example 2. The median of the heights of the basketball players of Example 1 is 75 inches, since there are 4 players shorter and 4 taller than that.

Example 3. The median of 5, 7, 11, and 19 is $\frac{1}{2}(7 + 11) = 9$ since there is an even number of items and the arithmetic mean of the middle two is 9.

In order to obtain the median of data that has been put in class marks, we take the class mark as the value of the item and count it the number of times indicated by the frequency.

Example 4. The median of the weights collected in classes in Section 9-1 is 149.5 since that is the class mark of the class which contains the middle items.

The item that occurs the most frequently is called the *mode* and is another measure of central tendency.

Example 5. The mode of the items given in Example 1 is 76 since that item occurs more times than any other.

Exercise 9-1

Find the arithmetic mean, the median, and the mode of the numbers given in each of Problems 1 through 4.

1. 66, 71, 75, 78, 78, 80, 84.

2. 24, 24, 25, 25, 27, 28, 28, 28, 35.

3. $2.35, $2.13, $2.13, $2.09, $2.08, $2.06.

4. $4100, $4600, $4900, $4900, $5200, $5700.

5. The hourly rates made by a group of workers were $1.27, $1.19, $1.18, $1.18, $1.18, $1.12, $1.04, $1.00, $.93, $.88, $.81, and $.70. Find the arithmetic mean, the median, and the mode of the rates. Why is the mode not a representative average?

6. The salaries of 8 men in a refinery were $4500, $4800, $5100, $5300, $5300, $5500, $5600, and $7900. Find the arithmetic mean, the median, and the mode of the salaries. Why is the arithmetic mean not a representative average?

7. The number of fish caught by the members of a party were 8, 10, 10, 10, 18, 25, 35, and 100. Find the arithmetic mean, the median, and the mode. Why are the arithmetic mean and the mode not representative averages?

8. The sizes of 7 ranches in terms of acres are 800, 1300, 1500, 1600, 1800, 2100, and 2100. Find the arithmetic mean, the median, and the mode. Is the mode a representative average? Why?

Arrange the numbers in each of Problems 9 through 12 in a frequency distribution with class width as indicated and then find the arithmetic mean, the median, and the mode.

9. Use class width of 5 with the following numbers: 15, 27, 24, 24, 29, 26, 33, 37, 33, 34, 34, 36, 37, 37, 37, 44, 43, 44, 48, 46, 47, 48, 49, 49.

10. Use class width of 9 with the following blood pressures: 107, 114, 122, 125, 125, 129, 136, 137, 140, 143, 143, 143, 152, 152, 159, 160, 161, 163, 165, 187.

11. Use class width of 30 with the following monthly salaries: $225, $230, $230, $240, $260, $275, $295, $305, $310, $310, $310, $350, $370, $390, $405, $415, $480, $490.

12. Use class width of 5 with the following ages: 43, 41, 37, 35, 33, 32, 30, 29, 28, 28, 28, 26, 26, 24, 24, 21, 20, 17, 16, 14.

9-3 MEASURES OF DISPERSION

In the last section, we discussed averages and we shall now study means of determining how the data is distributed about the average. There are several measures of dispersion that take into account the amount each item deviates from the arithmetic mean. One of them is called the *standard deviation* and is given by

$$\sigma = \sqrt{\frac{(x_1 - M)^2 + (x_2 - M)^2 + \ldots + (x_n - M)^2}{n}}$$

provided $x_1, x_2, \ldots, x_n$ are the n items and M is their arithmetic mean.

Example 1. Find the standard deviation of the following grades made by a student while studying plane trigonometry: 47, 58, 62, 67, 73, 75, 83, 85, 88, 88, 89, 93, 95, 97, 100.

Solution. The arithmetic mean is needed in finding the standard deviation. It is readily seen to be 80 since the total is 1200 and there are 15 items. The remainder of the work is easier to follow if placed in tabular form.

Item	*Item − Mean = Deviation*	*Deviation*^{2}
47	−33	1089
58	−22	484
62	−18	324
67	−13	169
73	−7	49
75	−5	25
83	3	9
85	5	25
88	8	64
88	8	64
89	9	81
93	13	169
95	15	225
97	17	289
100	20	400
1200	0	3466

There are 15 items and the sum of the squares of the deviations is 3466; hence, the standard deviation is

$$\sigma = \sqrt{\frac{3466}{15}}$$
$$= 15.2$$

The class mark is used as the value of the item if the items are arranged in a frequency distribution; furthermore, each item is used as many times as it occurs in the frequency distribution.

Example 2. Arrange the items given in Example 1 in class widths of 9 and then find the standard deviation.

Solution. We shall first make the frequency distribution, then compute the arithmetic mean and the standard deviation.

Class	47-55	56-64	65-73	74-82	83-91	92-100
Tally	I	II	II	I	~~IIII~~	IIII
Frequency	1	2	2	1	5	4

We must now multiply each class frequency by the class mark, find the sum of these products, and divide that sum by the number of items to obtain the arithmetic mean. Thus

$$M = \frac{1(51) + 2(60) + 2(69) + 1(78) + 5(87) + 4(96)}{15}$$
$$= 80.4$$

We shall arrange the remainder of the work in a table.

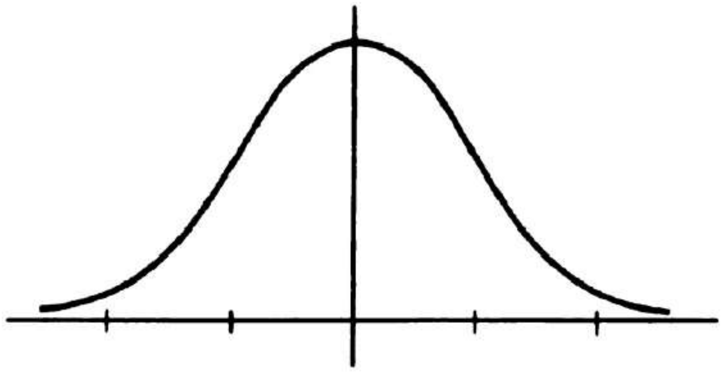
Class mark	*Frequency*	*Deviation*	*(Freq.) (Dev.)²*
51	1	−29.4	(1) (864.36) = 864.36
60	2	−20.4	(2) (416.16) = 832.32
69	2	−11.4	(2) (129.96) = 259.92
78	1	−2.4	(1) (5.76) = 5.76
87	5	6.6	(5) (43.56) = 217.80
96	4	15.6	(4) (243.36) = 973.44
			Sum = 3153.60

Consequently

$$\sigma = \sqrt{\frac{3153.60}{15}}$$
$$= 14.5$$

9-4 THE NORMAL CURVE

If e is approximately 2.718, x is the deviation of an item from the arithmetic mean of the items, σ is the standard deviation from the mean, and C is the value of y for $x = 0$, then the graph of $y = Ce^{-\frac{1}{2}(x/\sigma)^2}$ is called the *normal frequency curve*. The frequency distribution associated with it is called a *normal distribution*. The graph of the curve is shown in the accompanying figure.



It is beyond the scope of this book to do so but it can be shown that for a normal distribution with arithmetic mean M and standard deviation σ:

about 67% of the items come between $M - \sigma$ and $M + \sigma$,

about 95% of them come between $M - 2\sigma$ and $M + 2\sigma$,

and about 99% between $M - 3\sigma$ and $M + 3\sigma$.

Example. If the data of Example 1 of Section 9-3 formed a normal distribution, then about .67 of the 15 items would come between $80 - 15.2 = 64.8$ and $80 + 15.2 = 95.2$, since $M = 80$ and $\sigma = 15.2$. By actual count 10 come in that range. Furthermore, about $.95(15) = 14.25$ items should be between $80 - 2(15.2) = 49.6$ and $80 + 2(15.2) = 110.4$. By count, 14 items are in that range. Finally, about $.99(15) = 14.85$ items should be between $80 - 3(15.2) = 34.4$ and $80 + 3(15.2) = 125.6$, and 15 items are in that range.

Exercise 9-2

1. Find the standard deviation for the following high temperatures that occurred in a northern city: January 62°, February 57°, March 68°, April 71°, May 77°, June 89°, July 91°, August 94°, September 86°, October 71°, November 69°, December 65°.

2. The weight in pounds of the 14 men on a college baseball team are 138, 146, 153, 158, 159, 163, 168, 168, 172, 177, 185, 188, 190, 215. Find the standard deviation.

3. The best times in seconds in the 100-yard dash of the 10 backfield men on a college football team are 9.5, 9.7, 9.9, 10.0, 10.0, 10.3, 10.4, 10.5, 11.0, 11.3. Find the standard deviation.

4. The grades on a mathematics of finance test were 31, 37, 52, 58, 65, 69, 74, 78, 83, 88, 94, and 99. Find the standard deviation.

5. Put the following numbers in class widths of 5 and find the standard deviation: 23, 27, 28, 30, 31, 33, 36, 37, 38, 39, 40, 40, 42, 43, 44, 46, 47, 51, 52, 57.

6. The salaries of the members of a mathematics faculty in a high school in 1938-1939 were \$1170, \$1170, \$1260, \$1305, \$1394, \$1440, \$1440, \$1494, \$1503, \$1575, \$1620, \$1620, \$1680, \$1680, \$1710, \$1725, \$1773. Arrange these salaries in class widths of \$175 beginning with \$1170 and then find the standard deviation.

7. The heights in inches of the 18 boys in a high school class are 59, 60, 61, 62, 62, 63, 64, 66, 66, 66, 67, 68, 68, 69, 70, 71, 72, 73. Arrange in a frequency distribution with class widths of 3 beginning with 59 and then find the standard deviation.

8. The shoe sizes of the members of a basketball team are 6, 6.5, 7, 7.5, 8, 9, 10, 10.5, 11, 12, 13, 14.5. Arrange these in a frequency distribution with class width of 1.5 beginning with 6 and then find the standard deviation.

9. If the arithmetic mean of the ages of the 150 members of the Kiwanis Club is 46 and the standard deviation is 8, describe the approximate distribution of the ages by telling about how many are between 38 and 54, 30 and 62, 22 and 70.

10. The arithmetic mean of the grades of 900 students on an English test was 79 and the standard deviation was 6. Give the approximate distribution of the grades.

11. The arithmetic mean of the intelligence quotients of the 1800 freshmen at a university one fall was 117 and the standard deviation was 7. Describe the I.Q. distribution.

12. The arithmetic mean of the weights of 1200 freshmen boys was 158 and the standard deviation was 14. Describe the distribution of the weights.

9-5 SIMPLE CORRELATION

We shall discuss and give a formula for a measure of the relation between two sets of quantities or of the relation of both to a third quantity. This measure as expressed numerically is called the *coefficient of correlation.* A high degree of correlation between two quantities may indicate that they are closely related to each other or that both are closely related to a third quantity. Thus, consistently high grades in German and chemistry might indicate that both are closely related to a high intelligence quotient.

To find a formula for the coefficient of correlation between two sets of items or quantities, we shall represent them by $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_n$ and their arithmetic means by M_x and M_y. Then

$$\begin{aligned}\sigma_x{}^2 &= \frac{(x_1 - M_x)^2 + (x_2 - M_x)^2 + \ldots + (x_n - M_x)^2}{n} \\ &= \frac{X_1{}^2 + X_2{}^2 + \ldots + X_n{}^2}{n} \\ &= \frac{\Sigma X^2}{n}\end{aligned}$$

where $X_1 = x_1 - M_x$, $X_2 = x_2 - M_x, \ldots, X_n = x_n - M_x$ and ΣX^2 represents the sum of the squares of the X's. Similarly

$$\begin{aligned}\sigma_y{}^2 &= \frac{(y_1 - M_y)^2 + (y_2 - M_y)^2 + \ldots + (y_n - M_y)^2}{n} \\ &= \frac{Y_1{}^2 + Y_2{}^2 + \ldots + Y_n{}^2}{n} \\ &= \frac{\Sigma Y^2}{n}\end{aligned}$$

Furthermore, we shall use ΣXY to represent $X_1 Y_1 + X_2 Y_2 + \ldots + X_n Y_n$. Now we can say that

$$r = \frac{\Sigma XY}{n\sigma_x \sigma_y}$$

is called the *coefficient of correlation* of the items $x_1, x_2, \ldots, x_n$ and the items $y_1, y_2, \ldots, y_n$. The correlation is said to be perfect if $r = 1$ or -1; furthermore, its value is never greater than 1 or less than -1. For r between 1 and -1, the correlation between the two sets of items increases as the absolute value of r increases.

Example. Find the coefficient of correlation for the following ages and corresponding incomes. The age is given in years and is followed in the second column by the annual income in hundreds of dollars.

Age x	*Income y*	$X = x - M_x$	$Y = y - M_y$	XY	X^2	Y^2
21	13	−21	−89	1869	441	7921
25	30	−17	−72	1224	289	5184
29	4	−13	−98	1274	169	9604
32	20	−10	−82	820	100	6724
35	28	−7	−74	518	49	5476
40	35	−2	−67	134	4	4489
44	82	2	−20	−40	4	400
48	113	6	11	66	36	121
52	141	10	39	390	100	1521
55	223	13	121	1573	169	14641
58	251	16	149	2384	256	22201
65	284	23	182	4186	529	33124
12⌊504	12⌊1224			14398	2146	111406
$M_x = 42$	$102 = M_y$					

Now

$$\Sigma XY = 14{,}398$$

$$\sigma_x^2 = \frac{\Sigma X^2}{n} = \frac{2146}{12} = 178.83, \ \sigma_x = 13.4$$

$$\sigma_y^2 = \frac{\Sigma Y^2}{n} = \frac{111406}{12} = 9283.83, \ \sigma_y = 96.4$$

Consequently

$$\begin{aligned} r &= \frac{\Sigma XY}{n\sigma_x \sigma_y} \\ &= \frac{14{,}398}{(12)\,(13.4)\,(96.4)} \\ &= \frac{14{,}398}{15501.12} \\ &= .93 \end{aligned}$$

9-6 CORRELATION WITH DATA IN CLASSES

At times the items are sufficiently numerous that a practical procedure in obtaining the coefficient of correlation requires that the items be put in

classes. If this is done each item is used as though its value were the class mark and each is used as many times as it occurs.

Example. Arrange the following data on ages and belt sizes in six classes and then find the coefficient of correlation. The age is given first in each case: 13, 22; 15, 34; 16, 29; 16, 26; 18, 33; 19, 29; 23, 34; 25, 33; 28, 28; 31, 31; 35, 29; 38, 42; 40, 34; 42, 31; 45, 34; 48, 40; 53, 31; 54, 32; 56, 33; 56, 44; 57, 51; 57, 30; 58, 36; 59, 34; 60, 37; 61, 30; 63, 39; 65, 31; 65, 42; 66, 31.

Solution. Since the ages range from 13 to 66 and we are to have six classes, the class width should be about $\frac{1}{6}$(66-13) and we shall use 9. The waist measurements vary from 22 to 51; hence, we should use about $\frac{1}{6}$(51-22) and shall use 5 as the class width. The data will be arranged in tabular form with the ages horizontal and the waist measurements vertical.

	13-21	22-30	31-39	40-48	49-57	58-66
22-26	II					
27-31	II	I	II	I	II	III
32-36	II	II		II	II	II
37-41				I		II
42-46			I		I	I
47-51					I	

We shall now replace this frequency distribution by one that shows each frequency and uses class marks instead of class boundaries. The frequencies are shown in the row and column headed by f_x and f_y. Thus the 6 below 17 and in row with f_x indicates that there were 6 persons in the group with age class mark 17 and the 11 across from 29 and in column with f_y shows that there were 11 persons in the group with waist measure class mark 29.

	17	26	35	44	53	62	f_y	Y	f_yY^2	XY
24	2						2	-10	200	500
29	2	1	2	1	2	3	11	-5	275	-20
34	2	2		2	2	2	10	0	0	0
39				1		2	3	5	75	210
44			1		1	1	3	10	300	240
49					1		1	15	225	165
f_x	6	3	3	4	6	8	30		1075	1095
X	-25	-16	-7	2	11	20				
f_xX^2	3750	768	147	16	726	3200	8607			
XY	750	80	0	0	165	100	1095			

Since the values of M_x and M_y are needed in determining $X = x - M_x$ and $Y = y - M_y$, we shall now evaluate them to the nearest integer. They are

$$M_x = \frac{6(17) + 3(26) + 3(35) + 4(44) + 6(53) + 8(62)}{30} = 42$$

and

$$M_y = \frac{2(24) + 11(29) + 10(34) + 3(39) + 3(44) + 1(49)}{30} = 34$$

It is now a matter of subtraction to obtain the entries in the X row and Y column. The entries in the f_xX^2 row and f_yY^2 column give the indicated products. Each entry in the column headed by XY takes into account the value of Y, its frequency, the distribution of this frequency among the X's, and the values of those X's. Thus, the second entry in the XY column is

$$-20 = -5[2(-25) + 1(-16) + 2(-7) + 1(2) + 2(11) + 3(20)]$$

and the sixth entry in the XY row is

$$100 = 20[3(-5) + 2(0) + 2(5) + 1(10)]$$

The value of XY is computed as a row and as a column in order to have a check on its value.

Now that the table is completed, the work of computing the coefficient of correlation is not difficult but does require that we determine σ_x and σ_y. They are

$$\sigma_x = \sqrt{\frac{\Sigma f_x X^2}{n}} = \sqrt{\frac{8607}{30}} = \sqrt{286.9} = 16.9$$

and

$$\sigma_y = \sqrt{\frac{\Sigma f_y Y^2}{n}} = \sqrt{\frac{1075}{30}} = \sqrt{35.83} = 6.0$$

Therefore

$$r = \frac{\Sigma XY}{n\sigma_x \sigma_y} = \frac{1095}{30(16.9)(6.0)} = .36$$

These figures indicate that there is no appreciable correlation between age and waist size.

Exercise 9-3

Find the coefficient of correlation between the sets of data given in each of Problems 1 through 8.

1. *Year*	1890	1900	1910	1920	1930	1940	1950
Scholastic population/10,000	110	170	218	270	337	340	361
U.S. population/1,000,000	63	76	92	106	123	131	151

2. *Year*	1953	1954	1955	1956	1957
Billions of U.S. income	302	299	324	344	361
Billions of internal revenue	69	70	66	75	80

3. *Year*	1900	1910	1920	1930	1940	1950
Millions of scholastics	22	24	29	32	30	31
Number of teachers/10,000	42	52	68	85	88	91

4. *Year*	1951	1952	1953	1954	1955	1956
Millions of autos in use	52	53	56	59	63	65
Million gals. of alcohol used	618	562	532	438	465	553

5. The I.Q. of each of 20 students is given and is followed by the total number of quality points earned by each in four years: 106, 130; 108, 124; 111, 132; 114, 134; 116, 121; 118, 118; 121, 147; 121, 155; 124, 170; 130, 130; 130, 150; 130, 190; 132, 240; 133, 129; 135, 217; 138, 300; 140, 317; 140, 186; 153, 380; 160, 390.

6. The grade made by each of 20 students on a mathematics test is followed by the grade on a history test. The grades are: 40, 95; 43, 98; 47, 89; 58, 90; 63, 87; 66, 84; 70, 83; 75, 75; 80, 90; 82, 83; 85, 80; 88, 93; 89, 96; 89, 60; 90, 73; 91, 79; 93, 40; 95, 98; 97, 83; 99, 64.

7. The heights and weights of 20 college men are given below. Each height is in inches and is followed by the corresponding weight in pounds. The numbers are: 62, 110; 63, 170; 66, 130; 68, 140; 69, 137; 70, 200; 70, 142; 71, 150; 72, 160; 72, 170; 73, 210; 73, 197; 74, 185; 74, 244; 75, 182; 76, 184; 76, 230; 77, 172; 79, 213; 80, 234.

8. The number of hours studied per week and the number of quality points made in a semester by each of 20 students one semester were: 11, 7; 12, 10; 13, 11; 13, 9; 15, 12; 17, 15; 18, 14; 20, 20; 21, 13; 21, 22; 23, 25; 24, 37; 25, 43; 26, 23; 27, 45; 27, 29; 28, 34; 29, 42; 30, 50; 30, 39.

9. Arrange the data of Problem 5 into a frequency distribution. Use class marks of 11 for the I.Q.'s beginning with 106 and class marks of 55 beginning with 118 for the number of quality points. Use each mean to the nearest integer and find the coefficient of correlation.

10. Arrange the grades of Problem 6 in a frequency distribution with class marks of 9 beginning with 40 for each type of grade. Find the coefficient of correlation. Use the mean to the nearest integer.

11. Arrange the sets of data in Problem 7 in class marks and find the coefficient of correlation. Use class marks of 5 inches and of 35 pounds beginning with the smallest of each measurement. Use each mean to the nearest integer.

12. Arrange the data of Problem 8 in class marks and find the coefficient of correlation. Use class marks of 5 hours and of 11 quality points beginning with 11 hours and 7 quality points. Use each mean to the nearest integer.

10

Simple interest

10-1 TERMINOLOGY

To avoid part of the difficulties often encountered in mathematics of finance, we shall be careful to define a considerable number of words and symbols, and we suggest that the student be equally careful to learn the meaning of each word or symbol defined.

If we borrow money on a commercial basis, we pay for the use of the sum borrowed. The sum that is borrowed is called the *principal*. The payment made for use of the principal is called *interest*. The sum of the principal and

interest is called the *amount* or *accumulated value*. The per cent of the principal that is charged for its use for a unit of time is called the *rate of interest*. We shall always use an annual rate unless otherwise specified. The length of time for which the principal is borrowed is called the *time* or *term* of the loan. The interest charged depends on the principal, the rate of interest, and the time of the loan. *Interest is always based on principal.*

Example. If \$30.00 is paid for the use of \$300.00 for 2 years at 5%, the interest is \$30.00, the principal is \$300.00, the time is 2 years, the rate is 5%, and the amount is \$330.00.

10-2 THE SIMPLE INTEREST FORMULAS

Simple interest is always based on the original principal. If we represent the principal by P, the annual rate by r, the time in years by t, the interest by I, and the amount by S, then

$$I = Prt \tag{1}$$

since, if the interest on \$1.00 for one year is \$$r$, then the interest on \$$P$ for one year is \$$Pr$, and the interest on \$$P$ for t years is \$$Prt$. Furthermore, the amount, or accumulated value, is obtained by adding the principal and interest. Consequently, we have

$$\begin{aligned} S &= P + I \\ S &= P + Prt \quad \text{by use of (1)} \\ S &= P(1 + rt) \quad \text{by factoring out } P \end{aligned} \tag{2}$$

(The student should realize that these are merely three forms of the same formula and not three formulas.)

Example. Find the interest and accumulated value if one borrows \$450.00 for 1 year and 8 months at 4% per year.

Solution. In this problem, we have $P = \$450.00$, $t = 1\frac{2}{3}$ years, and $r = 4\%$ per year; hence, $I = Prt$ becomes

$$\begin{aligned} I &= \$450\,(.04)\,(\tfrac{5}{3}) \\ &= \$30.00. \end{aligned}$$

Consequently

$$\begin{aligned} S &= P + Prt \\ &= \$450 + \$30 \\ &= \$480 \end{aligned}$$

10-3 ORDINARY AND EXACT INTEREST

If the time involved is measured in days, there are several methods that may be used in computing interest. If a year is considered as 365 days, the interest is called *exact interest*. If a year is considered as 360 days, the interest is called *ordinary interest*.

Not only are there two numbers used as denominators in computing the fractional part of a year but there are also two methods of determining the numerator. One of them consists of counting the exact number of days between the two dates and is called *exact time*. The other uses each month as though it were 30 days regardless of the number of days actually in the month. If each month is counted as 30 days, the time is called *approximate time*. In determining the fractional part of a year, either of these numerators may be used with either denominator. Hence, there are four ways of computing fractional parts of a year:

(1) I_o, exact time. This indicates ordinary interest for exact time; that is, the time figured is $\frac{1}{360}$ of the exact number of days. (This is called the "Banker's Rule.")

(2) I_e, exact time. This indicates exact interest for exact time; that is, the time is figured as $\frac{1}{365}$ of the exact number of days.

(3) I_o, approximate time. This indicates ordinary interest for approximate time; that is, the time is figured as $\frac{1}{360}$ of the approximate number of days.

(4) I_e, approximate time. This indicates exact interest for approximate time; that is, the time is figured as $\frac{1}{365}$ of the approximate number of days.

The table given on the next page shows the number of the day of the year for each date except that, for leap years, each number for a date after February 28 must be increased by one.

Example 1. The number of days from May 29 until October 7 is 131 since May 29 is the 149th day of the year and October 7 is the 280th.

Example 2. The number of days from November 15, 1961, until February 10, 1962, was 87 since 46 days (365 − 319) remain in 1961 and February 10 is the 41st day of 1962. Notice that the first day of the period is counted but the last is not.

NUMBER OF EACH DAY OF THE YEAR												
Day of Month	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1	1	32	60	91	121	152	182	213	244	274	305	335
2	2	33	61	92	122	153	183	214	245	275	306	336
3	3	34	62	93	123	154	184	215	246	276	307	337
4	4	35	63	94	124	155	185	216	247	277	308	338
5	5	36	64	95	125	156	186	217	248	278	309	339
6	6	37	65	96	126	157	187	218	249	279	310	340
7	7	38	66	97	127	158	188	219	250	280	311	341
8	8	39	67	98	128	159	189	220	251	281	312	342
9	9	40	68	99	129	160	190	221	252	282	313	343
10	10	41	69	100	130	161	191	222	253	283	314	344
11	11	42	70	101	131	162	192	223	254	284	315	345
12	12	43	71	102	132	163	193	224	255	285	316	346
13	13	44	72	103	133	164	194	225	256	286	317	347
14	14	45	73	104	134	165	195	226	257	287	318	348
15	15	46	74	105	135	166	196	227	258	288	319	349
16	16	47	75	106	136	167	197	228	259	289	320	350
17	17	48	76	107	137	168	198	229	260	290	321	351
18	18	49	77	108	138	169	199	230	261	291	322	352
19	19	50	78	109	139	170	200	231	262	292	323	353
20	20	51	79	110	140	171	201	232	263	293	324	354
21	21	52	80	111	141	172	202	233	264	294	325	355
22	22	53	81	112	142	173	203	234	265	295	326	356
23	23	54	82	113	143	174	204	235	266	296	327	357
24	24	55	83	114	144	175	205	236	267	297	328	358
25	25	56	84	115	145	176	206	237	268	298	329	359
26	26	57	85	116	146	177	207	238	269	299	330	360
27	27	58	86	117	147	178	208	239	270	300	331	361
28	28	59	87	118	148	179	209	240	271	301	332	362
29	29		88	119	149	180	210	241	272	302	333	363
30	30		89	120	150	181	211	242	273	303	334	364
31	31		90		151		212	243		304		365

Example 3. If one borrows \$730 at 4% per year on Feburary 15, 1961, and repays it on May 21, 1961, find the four types of interest described above.

Solution. The exact number of days is 95 since February 15 is the 46th day of the year and May 21 is the 141st. Hence, if exact time is used, we have $t = 95$ days. Consequently

$$I_o, \text{ exact time} = \$730\ (.04)\frac{95}{360} = \$7.71$$

$$I_e, \text{ exact time} = \$730\ (.04)\ \frac{95}{365} = \$7.60$$

The approximate time is 3 months and 6 days, since the loan is made on February 15 and repaid on May 21. Hence, if approximate time is used, we have $t = 96$ days. Consequently

$$I_o, \text{ approximate time} = \$730\ (.04)\ \frac{96}{360} = \$7.79$$

$$I_e, \text{ approximate time} = \$730\ (.04)\ \frac{96}{365} = \$7.68$$

Regardless of whether exact or approximate time is used, we have

$$I_e = \frac{(Pr)\ (\text{number of days})}{365}$$

$$I_o = \frac{(Pr)\ (\text{number of days})}{360}$$

Hence, if the same method of counting time is used in both cases, we have

$$\frac{I_e}{I_o} = \frac{(Pr)\ (\text{number of days})/365}{(Pr)\ (\text{number of days})/360} = \frac{1/365}{1/360} = \frac{360}{365} = \frac{72}{73}$$

This equation gives the relation between exact and ordinary interest and can be used to find either if the other is known. It shows that, for a given method of counting time, exact interest is always less than ordinary interest.

Since all months except February have 30 or 31 days, the exact time is greater than or equal to the approximate time provided the latter part of February is not included in the time. If this period is included, the exact time may be greater than, equal to, or less than the approximate time.

Example 4. If three time intervals begin on February 15 and end on August 15, July 15, and May 15, the approximate times are 6 months =

180 days, 5 months = 150 days, and 3 months = 90 days, respectively. Except in leap years, the corresponding exact times are 181 days, 150 days, and 89 days, respectively.

Exercise 10-1

Find the interest and amount in each of the following problems:

	Principal	*Time*	*Rate*
1.	$1026.42	6 months	.06
2.	$ 122.30	3 months	.055
3.	$ 835.14	8 months	.04
4.	$ 400.00	5 months	.07

Compute the number of days between the following dates, using exact time and approximate time. (Remember that 1964 is a leap year.)

5. Feb. 15, 1962—June 15, 1962
6. Oct. 12, 1962—Feb. 19, 1963
7. Jan. 4, 1964—Apr. 30, 1964
8. Nov. 25, 1963—May 18, 1964

Find the interest, by four methods, in Problems 9 through 12.

	Principal	*Period*	*Rate*
9.	$1000	Feb. 15, 1962—June 15, 1962	$6\frac{1}{2}\%$
10.	$1550	Oct. 12, 1962—Feb. 19, 1963	5 %
11.	$ 490.12	Jan. 4, 1962—Apr. 30, 1962	4 %
12.	$ 530.25	Nov. 25, 1962—May 18, 1963	$3\frac{1}{2}\%$

If the *exact* interest on each debt is as given, find the corresponding *ordinary* interest in each of Problems 13 through 16.

13. $17.28
14. $162.19
15. $322.30
16. $90.80

If the *ordinary* interest on each debt is as given, find the corresponding *exact* interest in each of Problems 17 through 20.

17. $17.28
18. $162.19
19. $322.30
20. $90.80

In each of Problems 21 through 24, find the interest and the amount for the conditions listed.

	Principal	*Period of investment*	*Rate*
21.	$1025.42	90 days	5 %
22.	$ 750	6 months	$4\frac{1}{2}\%$
23.	$5268.19	30 days	3 %
24.	$2500	8 months	6 %

10-4 SOLUTION FOR P, r, and t

The student should realize that formula (1) of Section 10-2 is an equation in the four unknowns I, P, r, and t and that it can be solved for any one of these quantities provided the other three are known. The formula is

$$I = Prt \tag{1}$$

We shall not solve it for P, r, and t since to do so would be to invite the student to memorize four forms of the same formula and to think of them as four different formulas. Instead of doing this, we shall give three examples that illustrate how formula (1) can be used to find P, r, or t.

Example 1. How long must one leave \$300 invested in order to earn \$28 interest at 3% per year?

Solution. Substituting in (1), we have

$$\begin{aligned} \$28 &= \$300\,(.03)\,t \\ &= \$9\,t \end{aligned}$$

Hence, dividing by the coefficient of t, we get

$$t = 3\tfrac{1}{9}$$

Therefore the time is $3\frac{1}{9}$ years since the rate is for a year.

Example 2. At what rate will \$150 produce interest of \$20.25 in $4\frac{1}{2}$ years?

Solution. Substituting in (1), we have

$$\begin{aligned} \$20.25 &= \$150\,r\,(4.5) \\ r &= \frac{\$20.25}{(\$150)\,(4.5)} \\ r &= \frac{\$20.25}{\$675} \\ &= 3\% \end{aligned}$$

This is the annual rate since the time is given in years.

Example 3. What principal is required to produce interest of \$38.50 in two years at $3\frac{1}{2}\%$ per year?

Solution. Substituting in (1), we have

$$\$38.50 = P(.035)\,(2)$$

Consequently

$$P = \frac{\$38.50}{(.035)\,(2)}$$

$$= \$550.00$$

Exercise 10-2

1. Solve the equation $I = Prt$ for P, r, and t.

Find the value of the one of I, P, r, or t that is missing in each problem.

	Principal	*Interest*	*Time*	*Rate*
2.	$ 541.20	$18.95	6 months	?
3.	139.57	1.38	4 months	?
4.	1042.27	20.85	3 months	?
5.	2000.00	21.67	60 days	?
6.	?	4.83	2 months	.07
7.	?	17.08	5 months	.06
8.	?	27.51	30 days	.055
9.	?	17.50	45 days	.09
10.	496.51	14.90	?	.06
11.	812.92	30.48	?	.0375
12.	1184.67	26.66	?	.045
13.	2155.80	28.74	?	.08

The following is an extract from a table printed on a United States Savings Bond which cost $75 and may be redeemed at the sum indicated on the appropriate date. Compute the simple interest rate earned if the bond is sold at any one of the indicated times.

	At the end of	*Redemption price*		*At the end of*	*Redemption price*
14.	2 years	$79.24	18.	6 years	$90.72
15.	3 years	82.00	19.	7 years	93.76
16.	4 years	84.84	20.	$8\frac{1}{2}$ years	98.44
17.	5 years	87.76	21.	$8\frac{11}{12}$ years	100.00

10-5 SIMPLE DISCOUNT—PRESENT VALUE

The principal that must be invested at a given rate for a given time in order to produce a definite amount or accumulated value is called the *present value*. Formula (2) of Section 10-2 is

$$S = P(1 + rt) \tag{2}$$

and it gives the relation between the four quantities, S, P, r, and t. Hence, if any three of them are known, the other one can be found. In particular, we can solve for P if S, r, and t are known. Doing this, by dividing each member of (2) by $(1 + rt)$, we have

$$P = \frac{S}{1 + rt} \tag{2'}$$

This is not a new formula but merely a variation of (2). If P is found by means of (2′), we say that S has been *discounted*. The difference between S and P is called the *simple discount* on S and is the same as the simple interest on P.

Example. What is the present value of \$645 due in $2\frac{1}{2}$ years if the interest rate is 3%? What is the simple discount?

Solution. We can use (2) or (2′) since each gives the relation between S, P, r, and t. We shall use (2) since (2′) is merely a variation of it. Substituting $S = \$645$, $r = .03$, and $t = 2.5$ in (2) gives

$$\begin{aligned} \$645 &= P[1 + (.03)\ (2.5)] \\ &= P(1.075) \end{aligned}$$

Hence

$$\begin{aligned} P &= \frac{\$645}{1.075} \\ &= \$600.00 \end{aligned}$$

and the simple discount is $\$645 - \$600 = \$45$

Exercise 10-3

By use of (1) and (2), solve for the missing quantities.

	Present value	*Simple discount*	*Accumulated value*	*Rate*	*Time*
1.	\$ 400.00	\$ 18.00	?	?	$1\frac{1}{2}$ years
2.	600.00	60.00	?	?	$2\frac{1}{2}$ years

	Present value	*Simple discount*	*Accumulated value*	*Rate*	*Time*
3.	600.00	60.00	?	4%	?
4.	500.00	52.50	?	6%	?
5.	?	126.00	$1026.00	?	$3\frac{1}{2}$ years
6.	?	31.50	256.50	?	$3\frac{1}{2}$ years
7.	?	126.00	1026.00	4%	?
8.	?	28.00	406.00	2%	?
9.	474.81	?	481.93	?	$\frac{1}{4}$ year
10.	790.29	?	795.56	?	$\frac{1}{6}$ year
11.	4827.93	?	4850.06	$5\frac{1}{2}$%	?
12.	2510.14	?	2566.62	$4\frac{1}{2}$%	?

10-6 NOTES—BANK DISCOUNT

A *promissory note* is a promise to pay a certain sum on a specified date. The sum that is borrowed is called the *face* of the note and is the amount written on the note. The value to which this accumulates on the day the note is due is called the *maturity value*. If the note bears no interest, the maturity value is the same as the face of the note. If the note bears interest, the maturity value is the sum of the principal and the interest.

It is not unusual when borrowing money from a bank to be required to pay a charge based on the total amount that is to be repaid instead of on the sum that is borrowed. If the maturity value, instead of the principal, is used in determining the charge for the use of the money, we say that the note is *discounted*. The amount charged is called the *bank discount*. This charge is deducted from the maturity value of the note, and hence is actually never received by the borrower. The rate used in discounting the note is called the *discount rate*. The sum of money received by the borrower is the difference between the maturity value and the bank discount, and is called the *proceeds* of the note. The *term* of the discount is the time between the date the note is discounted and the maturity date. To determine the time, the practice is to use the exact number of days as the numerator and 360 as the denominator of the fraction (Banker's Rule). *Bank discount is always based on maturity value.* Simple interest and bank discount are computed in the same way but the interest rate is always higher than the discount rate since interest is based on the face of the note whereas discount is based on maturity value.

Thus, if we represent

maturity value by S
proceeds by P_b
time of discount by t years
rate of discount per year by d
bank discount by D, then

(3) $$D = Sdt$$

and

(4) $$P_b = S - D$$

The form can be changed to

$$P_b = S - Sdt \quad \text{by use of (3)}$$

and to

$$P_b = S(1 - dt) \quad \text{by factoring out } S$$

Formulas (3) and (4) are very similar to (1) and (2) of Section 10-2, but must not be confused with them. If the student remembers that interest is based on the principal, and bank discount on the maturity value, confusion is not likely to occur. As in the case of interest rates, all discount rates in this book will be on an annual basis unless otherwise specified.

Example 1. Find the bank discount and proceeds on a note whose maturity value is \$480.00 if discounted at 4% ninety days before it is due.

Solution. Since bank discount is based on maturity value, the rate of discount is 4%, and the time is $90/360 = \frac{1}{4}$ year. Formula (3) becomes

$$\begin{aligned} D &= \$480(.04)\left(\tfrac{1}{4}\right) \\ &= \$4.80 \end{aligned}$$

Consequently, by use of (4), we have

$$\begin{aligned} P_b &= S - D \\ &= \$480 - \$4.80 \\ &= \$475.20 \end{aligned}$$

Example 2. Find the bank discount and proceeds on a 120-day note for \$720 bearing 5% interest if discounted at 4% 90 days before it is due.

Solution. We must find the accumulated value before we can find the bank discount; hence, we shall start out by determining the interest.

$$
\begin{aligned}
I &= Prt \\
&= \$720(.05)\ (1/3) \\
&= \$12.00
\end{aligned}
$$

Hence

$$
\begin{aligned}
S &= P + I \\
&= \$720 + \$12 \\
&= \$732
\end{aligned}
$$

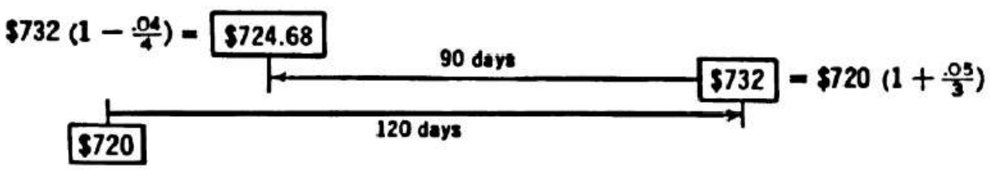
and

$$
\begin{aligned}
D &= Sdt \\
&= (\$732)\ (.04)\ (\tfrac{1}{4}) \\
&= \$7.32
\end{aligned}
$$

Therefore

$$
\begin{aligned}
P_b &= S - D \\
&= \$732 - \$7.32 \\
&= \$724.68
\end{aligned}
$$

The following diagram may help some students to visualize the situation that is involved in this problem:



Example 3. A note for \$540 is made out on April 16, is due on July 15, and bears interest at 5%. Find the discount and proceeds if it is discounted on June 25 at 6%.

Solution. The term of the note is 90 days or $\frac{1}{4}$ year since 14 days remain in April, there are 31 in May, 30 in June, and the note is due on July 15. Hence

$$
\begin{aligned}
I &= Prt \\
&= \$540(.05)\ (\tfrac{1}{4}) = \$6.75
\end{aligned}
$$

Therefore

$$
\begin{aligned}
S &= P + I \\
&= \$546.75
\end{aligned}
$$

and

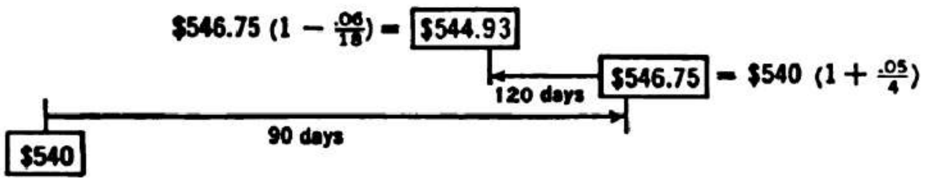
$$
\begin{aligned}
D &= Sdt \\
&= (\$546.75)\ (.06)\ (\tfrac{20}{360}) = \$1.82
\end{aligned}
$$

since it is discounted for 5 days in June and 15 days in July.

Finally

$$
\begin{aligned}
P_b &= S - D \\
&= \$546.75 - \$1.82 = \$544.93
\end{aligned}
$$

The problem is shown in diagram form below.



Example 4. Mr. Jones went to a bank to borrow \$1000 for 3 months and found that the bank charged an 8% discount rate. Since he needed \$1000 cash, he had to make the note for more than that. What was the face of the note?

Solution. In this problem, $P_b = \$1000$, $d = 8\%$, $t = \frac{1}{4}$, and we are to find S. Hence

$$P_b = S(1 - dt)$$

becomes

$$\$1000 = S\left(1 - \frac{.08}{4}\right) = .98S$$

and

$$S = \frac{\$1000}{.98} = \$1020.41$$

The reader will notice that if the formula $P_b = S(1 - dt)$ is solved for S, by dividing by $(1 - dt)$, we obtain

$$S = \frac{P_b}{1 - dt} \tag{4'}$$

Hence, the solution of problems such as Example 4 where P_b, d, and t are known can always be found by substituting directly in formula (4′).

10-7 INTEREST RATE AND THE CORRESPONDING BANK DISCOUNT RATE

It seems fairly obvious that the interest rate necessary to produce a given amount of interest should be higher than the discount rate necessary to produce the same amount of discount provided the principal and proceeds are equal. We shall now attack the problem of determining a relation between r, t, and d.

In order to do this, we shall use (2) and (4) with $P_b = P$ since we want the proceeds on S at the discount rate d to be equal to the principal that produces S at the interest rate r for all values of t. The equations referred to by number above are

$$S = P(1 + rt) \tag{2}$$

$$P = S(1 - dt) \tag{4}$$

since $P_b = P$.

If we solve (2) for P, we get

$$P = \frac{S}{1 + rt}$$

Hence, equating this expression for P and that given by (4), we have

$$\frac{S}{1 + rt} = S(1 - dt)$$

Consequently

$$\begin{aligned}
\frac{1}{1 + rt} &= (1 - dt) && \text{dividing by } S \\
1 &= (1 + rt)\,(1 - dt) && \text{multiplying by } 1 + rt \\
&= (1 - dt) + rt(1 - dt) && \text{expanding} \\
-rt(1 - dt) &= -dt && \text{collecting and transposing} \\
r(1 - dt) &= d && \text{dividing by } -t
\end{aligned}$$

Finally, dividing by $(1 - dt)$, we have

$$r = \frac{d}{1 - dt} \tag{5}$$

It is now easy to see that the interest rate that corresponds to a given discount rate is greater than the discount rate.

The relation (5) can be solved for d in terms of r and t, but that would give an unnecessary formula to be remembered. Instead of doing this we advise the student to know (5) and be able to solve it for d as often as necessary. We should notice that r and d are very nearly equal if t is small and that the r which corresponds to a given d increases rapidly as t increases.

Example 1. What interest rate corresponds to $d = 6\%$ if (a) $t = 6$ months; (b) $t = 2$ years?

Solution. (a) By substituting $d = .06$ and $t = \frac{1}{2}$ in (5), we get

$$r = \frac{.06}{1 - .06(\frac{1}{2})}$$
$$= \frac{.06}{.97}$$
$$= 6.19\%$$

(b) By putting $d = .06$ and $t = 2$ in (5), we have

$$r = \frac{.06}{1 - (.06)\,(2)}$$
$$= \frac{.06}{.88}$$
$$= 6.82\%$$

Example 2. What discount rate is equivalent to an interest rate of 5% if $t = 3$ months?

Solution. If we substitute $r = .05$ and $t = \frac{1}{4}$ in (5), we get

$$.05 = \frac{d}{1 - \frac{1}{4}d}$$
$$.05(1 - \tfrac{1}{4}d) = d$$
$$.05 - .0125\,d = d$$
$$.05 = 1.0125\,d$$
$$d = \frac{.05}{1.0125}$$
$$= 4.94\%$$

Exercise 10-4

Determine the missing quantities in Problems 1 through 8.

	d	r	t		d	r	t
1.	.06	?	$\frac{1}{2}$	5.	?	.05	$\frac{1}{4}$
2.	.08	?	$\frac{1}{3}$	6.	?	.06	1
3.	.04	?	$\frac{1}{4}$	7.	?	.035	$\frac{1}{6}$
4.	.05	?	$\frac{1}{3}$	8.	?	.045	$\frac{1}{3}$

9. Jones signs a 90-day note for $1000. The bank charges .08 discount. What are the proceeds of the note?

10. A note for \$500 due in 90 days with interest at .06 is dated November 1, 1964. The note was sold on January 15, 1965, to a bank, which discounted it at .08. What did the seller receive for the note?

11. A note for \$250 dated June 15 is due September 13. It draws interest at .05. It was discounted July 10 at .06. What were the proceeds?

12. A non-interest-bearing note for \$540.26 due July 1 was discounted on May 15 at 5%. What were the proceeds?

13. Smith needs \$980 to complete payment on an automobile. A bank which charges 6% discount agrees to lend him the money for 60 days. For how much does the bank make the note? What interest rate is Smith paying?

14. A note for \$1000 dated August 1 is due October 30. It draws interest at 6%. On September 15, I am offered \$1010 for the note. What interest rate will I earn on my money if I sell it?

15. The maturity value of a note is \$1250. Thirty days before maturity date it is sold for \$1243.75. Find the rate at which it was discounted.

16. A 90-day note for \$2500 bears interest at 8%. Forty-five days before it is due it is sold for \$2511.75. Find the discount rate.

10-8 PERPETUITIES

A sequence of equal payments at equal intervals and continuing (theoretically) forever is called a *perpetuity*. There is no such thing as the accumulated or maturity value of a perpetuity since there is no last payment. The investment that must be made at a given rate of interest to furnish a given return at the end of each period forever is called the *present value* of the perpetuity. We shall now obtain a formula for the present value of a perpetuity. If we invest \$$P$ now at rate r per period, the interest at the end of each period is $I = Pr$ since t is unity. If we solve this for P, we see that

$$P = \frac{I}{r} \tag{6}$$

is the present value of a perpetuity of I at the end of each period if money is worth r per period.

Example 1. Find the present value of a perpetuity of \$75 at the end of each year if money is worth 3% per year.

Solution. If we use $I = \$75$ and $r = .03$, the formula for the present value of a perpetuity gives

$$P = \frac{\$75}{.03}$$
$$= \$2500$$

Example 2. A cemetery advertises perpetual care on all its lots. If the upkeep on a lot costs \$6 per year, how much should be charged per lot for maintenance if money can be invested at 4%?

Solution. In this problem, assuming that the payments are made at the end of each year, $I = \$6$ and $r = .04$. Substituting in the formula for the present value of a perpetuity, we have

$$P = \frac{\$6}{.04} = \$150$$

Hence, the amount charged for maintenance should be \$150.

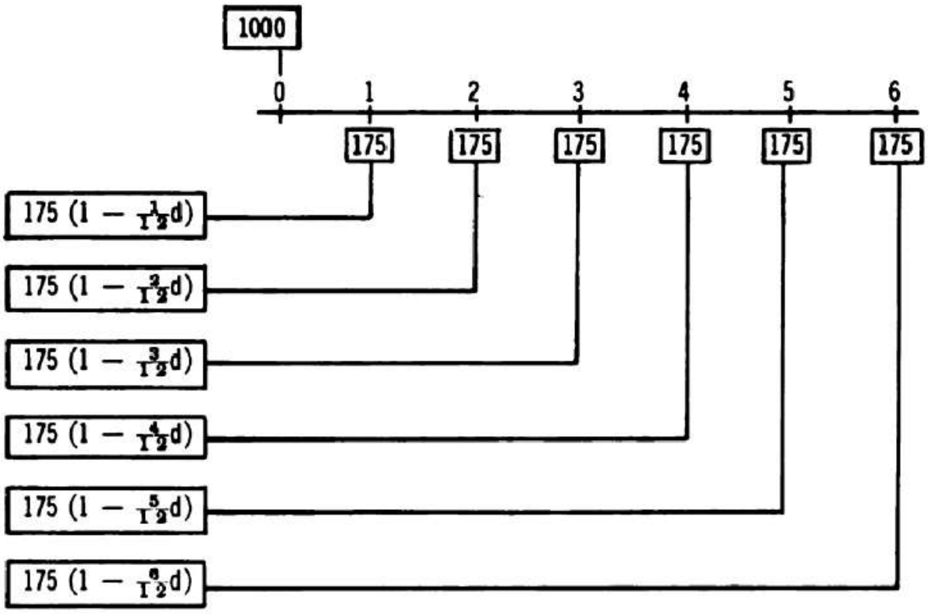
10-9 INSTALLMENT PAYMENTS

It is not unusual for us to want to have possession of an article when we do not have money to pay for it. Further, some of us often borrow money from a bank or a finance company to liquidate past obligations. This method of procedure has the advantage of permitting us to have desired articles when we have less cash than the price of the articles, and has the disadvantage of causing us to pay a high rate of discount or interest. The seller is often justified in the rate charged because it includes the expenses entailed in bookkeeping, and loss of discount or interest, as well as possible loss of principal in case payments are not completed.

Under such circumstances the dealer or bank or finance company may add a sum to the balance due or the face of the loan. This sum is sometimes called a *carrying charge* and sometimes called *interest*. The total is then divided into a specified number of installments which are due at equal intervals of time. We shall consider the sum of the discounted values of the installment payments to be equal to the balance due.

Example 1. On a personal loan of \$1000, a bank adds a charge of 5%. The total of \$1050 is then divided into 6 equal monthly installments of \$175 which are to be paid at the end of each month. What rate of discount is being charged?

Solution. In order to solve this problem we will set up an equation which states that, on the date the loan was made, the amount borrowed is equal to the sum of the discounted values of the installment payments.



Each payment has been discounted by using the proper time in the equation $P = S(1 - dt)$. Hence, equating the value of the original debt and the discounted values of the payments, we have

$$\begin{aligned} 1000 &= 175(1 - \frac{1}{12}d) + 175(1 - \frac{2}{12}d) + 175(1 - \frac{3}{12}d) \\ &\quad + 175(1 - \frac{4}{12}d) + 175(1 - \frac{5}{12}d) + 175(1 - \frac{6}{12}d) \\ &= 6(175) - \frac{175d}{12}(1 + 2 + 3 + 4 + 5 + 6) \\ &= 1050 - \frac{175d}{12}(21) \end{aligned}$$

Therefore, transposing all terms except 1050 and reducing $\frac{21}{12}$ to 1.75, we get

$$(175)d\,(1.75) = 50$$

Hence $$306.25d = 50$$

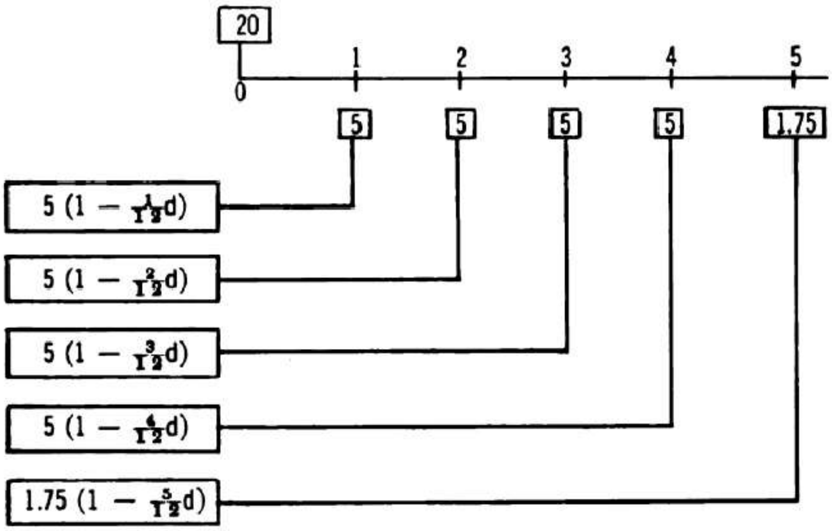
$$d = \frac{50}{306.25} = .16326 \text{ or } 16.33\%$$

If the equivalent rate of interest is desired, it may be found by substituting in the formula $r = \dfrac{d}{1 - dt}$ where $d = .16326$ and $t = \frac{1}{2}$. Thus

$$r = \frac{.16326}{1 - (.16326)(\frac{1}{2})} = \frac{.16326}{1 - .08163} = \frac{.16326}{.91837} = .1778$$
$$= 17.78\%$$

Example 2. On a balance of \$20 a store adds a carrying charge of \$1.75, and requires the customer to make a payment of \$5 at the end of each of the first four months and a final payment of \$1.75 at the end of the fifth month. Find the rate of discount charged.

Solution.



We discount each payment to the date of purchase and equate the value of the debt and the discounted value of the payments. Thus

$$20 = 5(1 - \frac{1}{12}d) + 5(1 - \frac{2}{12}d) + 5(1 - \frac{3}{12}d) + 5(1 - \frac{4}{12}d)$$
$$+ 1.75(1 - \frac{5}{12}d)$$

$$20 = 21.75 - \frac{5d}{12}(1 + 2 + 3 + 4) - \frac{8.75}{12}d \quad \text{removing parentheses and collecting}$$

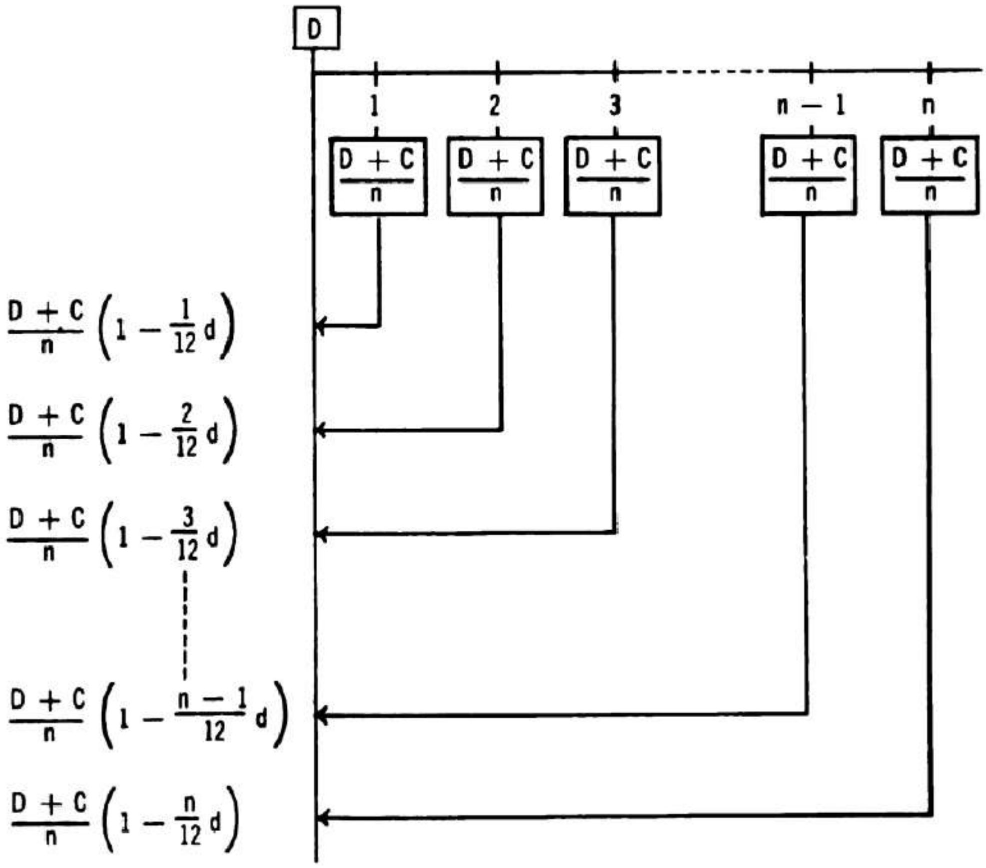
$$\frac{50}{12}d + \frac{8.75}{12}d = 1.75 \quad \text{transposing}$$

$$\frac{58.75}{12}d = 1.75 \quad \text{collecting}$$

Hence $\quad d = \frac{21}{58.75} = .3574 = 35.74\%$

If we represent the amount borrowed or owed by D, the carrying charge or interest added by C, and the number of equal monthly payments by n, we can obtain a formula for d in the following manner.

Since $\frac{D + C}{n}$ is the amount of each payment, we can make the following diagram for the equation of value.



Hence

$$D = \frac{D+C}{n}\left(1 - \frac{1}{12}d\right) + \frac{D+C}{n}\left(1 - \frac{2}{12}d\right) + \frac{D+C}{n}\left(1 - \frac{3}{12}d\right) +$$
$$\ldots + \frac{D+C}{n}\left(1 - \frac{n-1}{12}d\right) + \frac{D+C}{n}\left(1 - \frac{n}{12}d\right)$$

Since there are n terms on the right, the equation becomes

$$D = n\left(\frac{D+C}{n}\right) - \left(\frac{D+C}{n}\right)\left(\frac{d}{12}\right)[1 + 2 + 3 + \ldots + (n-1) + n]$$

The numbers in the brackets are the terms of an arithmetic progression which can be summed by the formula $s = \frac{n}{2}(a + l)$ where s is the sum, n is the number of terms, a is the first term and l the *nth* term. The sum is

$$s = \frac{n}{2}(1 + n) = \frac{n(n+1)}{2}$$

Substituting this sum for the quantity in the brackets, we have

$$D = D + C - \left(\frac{D+C}{n}\right)\left(\frac{d}{12}\right)\frac{(n)(n+1)}{2}$$

$$0 = C - \left(\frac{D+C}{24}\right)(n+1)d$$

Solving for d, we have

$$d = \frac{24C}{(n+1)(D+C)}$$

Example 3. To an unpaid balance of \$50, a store adds a carrying charge of \$5. The total of \$55 is divided into 11 monthly installments of \$5 each. Use the formula to find the discount rate that is charged.

Solution. In this problem $D = 50$, $C = 5$, $n = 11$. Hence

$$d = \frac{24(5)}{12(50+5)} = \frac{120}{660} = .1818 = 18.18\%$$

Exercise 10-5

1. Find the present value of a perpetuity of \$1000 per year if money is worth 4%; if money is worth 5%.

2. Find the present value of a perpetuity of \$2500 per year if money is worth $3\frac{1}{2}\%$; if money is worth $4\frac{1}{2}\%$.

3. Find the present value of a perpetuity of \$1500 per year if money is worth $2\frac{1}{2}\%$; if money is worth 6%.

4. Find the present value of a perpetuity of \$5000 per year if money is worth 2%; if money is worth 8%.

5. Find the present value of a perpetuity of \$6000 per year if money is worth 3%; if money is worth 5%.

6. It costs \$100,000 per year for a university to maintain its buildings and grounds. How much endowment should it have to take care of this expense if funds are kept invested at $2\frac{1}{2}\%$?

7. A charity bed in a hospital costs \$1400 per year. How much money should be put in trust to endow 10 beds if the hospital can invest the money at 4%?

8. A patron of a university wishes to endow a professorship. The annual income necessary is \$10,000. If the institution can invest its endowment funds at 4%, how much must the patron donate?

An automobile finance company lends money on the following plan: One-half of 1 % per month is added to the original principal for as many months as the borrower wishes to take to pay for his car. The amount obtained is then divided into equal monthly installments, to be paid at the end of each month. Find the rate of discount on each of the following loans; then find the rate of interest.

	Principal	*Time*		*Principal*	*Time*
9.	\$ 800	6 months	11.	\$1600	15 months
10.	\$1200	12 months	12.	\$2000	24 months

Following is a copy of a loan company advertisement that appeared in a Dallas newspaper. Find the rate of discount that is being charged for each loan.

	Amount of loan	*Twelve-months plan total cost for 1 year (Deducted from amount of loan)*	*Monthly payment*
13.	\$ 96.00	\$11.60	\$ 8.00
14.	168.00	20.80	14.00
15.	360.00	44.00	30.00
16.	600.00	72.00	50.00

The Atlantic Finance Company advertises rates on loans as shown at the top of the next page. Find the discount rate charged in each case. Then find the rate of interest.

	Amount borrowed	*Fifteen monthly payments*
17.	\$ 80.00	\$ 6.15
18.	100.00	7.68
19.	150.00	11.52
20.	200.00	15.36
21.	300.00	23.04
22.	500.00	38.39
23.	950.00	72.94
24.	1200.00	92.00

A large retail credit store requires a down payment on each purchase. A carrying charge is added to the balance and payments required according to the following table. Find the discount rate earned by the company in each case if the final payment is made one month after the last regular monthly payment.

	Balance	*Carrying charge*	*Number and amount of monthly payments*	*Final payment*
25.	\$ 15.00	\$ 1.75	3 @ \$ 5	\$ 1.75
26.	25.00	2.60	5 @ \$ 5	2.60
27.	37.50	4.00	8 @ \$ 5	1.50
28.	45.00	4.50	9 @ \$ 5	4.50
29.	52.50	5.50	9 @ \$ 6	4.00
30.	65.00	6.50	10 @ \$ 7	1.50
31.	85.00	8.50	11 @ \$ 8	5.50
32.	115.00	12.00	12 @ \$10	7.00
33.	145.00	15.00	14 @ \$11	6.00
34.	175.00	19.00	14 @ \$13	12.00
35.	200.00	21.00	15 @ \$14	11.00
36.	250.00	28.00	16 @ \$17	6.00

SUMMARY

The following symbols and definitions are used in this chapter:

P = principal
r = rate of interest
t = time
I = interest
S = amount

d = rate of bank discount
D = bank discount
P_b = proceeds
I_o = ordinary interest
I_e = exact interest

These symbols are related by the following equations:

Simple interest	*Bank discount*
$I = Prt$	$D = Sdt$
$S = P + I$	$P_b = S - D$
$S = P(1 + rt)$	$P_b = S(1 - dt)$
$P = \dfrac{S}{1 + rt}$	$S = \dfrac{P_b}{1 - dt}$

The simple interest rate and bank discount rate are connected by the following relation:

$$r = \frac{d}{1 - dt}$$

The relation between ordinary and exact interest is given by:

$$I_e = \frac{72}{73} I_o$$

If a debt is paid off in equal monthly installments, and if:

D = amount of debt
C = carrying charge
n = number of monthly payments
d = rate of discount

then

$$d = \frac{24\,C}{(n + 1)\,(D + C)}$$

Exercise 10-6 (Review)

1. Find the interest and amount in each of the following problems:

	Principal	*Time*	*Rate*		*Principal*	*Time*	*Rate*
(a)	$2016.28	7 mo.	.035	(c)	$750.00	3 yr.	.06
(b)	324.50	2 mo.	.08	(d)	986.30	4 mo.	.05

2. Find, by four methods, the interest on $128.50 from July 1, 1963, to Dec. 15, 1963 if $r = 4\frac{1}{2}\%$.

3. Mr. Torrence wishes to obtain \$1475 cash for 60 days from a bank that charges 8% discount. For what amount should he make the face of the note?

4. On April 1, 1963, Mr. George Taylor signed a 90-day 6% interest-bearing note for \$1000. (a) Find the maturity value. (b) On May 15, this note was discounted by a bank that charged a 6% discount rate. Find the proceeds.

5. A non-interest-bearing note for \$425 is due Dec. 15, 1964. It was discounted on Nov. 1, 1964. The proceeds were \$415. (a) Find the discount rate. (b) Find the corresponding interest rate.

6. On Nov. 1, 1962, Mr. Stephens bought 100 shares of common stock at \$2 per share. He sold this stock at \$5.50 per share on May 15, 1963. What rate of interest did he earn on his investment?

An automobile company advertises that it will repair your car and let you pay the bill in monthly installments. Compute the rate of discount charged by the company in the following instances:

	Amount of repairs	*Monthly payments*
7.	\$ 50	6 @ \$ 8.57
8.	\$100	8 @ \$13.22
9.	\$150	12 @ \$13.60
10.	\$175	12 @ \$16.84
11.	\$200	12 @ \$18.09

12. A sum of money doubles itself in 15 years at simple interest. Find the simple-interest rate.

13. A note for \$600 is due in 8 months, another note of \$575 is due in 2 months. If money is worth 5% simple interest, compare the value of the two notes (a) 3 months from now; (b) 9 months from now; (c) now.

What simple interest rates correspond to the following bank discount rates for the times indicated?

	d	t
14.	.03	60 days
15.	.05	90 days
16.	.065	1 year
17.	.08	6 months
18.	.06	120 days

What bank discount rates correspond to the following simple interest rates for the times indicated?

	r	t
19.	.03	60 days
20.	.05	90 days
21.	.065	1 year
22.	.08	6 months
23.	.06	120 days

Find the rate of discount on the following:

	Unpaid balance	*Carrying charge*	*Monthly payments*	*Final payment*
24.	$275	$33	17 @ $18.00	$ 2.00
25.	295	36	17 @ $19.00	8.00
26.	310	37	17 @ $19.50	15.50
27.	325	40	17 @ $20.50	16.50
28.	350	42	17 @ $22.00	18.00

11

Compound interest

11-1 DEFINITIONS

If, instead of being paid when due, the interest on an investment is added to the principal and this sum used as a new principal, we say that *compound interest* is being used. The interval between consecutive times of adding the interest to the principal is called the *conversion period*. Unless otherwise stated, the rate of interest will be the rate per conversion period. The time interval between the date on which the principal was invested and the date on which it is repaid is called the *term* of the invest-

ment. The sum to which the principal and interest on it grow during the term is called the *maturity value* or *accumulated value* of the principal. The sum that is invested is called the *present value* or the *principal*.

Example 1. If \$100.00 is borrowed for three years at 5% per year compound interest and accumulates to \$115.76, we say that the present value is \$100.00, the term is three years, the interest rate is 5%, the conversion period is one year, and the maturity value is \$115.76.

Example 2. Find the accumulated value of \$1000 invested at 4% compounded annually for 4 years.

Solution.

Year	*Principal*	*Interest*	*Accumulated value*
1	1000	1000(.04)	$1000 + 1000(.04) =$ $1000(1 + .04) = \$1040$
2	$1040 = 1000(1.04)$	$1000(1.04)(.04)$	$1000(1 + .04)(1 + .04) =$ $1000(1 + .04)^2 = 1081.60$
3	$1081.60 = 1000(1.04)^2$	$1000(1.04)^2(.04)$	$1000(1 + .04)^2(1 + .04) =$ $1000(1 + .04)^3 = 1124.86$
4	$1124.86 = 1000(1.04)^3$	$1000(1.04)^3(.04)$	$1000(1 + .04)^3(1 + .04) =$ $1000(1 + .04)^4 = 1169.86$

11-2 THE COMPOUND INTEREST FORMULA

We shall now derive a formula for the accumulated value at compound interest. To be specific, we shall find the accumulated value, S, after n conversion periods at rate i per period of an investment whose present value is P.

If P is invested at rate i for one period, the interest is Pi and the accumulated value at the end of the period is $P + Pi = P(1 + i)$. Since the investment is at compound interest, $P(1 + i)$ is the principal for the second period; hence, the interest is $P(1 + i)\,i$ and the accumulated value at the end of the second period is:

$$P(1 + i) + P(1 + i)i = P(1 + i)\,(1 + i) = P(1 + i)^2$$

In fact, the principal at the beginning of any period is multiplied by $1 + i$ to obtain the accumulated value at the end of the period. Hence, if P is

invested for n periods at rate i per period, the factor $1 + i$ enters n times (once for each period) and the accumulated value is $P(1 + i)^n$. Thus

Period	*Principal*	*Interest*	*Accumulated value*
1	P	Pi	$P + Pi = P(1 + i)$
2	$P(1 + i)$	$P(1 + i)i$	$P(1 + i) + P(1 + i)i$ $= P(1 + i)(1 + i)$ $= P(1 + i)^2$
3	$P(1 + i)^2$	$P(1 + i)^2 i$	$P(1 + i)^2 + P(1 + i)^2 i$ $= P(1 + i)^2(1 + i)$ $= P(1 + i)^3$
. . .	. . .	. . .	. . .
$n - 1$	$P(1 + i)^{n-2}$	$P(1 + i)^{n-2} i$	$P(1 + i)^{n-2} + P(1 + i)^{n-2} i$ $= P(1 + i)^{n-2}(1 + i)$ $= P(1 + i)^{n-1}$
n	$P(1 + i)^{n-1}$	$P(1 + i)^{n-1} i$	$P(1 + i)^{n-1} + P(1 + i)^{n-1} i$ $= P(1 + i)^{n-1}(1 + i)$ $= P(1 + i)^n$

Hence

$$S = P(1 + i)^n \tag{7}$$

is the amount to which P accumulates in n periods at rate i per period at compound interest.

The student should recognize the fact that formula (7) is an equation involving the four quantities S, P, i, and n; hence, a value is determined for the fourth unknown if a value is assigned to each of the other three. The equation is solved for S in (7); it will be solved for i and for n in later sections, and we shall now solve it for P. This can be done by dividing each member of (7) by the coefficient, $(1 + i)^n$, of P. Thus, we have

$$P = \frac{S}{(1 + i)^n}$$

or

$$P = S(1 + i)^{-n} = Sv^n \tag{7'}$$

where
$$v = \frac{1}{1 + i}$$

The student should realize that (7′) is merely another form of (7) and not a new formula. The factor $(1 + i)^{-n}$ or v^n is sometimes called the *discount factor*.

Example 1. If \$150.00 is invested for 7 years at compound interest, what is the accumulated value provided money is worth 4% per year?

Solution. We know the values of P, i, and n and can find S by use of (7). Thus

$$S = \$150(1.04)^7$$

The value of $(1.04)^7$ can be obtained from Table III by looking across from 7 and under 4%. It is 1.31593178; hence

$$\begin{aligned} S &= \$150(1.31593178) \\ &= \$197.39 \end{aligned}$$

This result can be checked by discounting \$197.39 for 7 years at 4%.

Example 2. How much must be invested on December 10, 1964, at 3½% converted annually in order to accumulate to \$5872 on December 10, 1968?

Solution. In this problem we want to find the value 4 years before the value is \$5872. We have $S = \$5872$, $i = 3.5\%$, and $n = 4$; consequently, by use of (7′), we have

$$\begin{aligned} P &= \$5872(1.035)^{-4} \\ &= \$5872(.87144223) \quad \text{by use of Table II} \\ &= \$5117.11 \end{aligned}$$

This result can be checked by accumulating \$5117.11 for four years at 3½%. Thus, we get

$$\begin{aligned} S &= \$5117.11(1.035)^4 \\ &= \$5117.11(1.14752300) \\ &= \$5872 \end{aligned}$$

Example 3. If the compound interest rate is 3% per 6 months, how much should be invested now in order to accumulate \$12,500 in 10 years?

Solution. In this problem, the periodic rate $i = .03$, the number of 6-month periods is $n = (2)(10) = 20$, and $S = \$12{,}500$. Substituting in (7′) we have

$$\begin{aligned} P &= \$12{,}500(1.03)^{-20} \\ &= \$12{,}500(.55367575) \quad \text{by use of Table II} \\ &= \$6920.95 \end{aligned}$$

Exercise 11-1

1. Find the amount of \$800 for 8 years at 6% compounded annually.

2. Find the compound amount of \$800 for 8 years if the interest rate is 3% per 6 months.

3. Find the amount of \$75 for 10 years at 3% compounded annually.

4. Find the compound amount of \$75 for 10 years if the interest rate is $\frac{3}{4}$% per 3 months.

5. Find the amount of \$1000 at 7% compounded annually for 2 years; 5 years; 10 years; 50 years.

6. Find the compound amount of \$1000 for 2 years; 5 years; 10 years if the interest rate is $\frac{7}{12}$% per month.

7. Find the amount of \$750 at 5% compounded annually for 3 years; 7 years; 20 years; 35 years.

8. Find the compound amount of \$750 for 3 years; 5 years; 20 years if the interest rate is $\frac{5}{12}$% per month.

9. How much should be invested now at $5\frac{1}{2}$% compounded annually in order to accumulate \$5000 in 10 years?

10. How much should be invested now at $2\frac{3}{4}$% per 6 months, compounded semi-annually, in order to accumulate \$5000 in 10 years?

11. Find the discounted value of \$1500 due in 4 years at 4% compounded annually.

12. Find the discounted value of \$1500 due in 4 years at 1% per quarter, compounded quarterly.

13. Find the discounted value of \$1750 due in 8 years at 6% compounded annually.

14. Find the discounted value of \$1750 due in 8 years at $\frac{1}{2}$% per month, compounded monthly.

15. How much money must be invested at 7% compounded annually in order to produce \$10,000 in 20 years?

16. How much money must be invested at $3\frac{1}{2}$% per 6 months, compounded semi-annually, in order to produce \$10,000 in 20 years?

17. A house is for sale at \$12,000 cash or \$6000 cash with additional payments of \$3000 at the end of one year and \$3100 at the end of three years. If interest is at 5% compounded annually, which is the better price for the buyer and by how much?

18. Work Problem 17 if the interest rate is $2\frac{1}{2}$% per 6 months, compounded semi-annually.

19. The quoted price of a lot is \$500 cash and \$500 at the end of each year for two years. If money is worth 5% compounded annually, find the equivalent cash price.

20. Work Problem 19 if money is worth $\frac{5}{12}$% per month, compounded monthly.

11-3 COMPARISON OF SIMPLE AND COMPOUND INTEREST

We shall now compare the accumulated value of P for n interest periods at simple interest and at compound interest. The comparison will be made by examining the formulas for simple interest and compound interest and then by comparing the graphs of these two functions. If r is used as the rate in each formula, we have

$$S_s = P(1 + rn) \quad \text{for simple interest}$$

and

$$S_c = P(1 + r)^n \quad \text{for compound interest}$$

If the latter is expanded by use of the binomial theorem, we have

$$S_c = P\left[1^n + n \cdot 1^{n-1} r + \frac{n(n-1)1^{n-2}}{2 \cdot 1} r^2 + \cdots + \frac{n(n-1)\cdots 3 \cdot 2}{(n-1)\cdots 3 \cdot 2 \cdot 1} 1 \cdot r^{n-1} + r^n\right]$$

$$= P\left[1 + nr + \frac{n(n-1)}{2 \cdot 1} r^2 + \cdots + \frac{n(n-1)\cdots 3 \cdot 2}{(n-1)\cdots 3 \cdot 2 \cdot 1} r^{n-1} + r^n\right]$$

Case I. If $n = 1$, then $S_s = P(1 + r)$; furthermore, each term in the right member of the equation for S_c is zero except for the first two. Consequently, $S_c = P(1 + r)$. Therefore, *for one interest period, simple and compound interest yield equal amounts for every given rate.*

Case II. If n is greater than one and is a whole number, then each term in the right-hand member of the expression for S_c is positive and $S_c = P(1 + nr + \text{positive terms}) = P\,(1 + nr) + P(\text{positive terms})$; furthermore, $S_s = P(1 + rn)$; consequently, *if n is greater than one and is a whole number, compound interest yields more than simple interest for any given rate.*

Case III. If $0 < n < 1$, the first two terms in the right member of the expression for S_c are positive. The remaining terms, beginning with the third, alternate in sign. It can be shown that the sum of these terms is negative, but numerically less than Pnr. Hence

$$S_c = P(1 + nr) + P(\text{negative sum numerically less than } nr)$$

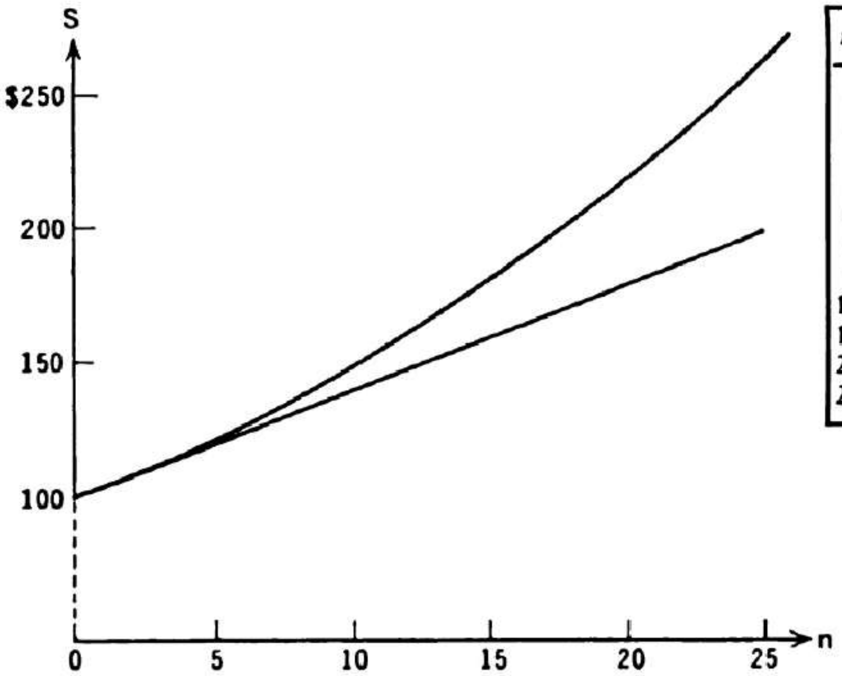
$$S_c < P(1 + nr) = S_s$$

and

$$S_c < S_s$$

Consequently, *if n is a fraction between 0 and 1, the compound interest formula gives a value which is less than that given by the simple interest formula for any given rate.*

The relation between accumulated value at simple interest and at compound interest at the same rate can be shown graphically also. The accompanying graphs are for $r = .04$ and $P = \$100.00$. The values for S at compound interest were obtained from Tables VIII and III and correspond to $n = \frac{1}{4}, \frac{1}{2}$, 1, 2, 4, 6, 9, 12, 16, 20, and 25. The values of S at simple interest were obtained by computation and correspond to the above values of n. Larger scales for the graphs would show the difference between S_c and S_s for small values of n (see table).



n	$100\,S_c$	$100\,S_s$
$\frac{1}{4}$	100.99	101.00
$\frac{1}{2}$	101.98	102.00
1	104.00	104.00
2	108.16	108.00
4	116.99	116.00
6	126.53	124.00
9	142.33	136.00
12	160.10	148.00
16	187.30	164.00
20	219.11	180.00
25	266.58	200.00

Two graphs that differ in no essential way from those in the figure can be obtained by using other values of r.

The values of $(1.04)^{1/4}$ and $(1.04)^{1/2}$, shown in the table, were obtained from Table VIII.

Compound interest is used on most loans except those that are for a short term. For example, the size of payment made on a house is ordinarily determined by use of compound interest.

11-4 SOLUTION FOR n BY LOGARITHMS AND BY INTERPOLATION

We shall now discuss two methods for solving (7) for n. The equation is

$$S = P(1 + i)^n \tag{7}$$

and the first method of solution will make use of logarithms. In order to do this, we must recall two of the theorems used in logarithmic computation. They are: (1) the logarithm of the product of two factors is equal to the sum of the logarithms of the factors, and (2) the logarithm of a power of a number is equal to the exponent of the power multiplied by the logarithm of the number.

If we equate the logarithms of the two members of (7), we have

$$\begin{aligned}\log S &= \log [P(1 + i)^n] \\ &= \log P + \log(1 + i)^n \qquad \text{by (1)} \\ &= \log P + n \log (1 + i) \qquad \text{by (2)}\end{aligned}$$

This is an equation that is linear in n and can be solved by adding $-\log S - n \log (1 + i)$ to each member and dividing by the coefficient of n. Thus

$$-n \log(1 + i) = \log P - \log S$$

or

$$n = \frac{\log P - \log S}{-\log(1 + i)}$$

$$n = \frac{\log S - \log P}{\log (1 + i)} \tag{7''}$$

This equation can be used as a formula for solving $S = P(1 + i)^n$ for n, but the student is advised to be able to solve it as often as necessary instead of memorizing (7″) since it is only a variation of (7).

Furthermore, (7) was derived only for integral values of n and, consequently, its solution for n as given by (7″) has not been justified except for n a whole number. It can be justified for fractional values of n, and we shall use it for them.

Example 1. How long will it take $2750 to accumulate to $5000 at $3\frac{1}{2}\%$ converted annually?

Solution. Since this is a problem in compound interest, we substitute in (7) and get

$$5000 = 2750(1.035)^n$$

Taking the logarithm of each member, we have

$$\begin{aligned}
\log 5000 &= \log 2750(1.035)^n \\
&= \log 2750 + \log 1.035^n \\
&= \log 2750 + n \log 1.035 \\
-n \log 1.035 &= \log 2750 - \log 5000 \\
n &= \frac{\log 2750 - \log 5000}{-\log 1.035} \\
&= \frac{\log 5000 - \log 2750}{\log 1.035} \\
&= \frac{3.69897 - 3.43933}{.01494} \quad \text{by use of Table I} \\
&= \frac{.25964}{.01494}
\end{aligned}$$

Thus, we have determined n as a fraction and can evaluate the fraction by the use of a calculating machine or by use of logarithms. If we use the latter, we have

$$\begin{aligned}
\log n &= \log \frac{.25964}{.01494} \\
&= \log .25964 - \log .01494 \\
&= (9.41437 - 10) - (8.17435 - 10) \\
&= 1.24002
\end{aligned}$$

Therefore $\quad n = 17.379$ years

If we prefer to express the time in terms of years, months, and days, we can do so by changing .379 years to months and days. Thus

$$\begin{aligned}
.379 \text{ years} &= 12(.379) \text{ months} \\
&= 4.55 \text{ months} \\
&= 4 \text{ months and } 30(.55) \text{ days} \\
&= 4 \text{ months and } 16 \text{ days}
\end{aligned}$$

Hence, \$2750 will accumulate to \$5000 in 17 years, 4 months, and 16 days at $3\frac{1}{2}\%$ converted annually.

(NOTE: This solution is based on the assumption that (7) is valid for fractional values of n.)

The other method of solving (7) for n consists of applying linear interpolation. The method will be illustrated by solving again the problem stated in Example 1.

Example 2. How long will it take \$2750 to accumulate to \$5000 at $3\frac{1}{2}\%$ converted annually?

Solution. Substituting in (7), we have $5000 = 2750(1.035)^n$

$$
\begin{aligned}
1.035^n &= \frac{5000}{2750} \quad \text{dividing by 2750} \\
&= 1.81818182 \\
&= 1.8182 \text{ to five significant figures}
\end{aligned}
$$

Now, if we look in the column headed by $3\frac{1}{2}\%$ in Table III, we find that $1.035^{17} = 1.79467555$, or 1.7947 to five significant figures and $1.035^{18} = 1.85748920$, or 1.8575 to five significant figures.

Hence, we must interpolate between these two numbers on the assumption that (7) is valid for fractional values of n. In order to do so more readily, we make the following table:

$$
\begin{array}{cll}
 & n & 1.035^n \\
1 & \left\{\begin{array}{l} 17 \\ n \\ 18 \end{array}\right. & \left.\begin{array}{l} \left.\begin{array}{l} 1.7947 \\ 1.8182 \end{array}\right\} \text{— } .0235 \\ 1.8575 \end{array}\right\} \text{— } .0628
\end{array}
$$

Therefore 1.035^n is $\frac{235}{628}$ of the way from 1.035^{17} toward 1.035^{18}, and we assume that n is that same part of the way from 17 toward 18. Consequently

$$
\begin{aligned}
n &= 17\tfrac{235}{628} \text{ years} \\
&= 17.374 \text{ years} \\
&= 17 \text{ years and } 12(.374) \text{ months} \\
&= 17 \text{ years and } 4.49 \text{ months} \\
&= 17 \text{ years, 4 months, and } 30(.49) \text{ days} \\
&= 17 \text{ years, 4 months, 14.7 days} \\
&= 17 \text{ years, 4 months, 15 days}
\end{aligned}
$$

Example 3. How long will it take \$2750 to accumulate to \$5000 at $1\frac{3}{4}\%$ per 6 months, compounded semi-annually?

Solution. Substituting in (7) we have $5000 = 2750(1.0175)^n$

$$
\begin{aligned}
1.0175^n &= \frac{5000}{2750} \quad \text{dividing by 2750} \\
&= 1.81818182 \\
&= 1.8182 \text{ to five significant figures}
\end{aligned}
$$

Now, if we look in the column headed by $1\frac{3}{4}\%$ in Table III, we find that $1.0175^{34} = 1.80372452$ or 1.8037 to five significant figures and $1.0175^{35} = 1.83528970$ or 1.8353 to five significant figures.

Now, making a table similar to that in Example 2, we have:

$$\begin{array}{cccc}
 & n & 1.0175^n & \\
1 & \left\{\begin{array}{l} 34 \\ n \\ 35 \end{array}\right. & \left.\begin{array}{r} \left.\begin{array}{r} 1.8037 \\ 1.8182 \end{array}\right\} \\ 1.8353 \end{array}\right\} & \begin{array}{l} \text{——}.0145 \\ \text{——}.0316 \\ \ \end{array}
\end{array}$$

Therefore 1.0175^n is $\frac{145}{316}$ of the way from 1.0175^{34} toward 1.0175^{35}, and we assume that n is the same part of the way from 34 toward 35. Hence

$$\begin{aligned} n &= 34\tfrac{145}{316} \text{ half years} \\ &= 34.459 \text{ half years} \\ &= 17.23 \text{ years} \end{aligned}$$

The reader should notice that this time is less than that of 17.37 years found in Example 2. This should be expected since the half-yearly rate here is one-half the annual rate in Example 2, and the number of conversion periods is doubled.

Exercise 11-2

1. Compare the accumulated value of \$100 at the end of 1, 2, 5, 10, 15, 20, 25 years at a compound interest rate of 6% with the corresponding amounts at 6% simple interest. Compare graphically as well as numerically.

2. Compare the maturity value of \$100 at the end of 1, 4, 8, 12, 16, 20, 24 years at a simple interest rate of 5% with the corresponding amounts at a 5% rate compounded annually. Compare graphically as well as numerically.

3. Compare the accumulated value of \$100 at the end of 1, 3, 6, 10, 15, 21 years at a simple interest rate of 4% with the corresponding amounts at 4% compounded annually. Compare graphically as well as numerically.

4. Compare the accumulated value of \$100 at the end of 1, 2, 4, 7, 11, 16, 22 years at a simple interest rate of 3% with the corresponding amounts at 3% compounded annually. Compare graphically as well as numerically.

5. How long will it take \$200 to accumulate to \$400 if $i = .03, .04, .05, .06$, compounded annually? Use logarithms and also interpolation.

6. Work Problem 5 if the half-yearly rates are $i = .015, .02, .025$, and $.03$, and interest is compounded semi-annually.

7. How long will it take \$350 to accumulate to \$1000 if $i = .04, .045, .055, .08$, compounded annually? Use logarithms and also interpolation.

8. Work Problem 7 if the quarterly rates are $i = 1\%$, $1\frac{1}{8}\%$, $1\frac{3}{8}\%$, 2%, and interest is compounded quarterly.

9. How long will it take \$728.50 to accumulate to \$2216.13 at 6% compounded annually?

10. How long will it take \$728.50 to accumulate to \$2216.13 at $\frac{1}{2}\%$ per month, compounded monthly?

11. How long will it take \$2362.50 to accumulate to \$12,758.67 at 5% compounded annually?

12. How long will it take \$2362.50 to accumulate to \$12,758.67 at $1\frac{1}{4}\%$ per quarter compounded quarterly? Use logarithms.

13. If \$3813.39 has a present value of \$1000 at $5\frac{1}{2}\%$ compounded annually t years before it is due, find t.

14. Work Problem 13 if the semi-annual rate is $2\frac{3}{4}\%$ and interest is compounded twice a year.

15. If \$7309.28 has a present value of \$5634.54 at 4% compounded annually t years before it is due, find t.

16. Work Problem 15 if the quarterly rate is 1% and interest is compounded four times per year.

11-5 SOLUTION FOR i BY LOGARITHMS AND BY INTERPOLATION

We shall now solve

$$S = P(1 + i)^n \tag{7}$$

for i by use of logarithms and then by interpolation. If we take the logarithm of each member, we have

$$\begin{aligned} \log S &= \log\,[P(1 + i)^n] \\ &= \log P + n \log\,(1 + i) \end{aligned}$$

Hence
$$n \log\,(1 + i) = \log S - \log P$$

and

$$\log(1 + i) = \frac{\log S - \log P}{n} \tag{7'''}$$

This equation can be used to obtain the value of $1 + i$ and then i can be found by subtracting 1 from each member. The student should not try to remember (7''') as a formula for solving (7) for i since it is merely a variation of (7). Instead of using (7''') as a formula, one need only solve (7) for i by use of logarithms as often as i is required.

Example 1. If, in 12 years, \$600 accumulates to \$882, what is the compound interest rate provided it is converted annually?

Solution. By use of (7), we have $882 = 600(1 + i)^{12}$.

Hence

$$\begin{aligned}
\log 882 &= \log 600 + 12 \log(1 + i) \\
-12 \log (1 + i) &= \log 600 - \log 882 \\
\log (1 + i) &= \frac{\log 882 - \log 600}{12} \\
&= \frac{2.94547 - 2.77815}{12} \\
&= \frac{.16732}{12} \\
&= 0.01394 \\
1 + i &= 1.0326 \\
i &= 3.26\%
\end{aligned}$$

Equation (7) can be solved for i in any particular case by use of interpolation. The procedure is exactly the same as in any other situation in which linear interpolation is used. We shall illustrate it by solving Example 1 again.

Example 2. If \$600 accumulates to \$882 in 12 years at compound interest converted annually, what is the rate?

Solution. We have $$882 = 600(1 + i)^{12}$$
by use of (7). Hence, dividing each member by 600, we get

$$\begin{aligned}
(1 + i)^{12} &= \frac{882}{600} \\
&= 1.47
\end{aligned}$$

The two entries nearest to 1.47 in the table of accumulated values at compound interest opposite 12 are 1.4258 and 1.5111 to five figures and they are under 3% and $3\frac{1}{2}\%$, respectively. Therefore, we set up the table:

$$.005 \left\{ \begin{array}{l} \left. \begin{array}{r} 1.03^{12} = 1.4258 \\ (1 + i)^{12} = 1.47 \end{array} \right\} .0442 \\ \quad 1.035^{12} = 1.5111 \end{array} \right\} .0853$$

Consequently, i is $\frac{442}{853}$ of the way from 3% toward 3.5%. Therefore, we shall compute this fractional part of .005 and add it to 3%. Thus

$$\begin{aligned} \frac{442}{853}(.005) &= \frac{2.210}{853} \\ &= .00259 \\ i &= .03 + .00259 \\ &= 3.26\% \end{aligned}$$

Example 3. If \$600.00 accumulates to \$882.00 in 12 years at a quarterly rate, compounded 4 times per year, what is the quarterly rate?

Solution. Since there are 48 quarters in 12 years, by use of (7), we have $882 = 600(1 + i)^{48}$ where i is the quarterly rate. Hence, dividing each member by 600, we get

$$\begin{aligned} (1 + i)^{48} &= \frac{882}{600} \\ &= 1.47 \end{aligned}$$

Looking in the table of accumulated values at compound interest opposite $n = 48$, we find that 1.47 lies between 1.4314 and 1.5192, which are the values to five significant figures for $\frac{3}{4}\%$ and $\frac{7}{8}\%$, respectively.

Hence, we set up the table:

$$.00125 \left\{ \begin{array}{l} \left.\begin{array}{r} (1.0075)^{48} = 1.4314 \\ (1 + i)^{48} = 1.47 \end{array}\right\} — .0386 \\ (1.00875)^{48} = 1.5192 \end{array} \right\} — .0878$$

Consequently, i, the quarterly rate, is $\frac{386}{878}$ of the way from $\frac{3}{4}\%$ toward $\frac{7}{8}\%$. Therefore, we compute this fractional part of $\frac{1}{8}\%$ and add it to $\frac{3}{4}\%$.

$$\begin{aligned} \frac{386}{878}(.00125) &= \frac{.4825}{878} \\ &= .00055 \\ i &= .0075 + .00055 \\ &= .805\% \end{aligned}$$

Exercise 11-3

1. At what rate, converted annually, will \$500 accumulate to \$800 in 15 years? Compare with the simple-interest rate.

2. At what quarterly rate will \$500 accumulate to \$800 in 15 years?

3. At what rate converted annually will \$1250 accumulate to \$2450 in 18 years?

4. At what semi-annual rate will \$1250 accumulate to \$2450 in 18 years?

5. At what rate converted annually will \$750 accumulate to \$1000 in 10 years? Compare with the simple-interest rate.

6. At what semi-annual rate will \$750 accumulate to \$1000 in $9\frac{1}{2}$ years?

7. Find the semi-annual rate at which \$1760 will accumulate to \$3600 in $28\frac{1}{2}$ years.

8. At what rate of interest converted annually will \$1682.95 accumulate to \$3000 in 15 years?

9. At what rate converted annually will \$4386.51, due 6 years hence, have a present value of \$3942.71? Compare with the simple interest rate.

10. At what monthly rate will \$4386.51, due 6 years hence, have a present value of \$3942.71? Use logarithms.

11. At what rate of interest, converted annually, will \$10,000, due 30 years from now, have a present value of \$3284.97?

12. At what rate of interest, converted annually, will \$20,000 due in 42 years have a value of \$5,000 6 years from now?

13. A note for \$2500 bears interest at 4% and is due in 10 years. What rate, compounded annually, will an investor make if he buys the note 4 years from now for \$3200?

14. An investor paid \$4113 for a 7-year \$4000 note that bears interest at $3\frac{1}{2}\%$ compounded annually. If he bought it 2 years after it was made out, what rate compounded annually did he make on his investment?

15. An investor wants to make 4% compounded annually. Can he afford to pay \$11,000 for a 5-year \$10,000 note bearing 6% interest compounded annually if he buys it the day it is made out?

16. A note for \$5000 is due in 6 years. It is purchased by an investor for \$3900. What semi-annual rate does he earn on his investment?

11-6 NOMINAL AND EFFECTIVE RATES

If the conversion period in compound interest is different from one year, the stated annual rate is called the *nominal rate*. This term is used because an annual rate is usually *named* in a transaction together with the number of times per year it is compounded. We will represent the nominal rate by j, and the number of times it is compounded per year by m. We often use the symbol $j_{(m)}$ to designate a situation of this sort since it shows

not only the rate but also the number of conversion periods per year. The symbol $\frac{j}{m}$ is another way of expressing the periodic rate. Henceforth, all rates quoted in connection with compound interest will be annual rates unless some other unit of time is specified. The annual rate at which the principal increases if the nominal rate is j compounded m times per year is called the *effective rate* and will be represented by e. This is the rate actually earned. If interest is compounded annually, the nominal or named rate is equal to the effective or earned rate. Further, if the periodic rate i is an annual rate, then $e = i$. If a given principal produces the same accumulated value at two or more different nominal rates, we say that the rates are *equivalent*.

Example 1. If \$100 is invested at 4% per year and interest is converted semi-annually, the nominal rate is 4%, the conversion period is 6 months, $m = 2$, and the compound amount at the end of 6 months is $S = \$100(1.02) = \$102$ since the rate during each 6-month period is 2%. Furthermore, the compound amount at the end of the second 6 months is $S = \$102(1.02) = \$104.04$. Now, it is obvious that the total interest earned on \$100 in one year at $j = 4\%$, $m = 2$, is \$4.04. Hence, the effective rate is $\frac{4.04}{100} = .0404$ or 4.04%.

The relation between the nominal and the effective rate is given by

$$1 + e = \left(1 + \frac{j}{m}\right)^m \tag{8}$$

since \$1 will accumulate to $\$(1 + e)$ in one year at the effective rate e and to $\$\left(1 + \frac{j}{m}\right)^m$ in one year at the nominal rate j compounded m times per year. The accumulated value of 1 at the end of one year at rate j compounded m times per year is $\left(1 + \frac{j}{m}\right)^m$ by (7) since the periodic rate is $i = \frac{j}{m}$ and the number of periods in one year is equal to m.

Example 2. What effective rate is equivalent to a nominal rate of 6% converted quarterly?

Solution. If we substitute in the relation (8) between effective and nominal rates, we get

$$\begin{aligned} 1 + e &= \left(1 + \frac{.06}{4}\right)^4 \\ &= (1.015)^4 \\ &= 1.0614 \quad \text{to five significant figures} \\ e &= .0614 \end{aligned}$$

Consequently, the effective rate is $e = 6.14\%$.

In order to compare two rates, we compare the corresponding effective rates.

Example 3. Is 4% converted 12 times per year or $4\frac{1}{2}\%$ converted twice per year the more desirable from the viewpoint of the lender?

Solution. The effective rate equivalent to $j = .04$, $m = 12$ is $e = 4.07\%$ since

$$\begin{aligned} 1 + e &= \left(1 + \frac{.04}{12}\right)^{12} \\ &= \left(1 + \frac{.01}{3}\right)^{12} \\ &\text{or } 1.0407 \quad \text{to five significant figures} \\ e &= .0407 \text{ or } 4.07\% \end{aligned}$$

The effective rate equivalent to $j = .045$, $m = 2$ is $e = 4.55\%$ since

$$\begin{aligned} 1 + e &= \left(\frac{1 + .045}{2}\right)^2 \\ &= (1.0225)^2 \\ &\text{or } 1.0455 \quad \text{to five significant figures} \\ e &= .0455 \text{ or } 4.55\% \end{aligned}$$

Consequently, $j = 4.5\%$, $m = 2$ yields a greater return than $j = 4\%$, $m = 12$.

Example 4. What nominal rate with $m = 4$ corresponds to an effective rate of 5%?

Solution. We are given $e = 5\%$, $m = 4$. Substituting in (8) we have

$$1 + .05 = \left(1 + \frac{j}{4}\right)^4$$

Extracting the fourth root of both sides, we have

$$(1 + .05)^{1/4} = 1 + \frac{j}{4}$$

$$1.01227224 = 1 + \frac{j}{4} \quad \text{by use of Table VIII}$$

$$.01227224 = \frac{j}{4} \quad \text{subtracting 1 from each side}$$

$$j = .04908896$$

$$= 4.91\% \quad \text{to the nearest hundredth of } 1\%$$

Hence, a rate of 4.91% compounded quarterly is equivalent to an effective rate of 5%.

11-7 ACCUMULATED AND PRESENT VALUES AT A NOMINAL RATE

If interest is converted m times per year, the number of conversion periods in t years will be mt. Hence, if we substitute $\frac{j}{m}$ for i and mt for n, in

(7) $$S = P(1 + i)^n$$

we obtain

(9) $$S = P\left(1 + \frac{j}{m}\right)^{mt}$$

This equation is precisely equation (7) with the periodic rate i expressed as a fraction of the nominal annual rate j and the number of periods n expressed as the product of the number of conversion periods m per year and the number of years t. Equation (9) is more readily used than (7) if the interest rate is nominal and the time is given in years.

Example 1. To what amount will \$250 accumulate in $4\frac{1}{2}$ years at 4% converted semi-annually?

Solution. In this problem, $P = \$250$, $t = 4\frac{1}{2}$, $j = 4\%$ and $m = 2$. Hence, substituting in (9), we have

$$S = \$250 \left(1 + \frac{.04}{2}\right)^{(2)\,(4\frac{1}{2})}$$
$$= \$250\,(1 + .02)^9$$
$$= \$250\,(1.19509257)$$
$$= \$298.77$$

We can solve (9) for P by multiplying both sides by $\left(1+\frac{j}{m}\right)^{-mt}$.
Doing this yields the following variation of (9):

(9′) $$P = S\left(1+\frac{j}{m}\right)^{-mt}$$

Example 2. What will be the value on January 10, 1964 of \$1000 due on January 10, 1972, provided money is worth 6% converted quarterly?

Solution. In this problem $S = \$1000$, $t = 8$, $j = .06$, and $m = 4$. Substituting in (9′) we have

$$\begin{aligned} P &= \$1000\left(1+\frac{.06}{4}\right)^{-(8)\,(4)} \\ &= \$1000(1+.015)^{-32} \\ &= \$1000\,(.62099292) \\ &= \$620.99 \end{aligned}$$

We shall illustrate one more use of (9) by solving it for j provided the values of S, P, m, and t are given. The procedure is the same as that followed in solving $S = P(1+i)^n$ for i except that one additional step is necessary.

Example 3. If \$800 accumulates to \$1200 in 7 years and 9 months, what is the nominal rate converted quarterly?

Solution. We are given that $S = \$1200$, $P = \$800$, $m = 4$, and $t = 7\frac{3}{4}$. Substituting in (9), we have

$$\begin{aligned} 1200 &= 800\left(1+\frac{j}{4}\right)^{(4)\,(7\frac{3}{4})} \\ &= 800\left(1+\frac{j}{4}\right)^{31} \end{aligned}$$

Therefore $$\left(1+\frac{j}{4}\right)^{31} = 1.5$$

Using interpolation in Table III, we look across from $n = 31$, and find the two entries which are nearer than any others to 1.5. Thus, to five significant figures, we have

$$.00125 \qquad \left\{ \begin{array}{l} (1.0125)^{31} = 1.4698 \\ \left(1+\frac{j}{4}\right)^{31} = 1.5 \\ (1.01375)^{31} = 1.5271 \end{array} \right\} \begin{array}{l} .0302 \\ .0573 \end{array}$$

Hence $\frac{j}{4}$ is $\frac{302}{573} = .527$ of the way from $1\frac{1}{4}\%$ to $1\frac{3}{8}\%$.

Consequently

$$\begin{aligned} \frac{j}{4} &= .0125 + (.527)(.00125) \\ &= .0125 + .000659 \\ &= .013159 \\ j_{(4)} &= .052636 \\ &= 5.26\% \quad \text{to the nearest hundredth of } 1\% \end{aligned}$$

Exercise 11-4

Determine the missing quantity in each of Problems 1 through 8.

	e	j	m		e	j	m
1.	.06	?	12	**5.**	?	.06	12
2.	.05	?	2	**6.**	?	.05	2
3.	.04	?	4	**7.**	?	.04	4
4.	.03	?	1	**8.**	?	.03	1

9. To what sum will \$450 accumulate in 4 years at 5% converted semi-annually?

10. Find the accumulated value of \$775 after $4\frac{1}{2}$ years if money is worth 5% converted semi-annually.

11. Find the accumulated value of \$960.85 after $12\frac{1}{4}$ years if money is worth 6% converted monthly.

12. Find the accumulated value of \$1268.50 after $8\frac{3}{4}$ years if money is worth 8% converted quarterly.

13. What is the present value of \$700 due in 6 years if money is worth 4% converted quarterly?

14. What is the present value of \$6257 due in $3\frac{1}{2}$ years if money is worth 3% converted semi-annually?

15. What is the present value of \$1000 due in 10 years if money is worth 3% converted monthly?

16. What is the present value of $1250 due in $7\frac{1}{4}$ years if money is worth 7% compounded monthly?

17. An investor has an opportunity to buy a note that has a face value of $1000, is due in 6 years, and draws interest at 4% converted quarterly. What price should he pay in order to make 5% converted semi-annually?

18. A note for $5200 bearing interest at 4% converted semi-annually was made out on June 22, 1957 and is due on June 22, 1972. What should an investor pay on December 22, 1968 in order to make 6% converted quarterly?

19. What should a buyer pay on June 10, 1968 for a $5120, 4% converted quarterly note made out on March 10, 1958 and due on December 10, 1971 in order to make 6% converted semi-annually?

20. A note for $1000 is due in 10 years and draws interest at 5% compounded semi-annually. What should be paid for it 6 years before it is due in order to earn 6% compounded quarterly?

21. At what nominal rate converted quarterly is $5297.39 the value of $6015 3 years and 6 months before it is due?

22. At what nominal rate converted monthly will $1500 accumulate to $2000 in 5 years?

23. At what nominal rate converted quarterly will $5000 be the value of $7500 10 years and 3 months before it is due?

24. A man owns a note for $5000 due 4 years hence. If he sells this note for $4200 3 years before it is due, what nominal rate converted semi-annually does he earn?

11-8 COMPOUND INTEREST FOR FRACTIONAL PERIODS

Heretofore, our discussion of compound interest has been limited to the situation in which the time is a whole number of interest periods, except in Section 11-4, where we determined fractional values of n by use of logarithms and interpolation. We shall now continue our extended conception of compound interest by agreeing that the accumulated value shall be the value obtained by use of (7) or (9) even though the term is not a whole number of interest periods.

Example 1. Find the accumulated value of $1000 invested at 5% converted annually for $2\frac{1}{2}$ years.

Solution. In this problem, we have $P = \$1000$, $j = .05$, $m = 1$, $t = 2\frac{1}{2}$, and substituting in (9) gives

$$S = \$1000(1.05)^{(2\frac{1}{2})}$$

Hence
$$\begin{aligned}\log S &= \log 1000 + 2.5 \log 1.05\\ &= 3 + 2.5\ (.02119)\\ &= 3.05298\\ S &= \$1129.74\end{aligned}$$

The value of S can be computed by the use of Tables III and VIII as follows:

$$\begin{aligned}S &= \$1000(1.05)^{(2\frac{1}{2})}\\ &= \$1000(1.05)^2(1.05)^{1/2}\\ &= \$1000(1.1025)(1.02469508)\\ &= \$1130.01\end{aligned}$$

This gives us an accurate answer, whereas the logarithmic method cannot give a result correct to six significant figures because the table used is accurate to only five places.

Example 2. Find the accumulated value of \$1500 invested for 4 years and 5 months at $4\frac{1}{2}\%$ converted quarterly.

Solution. In this problem, $P = \$1500$, $j = .045$, $m = 4$, and $t = 4\frac{5}{12}$. Substituting in (9) gives

$$\begin{aligned}S &= \$1500\left(1 + \frac{.045}{4}\right)^{4(4\frac{5}{12})}\\ &= \$1500(1.01125)^{17\frac{2}{3}}\\ &= \$1500(1.01125)^{17}(1.01125)^{2/3}\end{aligned}$$

Looking in Table VIII under $1\frac{1}{8}\%$ for the value of $(1.01125)^{2/3}$ reveals that no such value appears. It is simple, however, to compute this value from that of $(1.01125)^{1/3}$, which is in the table, for $(1.01125)^{2/3} = [(1.01125)^{1/3}]^2$. Hence

$$\begin{aligned}S &= \$1500(1.01125)^{17}\ [(1.01125)^{1/3}]^2\\ &= \$1500(1.20946997)(1.00373602)^2\\ &= \$1814.205(1.0074860)\\ &= \$1827.78\end{aligned}$$

It should be noted that we may always find the accumulated value at compound interest by substituting in

(9) $$S = P\left(1 + \frac{j}{m}\right)^{mt}$$

since (7) is a special case of it with $j = i$, $m = 1$, and $t = n$. It is also always possible to find the present value at compound interest by substituting in the same formula and solving for P.

Quite often a combination of compound and simple interest is used if the term is not a whole number of interest periods. This gives an approximation to the value of $P\left(1 + \frac{j}{m}\right)^{mt}$. If this approximation is used, *we employ simple interest for any time less than an interest period.*

To obtain an approximation to the accumulated value at compound interest for a time that is not a whole number of interest periods:

(a) *Compute the compound amount for the largest whole number of interest periods that is less than the given time.*
(b) *Accumulate the compound amount obtained in* (a) *at simple interest for the remaining fractional part of a period.*

Example 3. If \$1000 is invested at 5% converted annually for $2\frac{1}{2}$ years, find the accumulated value by the approximate method.

Solution. The largest whole number of interest periods of one year each contained in $2\frac{1}{2}$ years is two. Hence, step (a) consists of evaluating

$$\begin{aligned} S &= \$1000(1.05)^2 \\ &= \$1000(1.1025) \\ &= \$1102.50 \end{aligned}$$

and step (b) requires that we accumulate \$1102.50 for $\frac{1}{2}$ year at 5% simple interest. Thus, the value after $2\frac{1}{2}$ years is

$$\begin{aligned} \$1102.50[1 + (.05)\tfrac{1}{2}] &= \$1102.50(1.025) \\ &= \$1130.06 \end{aligned}$$

Example 4. Find the accumulated value of \$1500 invested for 4 years and 5 months at $4\frac{1}{2}$% converted quarterly. Use the approximate method.

Solution. The largest number of whole interest periods of one quarter-year each contained in 4 years and 5 months is 17. Hence, step (a) consists in evaluating

$$\begin{aligned} S &= \$1500(1.01125)^{17} \\ &= \$1500(1.20946997) \\ &= \$1814.20 \end{aligned}$$

and step (b) requires that we accumulate \$1814.20 for $\frac{2}{3}$ of 1 quarter at a quarterly simple interest rate of $1\frac{1}{8}\%$. Thus, the value after 4 years and 5 months is

$$\begin{aligned} S &= \$1814.20\,[1 + (.01125)\,\tfrac{2}{3}] \\ &= \$1814.20(1 + .0075) \\ &= \$1827.81 \end{aligned}$$

To find an approximation to the present value of an amount S which is due in a time that is not a whole number of interest periods:

(a) *Compute the present value for the largest number of interest periods contained in the term.*

(b) *Discount the value obtained in* (a) *at simple interest for the fractional part of a period necessary to bring the total time to the term of the investment.*

From the previous examples and the above statement of how to find the approximate present value, we conclude that the formula

$$S = P\left(1 + \frac{j}{m}\right)^{l}\left[1 + \left(\frac{j}{m}\right)f\right] \tag{10}$$

where l is the largest integer in mt and f is the fraction $mt - l$ really describes the approximate method. In Example 4, $mt = (4)(4\frac{5}{12}) = 17\frac{2}{3}$. The largest integer in $17\frac{2}{3}$ is $l = 17$. The fraction $f = 17\frac{2}{3} - 17 = \frac{2}{3}$.

Example 5. What is the present value of \$1000 due in 3 years and 8 months if money is worth $3\frac{1}{2}\%$ conveited semi-annually?

Solution. In this problem $S = \$1000$, $j = .035$, $m = 2$, $t = 3\frac{8}{12} = 3\frac{2}{3}$ and we want to find P. In order to substitute in (10) we must find l and f. Since l is the largest integer in $mt = 2(3\frac{2}{3}) = 7\frac{1}{3}$ it is 7 and $f = 7\frac{1}{3} - 7 = \frac{1}{3}$. Substituting, we have

$$\begin{aligned} \$1000 &= P\left(1 + \frac{.035}{2}\right)^{7}\left[1 + \left(\frac{.035}{2}\right)\left(\frac{1}{3}\right)\right] \\ &= P(1 + .0175)^{7}\left[1 + (.0175)\left(\frac{1}{3}\right)\right] \\ &= P(1.1291222)(1.0058333) \\ &= P(1.1357087) \\ P &= \frac{\$1000}{1.1357087} \\ &= \$880.51 \end{aligned}$$

Exercise 11-5

Find the maturity value of each note described in Problems 1 through 8. Use the method specified by your instructor.

	Principal	*Rate*	*Due in*
1.	$ 600.00	$j = .04,\ m = 1$	3 years, 3 months
2.	800.00	$j = .06,\ m = 2$	5 years, 2 months
3.	5000.00	$j = .035,\ m = 1$	4 years, 5 months
4.	6217.00	$j = .05,\ m = 1$	3 years, 7 months
5.	4926.00	$j = .06,\ m = 4$	6 years, 8 months
6.	750.00	$j = .03,\ m = 4$	10 years, 4 months
7.	400.00	$j = .05,\ m = 4$	12 years, 10 months
8.	3218.25	$j = .045,\ m = 2$	7 years, 9 months

Find the present value of each note described in Problems 9 through 16. Use the method specified by your instructor.

	Maturity value	*Rate*	*Due in*
9.	$ 700.00	$j = .05,\ m = 1$	4 years, 2 months
10.	1000.00	$j = .06,\ m = 2$	2 years, 7 months
11.	2714.00	$j = .04,\ m = 4$	1 year, 5 months
12.	3965.00	$j = .06,\ m = 1$	5 years, 9 months
13.	1472.00	$j = .04,\ m = 1$	8 years, 4 months
14.	760.00	$j = .06,\ m = 2$	3 years, 5 months
15.	1000.00	$j = .05,\ m = 4$	7 years, 8 months
16.	4721.00	$j = .03,\ m = 4$	9 years, 4 months

11-9 EQUATIONS OF VALUE

An equation of value is an equation which states that the value of one set of obligations is equal to the value of a second set on a specified date. The particular date that is chosen is immaterial but it is essential that the same date be used for all of the obligations. The date selected for use in an equation of value is called the *comparison date* or *focal date*, and all obligations under consideration must be evaluated on this date in order to set up the equation. It is desirable to choose the comparison date so as to have as simple an equation as possible. Furthermore, if compound interest is used, the particular date chosen as the comparison date does

not affect the solution obtained for the equation because, if two dates were chosen, the resulting equations differ only in that one could be obtained from the other by multiplying both sides of it by a power of

$$\left(1 + \frac{j}{m}\right).$$

Example. What amount paid in 2 years with an equal amount paid in 5 years will equitably repay a note of $500 due in 4 years with interest at 6% converted quarterly and another of $800 due in 3 years with interest at 4% converted annually, provided money is worth 5% compounded semi-annually?

Solution 1. We shall make a table of the two sets of obligations with 5 years from now as the comparison date. If each payment is X, we can make the following table:

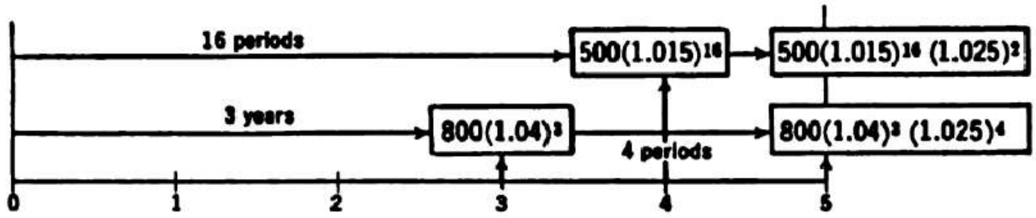
DEBTS OR OLD OBLIGATIONS	DEBTS OR OLD OBLIGATIONS	PAYMENTS OR NEW OBLIGATIONS
Value when due	*Value 5 years from now*	*Value 5 years from now*
$\$500(1.015)^{16}$	$\$500(1.015)^{16}(1.025)^{2}$	$X(1.025)^{6}$
$\$800(1.04)^{3}$	$\$800(1.04)^{3}(1.025)^{4}$	X

The entries in the table can be justified as follows: The $500 is due in 4 years with $j = 6\%$, $m = 4$ and must be accumulated for another year at $j = 5\%$, $m = 2$; the $800 is due in 3 years with $j = 4\%$, $m = 1$ and must be accumulated for 2 more years at $j = 5\%$, $m = 2$; the first $\$X$ is due in 2 years and must be accumulated for 3 years at $j = 5\%$, $m = 2$; and the second $\$X$ is due in 5 years. Hence, setting the two sets of values 5 years from now equal to one another, we have

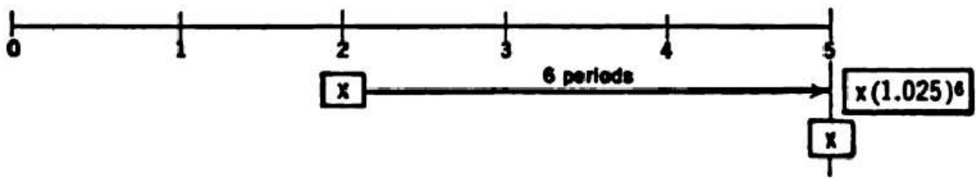
$$(1) \qquad X + X(1.025)^{6} = \$500(1.015)^{16}(1.025)^{2} + \$800(1.04)^{3}(1.025)^{4}$$

$$\begin{aligned}
X(1 + 1.15969342) &= \$500(1.26898555)(1.05062500) \\
&\quad + \$800(1.12486400)(1.10381289) \\
2.15969342\,X &= \$666.61 + \$993.31 \\
&= \$1659.92 \\
X &= \frac{\$1659.92}{2.15969342} \\
&= \$768.59
\end{aligned}$$

Solution 2. Some students may find it easier to write out equation (1) of Solution 1 by use of a graphical scheme. Thus, using 5 years from the present as the comparison date, the debts (old obligations) have a value of $500(1.015)^{16}(1.025)^2 + 800(1.04)^3(1.025)^4$ as seen from this diagram.



Furthermore, the new obligations have a value 5 years from now of $X(1.025)^6 + X$ as seen from the diagram below.



Hence, equating the values of the old and new sets of obligations 5 years from now, we have

$$X\,(1.025)^6 + X = 500(1.015)^{16}(1.025)^2 + 800(1.04)^3(1.025)^4$$

This is equation (1) of Solution 1. Hence

$$X = \$768.59$$

as obtained in Solution 1.

Exercise 11-6

1. Mr. Hall owns two notes and needs some cash. One note is for \$800 with 4% interest converted semi-annually and is due in 5 years. The other is for \$1150 with interest at 5% converted annually and is due in 3 years. He goes to a banking firm that agrees to give him a certain sum of cash and the same sum at the end of 6 months. Determine that sum under the assumption that money is worth 6% converted semi-annually.

2. Mr. Phillips owes \$700 with interest at 5% converted semi-annually due in 4 years and \$1600 with interest at 6% converted quarterly due in 6 years. If money is worth 4%, what equal payments 2 and 3 years from now will equitably discharge the debts?

3. A firm owes \$17,500 with interest at 8% converted quarterly which is due in 3 years and \$26,850 due in 4 years with interest at 5% converted annually. What equal payments made now and 1 year from now will equitably discharge these debts if money is worth 6% converted semi-annually?

4. Mr. Kirkland owes \$12,000 due in 5 years with interest at 6% converted semi-annually. What equal payments made in 1 year, 2 years, and 3 years will properly discharge this debt if money is worth $j = 5\%$, $m = 4$?

5. I owe a friend \$500 with interest at $j = 4\%$, $m = 2$ due in 2 years, and \$900 with interest at 5% converted annually due in 3 years. What equal sums paid now and in 6 months will equitably pay off these debts if money is worth $j = 4\%$, $m = 4$?

6. Tom owes \$9200 due without interest on July 28, 1975 and \$7300 with interest at $j = 4\%$, $m = 2$ from October 28, 1957 until due on April 28, 1970. What equal sums paid on April 28, 1963, July 28, 1964, and October 28, 1964 will equitably discharge his debts if money is worth $j = 4\%$, $m = 4$?

7. A farm is worth \$17,000 cash. The buyer pays \$4000 cash, \$5000 at the end of 2 years, \$4000 at the end of 4 years, and a final payment at the end of 6 years. Determine the size of the final payment if money is worth $j = 5\%$, $m = 2$.

8. A house is worth \$12,000. The buyer pays \$7000 cash and makes equal payments at the end of 1, 2, and 3 years. Find the size of these payments if money is worth $j = 5\%$, $m = 2$.

9. A firm borrows \$20,000 and repays it by making equal payments at the ends of 6 months, 1 year, $1\frac{1}{2}$ years, 2 years, and $2\frac{1}{2}$ years. How much is each of these payments if money is worth $j = 3\%$, $m = 2$?

10. A firm borrows \$42,500 and repays it by making equal monthly payments for 6 months. What is each payment if the first is made 1 month after the debt is incurred? Assume that money is worth $j = 6\%$, $m = 12$.

11. A firm owes \$25,000 due in 5 years without interest. It pays off the obligation by making 3 equal semi-annual payments. What is each payment if the first is made 6 months after the debt is incurred? Assume that money is worth $j = 4\%$, $m = 2$.

12. A firm owes \$20,000 due in 3 years with interest at 4% compounded annually. The obligation is repaid by making a payment after 1 year, twice that sum after 2 years, and three times the original payment after 3 years. Determine each of these payments if money is worth 5% compounded annually.

SUMMARY

The symbols e and j are used to represent effective and nominal rate, respectively. The symbol m is used to indicate the number of times per year that interest is converted, and the symbol t is used to represent the number of years in the term of the investment. When the term is expressed in periods, the symbol i is used for the periodic rate, and the symbol n for the number of periods.

The relation between nominal and effective rates is shown to be

$$1 + e = \left(1 + \frac{j}{m}\right)^m \tag{8}$$

The present value of an investment is represented by P and the accumulated value by S. They are connected by

$$S = P(1 + i)^n \tag{7}$$

and

$$S = P\left(1 + \frac{j}{m}\right)^{mt} \tag{9}$$

If the term of the investment is not equal to an integral number of conversion periods, an approximate relation between P and S is given by

$$S = P\left(1 + \frac{j}{m}\right)^l \left[1 + \left(\frac{j}{m}\right)f\right] \tag{10}$$

where l is the largest integer in mt and f is the fraction $mt - l$. Instead of remembering this formula, the student should reread the discussion preceding (10) in Section 11-8.

Exercise 11-7 (Review)

In Problems 1 through 4, find the accumulated value if the given sum is invested for the stated term at the specified rate.

1. $1500 for 5 years at 6% compounded quarterly.
2. $4600 for 2 years at 5% effective.
3. $1225 for 10 years at 4% compounded semi-annually.
4. $625 for 1 year at 6% compounded monthly.

In Problems 5 through 8, find the present value.

5. $2500 due in 6 years, if money is worth 7% compounded semi-annually.
6. $1400 due in 8 years, if money is worth 3% compounded quarterly.
7. $10,000 due in 5 years, if money is worth $4\frac{1}{2}$% effective.
8. $750 due in 3 years, if money is worth $5\frac{1}{2}$% compounded semi-annually.

9. If $j_{(2)} = .06_{(2)}$, find e.

10. If $j_{(4)} = .04_{(4)}$, find e.

11. If $e = .05$, find $j_{(12)}$.

12. If $e = .03$, find $j_{(4)}$.

13. If $3000 accumulates to $4050 in 11 years, (a) find e; (b) find $j_{(2)}$; (c) find $j_{(4)}$.

14. If $2785 accumulates to $3540.14 in 6 years, find (a) e; (b) $j_{(12)}$.

15. If the present value of $1425.50 due in 4 years is $1276.11, (a) find e; (b) find $j_{(6)}$.

16. If the present value of $2000 due in 20 years is $582.40, (a) find e; (b) find $j_{(4)}$; (c) find $j_{(12)}$.

17. Find the amount of $3000 due in $5\frac{1}{2}$ years with interest at 5%, compounded annually (a) if compound interest is used for the fractional period; (b) if simple interest is used for the fractional period.

18. Find the amount of $750 due in 3 years and 2 months with interest at 4% compounded quarterly if for the fractional period (a) compound interest is used; (b) simple interest is used.

19. A owes B three sums: (1) $5000 due in 4 years with interest at 5% compounded semi-annually; (2) $4500 due in 6 years with interest at $4\frac{1}{2}$% compounded annually; (3) $3800 due in 2 years without interest. If B wishes to earn 6% compounded quarterly, for how much should he be willing to let A settle these three debts (a) now? (b) 3 years hence? (c) 4 years hence?

20. A owes B two debts: (1) $2500 due in $4\frac{1}{2}$ years without interest; (2) $1500 due in 6 years with interest at 3% compounded semi-annually. A wishes to settle these debts by making equal payments at the end of 3 and 5 years. If B agrees on a rate of 5% compounded semi-annually, what is the amount of each payment?

12

Simple annuities

12-1 TERMINOLOGY

A sequence of equal payments made at equal intervals of time is called an *annuity*. Thus the payments for rent, for board, or on an automobile form an annuity. An annuity is called a *contingent annuity* if the payments are dependent on some condition. An annuity is called an *annuity certain* if the payments are to be made regardless of what conditions may arise. We shall consider only annuities certain in this chapter. (Contingent annuities will be discussed in Chapter 16.) The term "annuity"

here will be understood to mean "annuity certain" since no other type of annuity is to be considered for the present.

Annuities are also classified according to the time the payment is made. If a payment is made at the end of each interval, the annuity is called an *ordinary annuity*. If a payment is made at the beginning of each interval, the annuity is called an *annuity due*.

The interval between payments is called the *payment period*. The interval between the beginning of the first payment period and the end of the last one is called the *term* of the annuity and, in general, is measured in years. The interval between consecutive conversions of interest is called the *conversion period* or *interest period*. The amount paid at the end or beginning of each payment period is called the *periodic rent* or *periodic payment*. A *simple annuity* is one in which the payment period and the conversion period coincide; in this chapter we shall study only simple annuities.

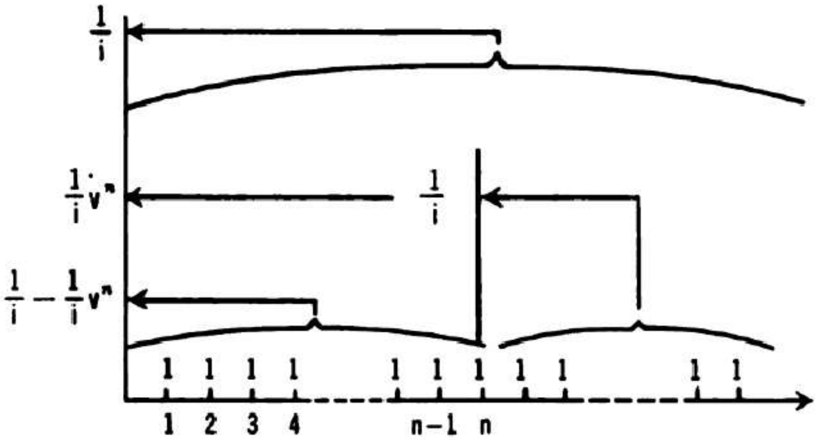
Example. If a series of payments of $50 is made at the end of each 3 months until 14 payments have been made, the periodic rent is $50, the payment period is 3 months, the term is 14 periods, and we have an ordinary annuity certain. It is certain since the payments are not dependent on any condition and it is ordinary because the payments are made at the ends of the intervals. If the interest rate is 6% converted quarterly, the conversion or interest period is 3 months, and the periodic rate is $1\frac{1}{2}\%$.

12-2 PRESENT VALUE OF AN ORDINARY ANNUITY

The value at the beginning of the term of all periodic payments is called the *present value* of the annuity. We shall use the symbol $a_{\overline{n}|i}$, read "*a* angle *n* at *i*," to represent the present value if the periodic payment is 1 and shall now derive a formula for it.

A. *As the difference of two perpetuities.* We shall now determine the present value of the sequence of payments if 1 is paid at the end of each payment period for a term of *n* periods and if money is worth *i* per payment period. If we recall that a perpetuity is an unending sequence of equal payments at equal intervals, we can think of an ordinary annuity as the difference between two perpetuities. The term of one of them begins now and that of the other begins *n* periods from now. We can find the present value of

the ordinary annuity as the difference of the present values of the two perpetuities. By referring to the accompanying diagram, the student will be able to visualize the method. Since the value at the beginning of the term of a perpetuity of 1 per period is the sum that must be invested in



order to furnish the payments, we see that $\frac{1}{i}$ is the present value of a perpetuity of 1 per period beginning now. Furthermore, $\frac{1}{i}$ is the value n periods from now of a perpetuity of 1 per period beginning n periods from now. The value now of this perpetuity can be obtained by discounting for n periods at rate i. Thus

$$\frac{1}{i}(1+i)^{-n}$$

is the value now of a perpetuity of 1 paid at the end of each period with term beginning n periods from now. Consequently, the present value of the annuity of 1 at the end of each period for n periods at rate i per period, if considered as the difference of two perpetuities, is

$$\frac{1}{i} - \frac{1}{i}(1+i)^{-n}$$

$$= \frac{1-(1+i)^{-n}}{i} = a_{\overline{n}|i}$$

Therefore, we know that

(1) $$a_{\overline{n}|i} = \frac{1-(1+i)^{-n}}{i}$$

is the present value of an ordinary annuity of 1 per period for n periods if money is worth i per period.

If the periodic rent is R per period instead of 1, the present value is

$$A = Ra_{\overline{n}|i} = R\frac{1-(1+i)^{-n}}{i} \qquad (1')$$

using the symbol A to represent the present value of an annuity of R per period.

Example. Find the present value of an ordinary annuity of \$50 per year for 16 years if money is worth 3% per year.

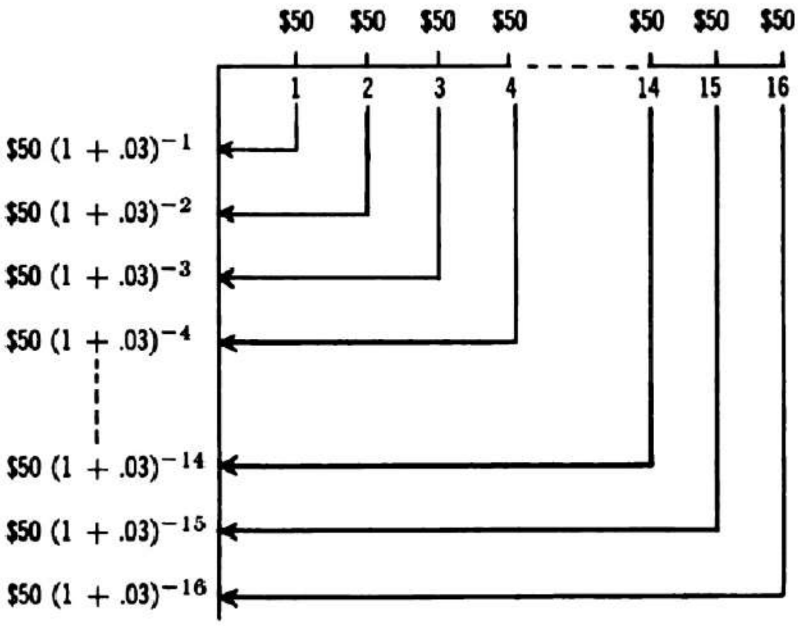
Solution. In this problem, $R = \$50$, $n = 16$, and $i = 3\%$; hence, the symbol for the present value is

$$A = \$50a_{\overline{16}|.03}$$

We can find the value of $a_{\overline{16}|.03}$ from Table IV by looking across from 16 and under 3%; it is 12.56110203. Therefore the present value is

$$\begin{aligned} A &= \$50a_{\overline{16}|.03} \\ &= \$50(12.56110203) \\ &= \$628.06 \end{aligned}$$

B. *As a geometric progression.* Now let us look at the above example from a different point of view. Using the diagram below we will discount each payment separately, for the proper number of periods.



The present value of the first payment is $\$50(1 + .03)^{-1}$.
The present value of the second payment is $\$50(1 + .03)^{-2}$.
The present value of the third payment is $\$50(1 + .03)^{-3}$.
. . . .
The present value of the sixteenth payment is $\$50(1 + .03)^{-16}$.
Consequently, the sum of those present values is

$$\$50a_{\overline{16}|.03} = \$50(1 + .03)^{-1} + \$50(1 + .03)^{-2} + \$50(1 + .03)^{-3} + \ldots + \$50(1 + .03)^{-16}$$

This series is a geometric progression of 16 terms with the first term $\$50(1 + .03)^{-1}$ and common ratio $(1 + .03)^{-1}$. Consequently, its sum is

$$\$50a_{\overline{16}|.03} = \frac{\$50(1.03)^{-1} - (1 + .03)^{-1} \cdot \$50(1 + .03)^{-16}}{1 - (1.03)^{-1}}$$

since the sum of a geometric progression is

$$\frac{\text{the first term} - (\text{the common ratio})\ (\text{the last term})}{1 - \text{the common ratio}}$$

We can simplify this expression for $\$50a_{\overline{16}|.03}$ by multiplying each term of the numerator and denominator by $(1 + .03)$. Thus we have

$$\$50a_{\overline{16}|.03} = \frac{\$50(1+.03)^{-1}(1+.03) - \$50(1+.03)(1+.03)^{-1}(1+.03)^{-16}}{(1 + .03)^{1} - (1 + .03)^{1}\,(1 + .03)^{-1}}$$

$$= \frac{\$50\ (1 + .03)^{0} - (1 + .03)^{0}\ (1 + .03)^{-16}}{(1 + .03)^{1} - (1 + .03)^{0}}$$

since exponents of like bases are added in multiplication. Furthermore, $(1 + .03)^{0} = 1$. Hence

$$\$50a_{\overline{16}|.03} = \$50\,\frac{1 - 1(1.03)^{-16}}{1 + .03 - 1}$$

$$= \$50\,\frac{1 - (1.03)^{-16}}{.03}$$

It should be noticed that this is precisely formula (1′) with $R = \$50$, $i = .03$, and $n = 16$.

Using the above example as a model, the student should now be able to prove that

$$a_{\overline{n}|i} = \frac{1 - (1 + i)^{-n}}{i} \qquad (1)$$

by discounting each payment of 1 for the proper number of interest periods and adding the results by use of the formula for the sum of a geometric progression.

Exercise 12-1

Find the present values of the ordinary annuities in Problems 1 through 8 by using geometric progressions.

1. $500 per year for 20 years at 5% compounded annually.
2. $825 per year for 15 years at $3\frac{1}{2}$% compounded annually.
3. $4000 per year for 5 years at 4% compounded annually.
4. $750 per year for 12 years at 8% compounded annually.
5. $300 per quarter for 50 quarters at 7% compounded quarterly.
6. $442.50 per 6 months for 15 years at 6% compounded semi-annually.
7. $50 per month for 5 years at 6% compounded monthly.
8. $750 per 6 months for 10 years at 5% compounded semi-annually.

9. A house is purchased for $5000 cash and a payment of $500 at the end of each year for 20 years. If money is worth $4\frac{1}{2}$% compounded annually, what is the equivalent cash price of the house?

10. A fraternity chapter purchases a house for $12,500 cash and a payment of $1200 at the end of each year for 25 years. If the mortgage is on the basis of $5\frac{1}{2}$% compounded annually, what is the equivalent cash price of the house?

11. An insurance company sells a man an annuity certain of $4000 per year for 20 years on a 3% basis. What was the purchase price of the annuity?

12. The owner of a piece of property has two offers for it. One is $5000 cash and $1000 at the end of each year for 10 years with interest at 4%; the other is $6000 cash and $800 at the end of each year for 12 years at 5%. Which is the better offer, and by how much?

13. A veteran promises to pay $2000 cash and $240 at the end of each quarter for 20 years in order to purchase a home. If money is worth 6% compounded quarterly, what is the equivalent cash price?

14. The owner of a piece of property has two offers for it. One is $5000 cash and $500 at the end of each 6 months for 10 years with interest at 4% compounded semi-annually; the other is $6000 cash and $200 at the end of each quarter for 12 years at 5% compounded quarterly. Which is the better offer, and by how much?

15. A man puts his farm up for sale at $20,000. He is offered $7500 cash and $1425 at the end of each 6 months for 5 years with interest at $4\frac{1}{2}$% compounded semi-annually. What is the difference between the offer and the price asked?

16. I have an automobile for sale for $1500. I am offered $400 cash and $48 at the end of each month for 2 years. What is the difference between the cash price and the offer if money is worth 6% compounded monthly?

12-3 ACCUMULATED VALUE OF AN ORDINARY ANNUITY

The value, at the end of the term, of the payments for periodic rent is called the *accumulated value* of the annuity. In order to find the accumulated value of an annuity, we can find the sum of the geometric progression formed by the accumulated values of the separate payments, or we can merely accumulate the present value at compound interest for the term of the annuity at the interest rate of the annuity. The term is n periods, the interest rate is i per period and the present value is

$$a_{\overline{n}|i} = \frac{1-(1+i)^{-n}}{i}$$

for an ordinary annuity of 1 per period. Hence, using the symbol $s_{\overline{n}|i}$ to represent this accumulated value,

$$\begin{aligned} s_{\overline{n}|i} &= a_{\overline{n}|i}(1+i)^n \\ &= \frac{1-(1+i)^{-n}}{i}(1+i)^n \\ &= \frac{(1+i)^n - 1}{i} \end{aligned}$$

Therefore

(2) $$s_{\overline{n}|i} = \frac{(1+i)^n - 1}{i}$$

is the accumulated value of an ordinary annuity of 1 per period for n periods at rate i per period.

If the periodic rent is R per period instead of 1, we have as the accumulated value

(2′) $$S = Rs_{\overline{n}|i} = R\frac{(1+i)^n - 1}{i}$$

Example. Find the accumulated value of an ordinary annuity of \$65 per period for 23 periods if money is worth 4% per period.

Solution. In this problem, $R = \$65$, $n = 23$, and $i = 4\%$; hence, the accumulated value is

$$S = \$65\, s_{\overline{23}|.04}$$

The value of $s_{\overline{23}|.04}$ can be found in Table V by looking across from 23 and under 4% and is 36.61788858. Consequently

$$\begin{aligned} S = \$65\, s_{\overline{23}|.04} &= \$65(36.61788858) \\ &= \$2380.16 \end{aligned}$$

Exercise 12-2

Find the accumulated value of the ordinary annuities in Problems 1 through 4 by using geometric progressions, and in Problems 5 through 8 by use of (2).

1. \$300 per year for 20 years at $4\frac{1}{2}\%$ compounded annually.
2. \$528.14 per year for 4 years at $5\frac{1}{2}\%$ compounded annually.
3. \$75 per month for 5 years at 5% compounded monthly.
4. \$750 semi-annually for 10 years at 7% compounded semi-annually.
5. \$2122.36 per year for 27 years at $2\frac{3}{4}\%$ compounded annually.
6. \$10,000 per year for 9 years at $2\frac{1}{4}\%$ compounded annually.
7. \$5000 semi-annually for 9 years at 4% compounded semi-annually.
8. \$125 quarterly for 8 years at 6% compounded quarterly.

9. A corporation sets aside \$51,825.47 at the end of each year for 15 years at 2% compounded annually to retire a bond issue at the end of the time. What amount of bonds can be retired?

10. A man, wishing to retire at age 60, begins at age 35 to set aside \$1000 at the end of each year at 3% compounded annually. If he dies just after making the 18th deposit, how much is in the fund? If he reaches age 60, how much will he have?

11. Work Problem 10 if \$500 is deposited at the end of each 6 months at 3% compounded semi-annually.

12. In order to purchase an automobile 3 years from now, John Joiner invests \$800 at the end of each year at 4% compounded annually. How much will he have in his fund at the end of 3 years?

13. Find the accumulated value of an annuity of \$750 per year for 15 years if money is worth 6% compounded annually. Show that this is equal to $(1.06)^{15}$ multiplied by the present value of the same annuity.

14. Find the accumulated value of an annuity of \$1200 per year for 10 years if money is worth $5\frac{1}{2}\%$ compounded annually. Show that this is equal to $(1.055)^{10}$ multiplied by the present value of the same annuity.

15. Find the accumulated value of an annuity of \$200 paid at the end of each 6 months for 5 years if money is worth 8% compounded semi-annually.

16. Find the accumulated value of an annuity of \$50 paid at the end of each month for 10 years if money is worth 6% compounded monthly.

17. A man deposits \$150 at the end of each 6 months for 12 years in a savings bank that pays $2\frac{1}{2}\%$ compounded semi-annually. How much does he have at the end of the time?

18. At the end of each month for 12 years, \$125 is invested with the Ajax Building and Loan Association, which pays 3% compounded monthly. What is the amount of the investment at the end of the 12th year?

19. A school teacher invests \$20 at the end of each month for 15 years in an investor's syndicate which pays 5% compounded monthly. If at the time of each deposit the syndicate deducts 2% of the deposit to cover its expenses of operation, how much was in the fund at the end of the 15th year?

20. In order to make a down payment on a house 5 years from now, Mr. Lake deposits \$1000 at the end of each year in a Building and Loan Association which pays 6% compounded annually. What amount can be paid down on the house 5 years from now?

12-4 DETERMINATION OF THE PERIODIC RENT

So far we have used formulas

$$A = R\frac{1-(1+i)^{-n}}{i} \tag{1'}$$

and

$$S = R\frac{(1+i)^{n}-1}{i} \tag{2'}$$

for determining the present value and accumulated value, respectively, of an ordinary annuity. It should be noticed that each of these formulas has four unknown quantities in it and, further, that any one of the four can be determined if the other three are known. Thus, if we know the present value or accumulated value, the periodic interest rate and the term of the annuity, we can determine the periodic rent. This type of problem is often encountered, and we shall illustrate its solution with two examples.

Example 1. A man buys a \$20,000 house. He makes a down payment of \$5000, and agrees to pay the balance with interest at 5% compounded annually by making payments at the end of each year for 20 years. Find the annual payment.

Solution. After the down payment of \$5000, the present value of the remaining balance is \$15,000. Hence, we know that $A = \$15{,}000$, $i = 5\%$, and $n = 20$. Substituting in formula (1′) we have

$$R\frac{1-(1.05)^{-20}}{.05} = \$15{,}000$$

or

$$R\, a_{\overline{20}|.05} = \$15{,}000$$

Solving for R, we have

$$R = \frac{\$15{,}000}{a_{\overline{20}|.05}} = \frac{\$15{,}000}{12.46221034} = \$1203.64$$

Hence, 20 annual payments of \$1203.64 are sufficient to pay the debt of \$15,000 with the interest thereon.

Example 2. A man wishes to accumulate \$25,000 at the end of 20 years by making semi-annual deposits in a fund that bears interest at 5% compounded semi-annually. Find the amount he must deposit at the end of each 6 months.

Solution. In this problem we are given $S = \$25{,}000$, $n = 2(20) = 40$ 6-month periods, and $i = \frac{.05}{2} = .025 =$ the periodic interest rate, and we want to find R. Substituting in (2′), we have

$$R\,\frac{(1+.025)^{40}-1}{.025} = \$25000$$

or

$$R\, s_{\overline{40}|.025} = \$25000$$

Solving for R, we have

$$R = \frac{\$25{,}000}{s_{\overline{40}|.025}} = \frac{\$25{,}000}{67.402554} = \$370.91$$

Hence, 40 semi-annual deposits of \$370.91 at 5% compounded semi-annually are sufficient to accumulate a fund of \$25,000 in 20 years.

12-5 THE RELATION BETWEEN $\frac{1}{a_{\overline{n}|i}}$ AND $\frac{1}{s_{\overline{n}|i}}$

In each of the last two examples where we were finding the periodic rent, it should be noticed that a division by $a_{\overline{n}|i}$ or $s_{\overline{n}|i}$ was required. We have included Table VII as a matter of convenience to simplify this type of computation. It gives the values of $\frac{1}{a_{\overline{n}|i}}$ for certain combinations of n and i, and enables us to perform a multiplication instead of a division when finding the periodic rent.

Example 1. In Example 1 of the last section it became necessary to evaluate

$$R = \frac{\$15{,}000}{a_{\overline{20}|.05}} = \frac{\$15{,}000}{12.46221034}$$

This was done by performing the actual division. This becomes unnecessary if we use Table VII for $R = \$15{,}000\left(\frac{1}{a_{\overline{20}|.05}}\right) = \$15{,}000(.08024259) = \$1203.64$. This value of $\frac{1}{a_{\overline{20}|.05}}$ is found by looking across from 20 under 5% in Table VII.

This same table can be used to determine the value of $1/s_{\overline{n}|i}$ because of the relation between $1/a_{\overline{n}|i}$ and $1/s_{\overline{n}|i}$. Since

$$(1 + i)^{-n}\, s_{\overline{n}|i} = a_{\overline{n}|i}$$

it follows that

$$(1 + i)^{-n} = a_{\overline{n}|i}/s_{\overline{n}|i}$$

by dividing by $s_{\overline{n}|i}$. If we substitute this expression for $(1 + i)^{-n}$ in

$$a_{\overline{n}|i} = \frac{1 - (1 + i)^{-n}}{i}$$

we get

$$a_{\overline{n}|i} = \frac{1 - \dfrac{a_{\overline{n}|i}}{s_{\overline{n}|i}}}{i}$$

$$1 = \frac{\dfrac{1}{a_{\overline{n}|i}} - \dfrac{1}{s_{\overline{n}|i}}}{i} \quad \text{dividing by } a_{\overline{n}|i}$$

Finally, multiplying by i, we have

$$i = \frac{1}{a_{\overline{n}|i}} - \frac{1}{s_{\overline{n}|i}}$$

or

(3) $$\frac{1}{s_{\overline{n}|i}} = \frac{1}{a_{\overline{n}|i}} - i$$

We can now find the value of $1/s_{\overline{n}|i}$ from the table of values of $1/a_{\overline{n}|i}$ by subtracting i from the entry for $1/a_{\overline{n}|i}$, regardless of the value of n.

Example 2. In Example 2 of the last section it became necessary to evaluate

$$R = \frac{\$25{,}000}{s_{\overline{40}|.025}} = \frac{\$25{,}000}{67.402554}$$

Again, as in Example 1, this was done by actual division. Now, by the use of Table VII, this becomes unnecessary since

$$R = \frac{\$25{,}000}{s_{\overline{40}|.025}} = \$25{,}000\left(\frac{1}{s_{\overline{40}|.025}}\right) = \$25{,}000(.01483623) = \$370.91$$

This value of $\frac{1}{s_{\overline{40|}.025}}$ is found by looking across from 40 under $2\frac{1}{2}\%$ in Table VII. There, we find .03983623. This is the value of $\frac{1}{a_{\overline{40|}.025}}$ and from it we subtract $i = .025$ to obtain the value of $\frac{1}{s_{\overline{40|}.025}}$.

Exercise 12-3

1. What is the value of $\frac{1}{a_{\overline{16|}.02}}$; of $\frac{1}{s_{\overline{16|}.02}}$?
2. What is the value of $\frac{1}{a_{\overline{24|}.035}}$; of $\frac{1}{s_{\overline{24|}.035}}$?
3. What is the value of $\frac{1}{a_{\overline{8|}.07}}$; of $\frac{1}{s_{\overline{8|}.07}}$?
4. What is the value of $\frac{1}{a_{\overline{50|}.015}}$; of $\frac{1}{s_{\overline{50|}.015}}$?
5. What is the periodic rent of the annuity, payable for 20 years at 5%, whose present value is \$1? Whose present value is \$1000?
6. What is the periodic rent of the annuity, payable for 20 years at 5%, whose amount is \$1? Whose amount is \$1000?
7. What annuity, payable for 15 years at $6\frac{1}{2}\%$, can be bought for \$1? For \$10,000?
8. How much must be set aside at the end of each year for 20 years at $4\frac{1}{2}\%$ in order to accumulate \$1? \$15,000?

Find the periodic rent on the ordinary annuities in Problems 9 through 16.

9. $A = \$8000$, $n = 20$, $i = .05$; $i = .04$.
10. $S = \$8000$, $n = 20$, $i = .05$; $i = .04$.
11. $A = \$4250$, $n = 10$, $i = .045$; $i = .035$.
12. $S = \$4250$, $n = 10$, $i = .045$; $i = .035$.
13. $A = \$2500$, $n = 15$, $i = .04$; $i = .03$.
14. $S = \$2500$, $n = 15$, $i = .04$; $i = .03$.
15. $A = \$10{,}000$, $n = 25$, $i = .055$; $i = .0225$.
16. $S = \$10{,}000$, $n = 25$, $i = .055$; $i = .0225$.
17. A man wishes to accumulate \$5000 in 7 years. How much should be set aside at the end of each 6 months (a) if he can invest his money at 4% compounded semi-annually? (b) If he can invest his money at 6% compounded semi-annually?

18. A bank lends an oil company \$1,000,000 which is to be repaid in 10 equal semi-annual installments at 6% compounded semi-annually. Find the amount of each installment.

19. An endowment policy of \$20,000 is payable at age 65 to Mr. McElvey. He has a choice of taking the cash or an annuity certain payable to him or his heirs at the end of each year for 15 years. If he chooses the annuity, what is the annual payment if $i = .045$?

20. Mr. Johnson purchases a business valued at \$25,000. He pays \$8000 cash and the balance in 4 equal annual payments with interest at 5%. Find the annual payment.

21. The cash price of a house is \$12,000. A down payment of \$3000 is required, and the balance is to be paid off in 20 equal annual installments. Find the amount of each installment (a) if $i = .045$; (b) if $i = .06$.

22. A fraternity buys a house whose cash value is \$30,000. The chapter has \$5000 cash. They obtain the balance of \$25,000 from Carl Thomas, a member, who takes a mortgage on the house and agrees to let them pay him back in 25 annual installments at 7%. After 12 annual payments have been made, Mr. Thomas cuts the balance of the loan by \$3000, and the interest rate to 4% on the unpaid balance. Find (a) the annual payment for the first 12 years; (b) the annual payment for the last 13 years.

23. A municipality issues \$2,500,000 in airport bonds which mature in 25 years. How much must be set aside each year at $1\frac{1}{2}\%$ to pay off the bonds when they mature?

24. A department store sets aside annually at 5% a fund to purchase a delivery truck at the end of 4 years at an estimated price of \$4000. How much should be placed in the fund each year?

12-6 DETERMINATION OF THE TERM

Quite often the present or accumulated value of an annuity, the rate, the time between payments, and the periodic payment are known and the term is to be determined. Under such conditions, an integral number of periodic payments may not pay off the liability; that is, k payments may not be enough and $k + 1$ payments may be too many to pay off the debt. The practice under such conditions is to make k payments of regular size and a payment of irregular size one payment interval later, except as noted in the next sentence. If the accumulated value is involved, k payments may not be enough to pay off the debt but k payments plus the interest on them for one interest period may be too much. If so, the partial payment is made at the end of the kth payment period.

This irregular payment can be determined by means of an equation of value. The focal or comparison date may be chosen at will.

Example 1. Determine the number of full payments of \$70 at the end of each 3 months and the irregular payment to be made 3 months after the last full payment in order to accumulate \$1500 if money is worth 6% converted quarterly.

Solution. In this problem, $R = \$70$, $i = \dfrac{.06}{4} = .015$, $S = \$1500$, and n, the number of quarterly payments, is to be found. Hence

$$\$70 s_{\overline{n}|.015} = \$1500$$

$$s_{\overline{n}|.015} = \frac{\$1500}{70} = 21.42857143$$

This value of $s_{\overline{n}|.015}$ is between that of $s_{\overline{18}|.015} = 20.48937572$ and $s_{\overline{19}|.015} = 21.79671636$.

Therefore, 18 regular payments and an irregular payment, 3 months later, must be made.

If 19 regular payments were made, the accumulated value would be

$$\$70 s_{\overline{19}|.015} = \$70(21.79671636)$$
$$= \$1525.77$$

This is obviously \$25.77 more than the required amount of \$1500. Since the last payment draws no interest, instead of the regular payment of \$70,

$$\$70 - \$25.77 = \$44.23$$

is the irregular payment that must be made 19 quarters or 4 years and 9 months after the beginning of the term in order to accumulate \$1500.

Example 2. Determine the number of full payments of \$150 at the end of each year, and the exact amount of the fractional payment to be made at the end of the next year in order to discharge a debt of \$2000 if money is worth 4% converted annually.

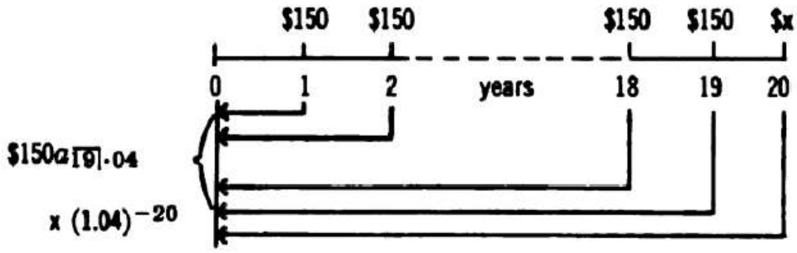
Solution. In this problem $R = \$150$, $A = \$2000$, and $i = .04$. Hence, using the present value formula we have

$$\$150 a_{\overline{n}|.04} = \$2000$$

$$a_{\overline{n}|.04} = 13.333333$$

Using the $a_{\overline{n}|.04}$ table, we see that 13.333333 lies between the values of $a_{\overline{19}|.04}$ and $a_{\overline{20}|.04}$. We shall now find the payment, x, that must be made

at the end of the 20th year, by the use of a line diagram and an equation of value with *now* as the comparison date. We have the following diagram



and the equation

$$\$150a_{\overline{19}|.04} + x(1.04)^{-20} = \$2000$$

Hence

$$\begin{aligned}(1.04)^{-20}x &= \$2000 - \$150a_{\overline{19}|.04}\\ x &= (\$2000 - \$150a_{\overline{19}|.04})\,(1.04)^{20}\\ &= [\$2000 - \$150(13.1339394)]\,(2.19112314)\\ &= (\$2000 - \$1970.09)\,(2.19112314)\\ &= (\$29.91)\,(2.19112314)\\ &= \$65.54\end{aligned}$$

Example 3. How long will it take payments of $600 per year to amount to $10,000 at 5% compounded annually?

Solution. In this problem, $R = \$600$, $S = \$10{,}000$, $i = .05$. Hence, using the formula $S = Rs_{\overline{n}|i}$ we have

$$\$600s_{\overline{n}|.05} = \$10{,}000$$

Therefore

$$s_{\overline{n}|.05} = \frac{\$10{,}000}{\$600} = 16.666667$$

and, by use of the table for $s_{\overline{n}|.05}$ we see that $s_{\overline{n}|.05}$ lies between

$$s_{\overline{12}|.05} = 15.91712652 \text{ and}$$
$$s_{\overline{13}|.05} = 17.71298285$$

Now

$$\$600s_{\overline{12}|.05} = \$600(15.91712652) = \$9550.28$$

and an extra payment of \$449.72 at the time of the 12th payment of \$600 would be sufficient to produce \$10,000. On the other hand

$$\$600 s_{\overline{13}|.05} = \$600(17.71298285)$$
$$= \$10,627.79$$

This is \$27.79 more than the full 13th payment of \$600 and means that one year's interest on \$9550.28 at 5%, or \$477.51, is sufficient to build the fund to \$10,027.79 without any 13th payment. Hence, an extra payment of \$449.72 should be made at the time of the 12th payment of \$600.

Exercise 12-4

1. How long will it take an annuity of \$862.50 per year to amount to \$28,519.39 at 5%?

2. How long will it take an annuity of \$350 per year to amount to \$4828.44 at $2\frac{1}{2}$%?

3. How long will it take to pay off a debt of \$5000 at 4% by paying \$616.45 at the end of each year?

4. How long will it take to pay off a debt of \$3600 at 6% by paying \$332.48 at the end of each year?

5. A debt of \$10,000 at 6% compounded annually is being paid off, principal and interest, by annual payments of \$1000. How many full payments are necessary? How much additional would be necessary to retire the debt at the time of the last full payment? At the end of the next year?

6. How long will it take payments of \$750 per year to accumulate to \$7500 if $i = .035$? Find the partial payment that must be made one year after the last full payment.

7. How long will it take to accumulate \$28,500 at 5% compounded annually if the annual deposit is \$1500? Find the final partial payment that must be made one year after the last full payment.

8. At the end of each year for 10 years, Mr. Harris has been paying \$1200, principal and interest, on a debt that originally was \$18,000. How much does he still owe after the 10th payment if $i = .045$? How many more full payments must he make? Find the final partial payment that must be made one year after the last full payment.

9. On a loan of \$1,500,000 at 3% an oil company repays \$200,000 annually for n years, and a partial payment at the end of $n + 1$ years. Find n and the partial payment.

10. A man and his wife decide to take a trip to Europe as soon as they can accumulate \$5000. If they save \$400 per year and invest it at 4%, how many full deposits of \$400 will be necessary? How much must they save the last year?

11. A house has a cash price of \$30,000. A purchaser pays \$12,000 cash and agrees to pay the balance in quarterly installments of \$500 each. If interest is at 4% compounded quarterly, how many full payments are necessary, and what is the final partial payment that must be made one quarter after the last full payment?

12. In Problem 11, what additional payment would be necessary, at the time the last full payment is made, to pay off the balance of the debt?

12-7 DETERMINATION OF THE RATE

If R, n, and either A or S are known, we can determine the periodic rate by interpolation in either the table for $a_{\overline{n}|i}$ or $s_{\overline{n}|i}$, respectively. If the payment and interest periods are one year, the interpolation yields the effective rate. Otherwise, the rate interpolated for must be multiplied by the number of interest periods per year to determine the nominal annual rate. Since the student is already familiar with linear interpolation, we shall simply illustrate with two examples.

Example 1. An ordinary annuity of \$1200 per year for 20 years accumulates to \$32,500. Find the effective rate.

Solution. In this problem, $R = \$1200$, $n = 20$ and $S = \$32,500$; hence

$$\$1200 s_{\overline{20}|i} = \$32,500$$

and

$$s_{\overline{20}|i} = \frac{32,500}{1200} = 27.08333$$

By the use of Table V, we find that $s_{\overline{20}|.03} = 26.87037$ and $s_{\overline{20}|.035} = 28.27968$. Consequently, the rate is between 3% and $3\frac{1}{2}$%. The table needed for the interpolation is

$$\tfrac{1}{2}\% \left\{ \begin{array}{l} \left. \begin{array}{l} s_{\overline{20}|.03} = 26.87037 \\ s_{\overline{20}|i} \quad = 27.08333 \end{array} \right\} .21296 \\ s_{\overline{20}|.035} = 28.27968 \end{array} \right\} 1.40931$$

Therefore, the rate is $\frac{.21296}{1.40931} = .1511$ of the way from 3% toward $3\frac{1}{2}\%$. Since $(.1511)(3\frac{1}{2}\% - 3\%) = (.1511)(.005) = .000756$, the rate is $3\% + .08\%$ or 3.08% to the nearest 1/100%.

Example 2. A loan of \$84 is to be repaid in 12 equal monthly installments of \$7.76 each. Find the monthly rate. Find the nominal rate compounded monthly.

Solution. In this problem, we have $A = \$84$, $n = 12$, and $R = \$7.76$; hence

$$\$7.76a_{\overline{12}|i} = \$84$$

and

$$a_{\overline{12}|i} = \frac{84}{7.76} = 10.82474$$

This value, from Table IV, lies between

$$a_{\overline{12}|.015} = 10.90751 \text{ and}$$
$$a_{\overline{12}|.0175} = 10.73955$$

The table needed is

$$\frac{1}{4}\% \left\{ \begin{array}{l} a_{\overline{12}|.015} = 10.90751 \\ a_{\overline{12}|i} \quad = 10.82474 \\ a_{\overline{12}|.0175} = 10.73955 \end{array} \right. \begin{array}{l} \left.\right\} .08277 \\ \left.\right\} .16796 \end{array}$$

Therefore, the monthly rate is $\frac{.08277}{.16796} = .4928$ of the way from $1\frac{1}{2}\%$ toward $1\frac{3}{4}\%$. Since $.4928(1\frac{3}{4}\% - 1\frac{1}{2}\%) = (.4928)(.0025) = .001232$, the rate is $1.5\% + .123\% = 1.623\%$. Since $i = \frac{j}{12} = 1.623\%$, the nominal rate compounded monthly is

$$j_{(12)} = 12(1.623\%) = 19.48\% \quad \text{to the nearest } 1/100\%$$

Exercise 12-5

1. Given $A = \$2684.91$, $R = \$880$, $n = 10$, find i.
2. Given $S = \$8731.42$, $R = \$523.90$, $n = 14$, find i.
3. Given $A = \$22{,}916.31$, $R = \$1850$, $n = 20$, find i.
4. Given $S = \$3471.62$, $R = \$600$, $n = 5$, find i.
5. Given $A = \$825$, $R = \$125$, $n = 8$, find i.

6. Given $S = \$1000$, $R = \$490$, $n = 2$, find i.

7. Given $S = \$6500$, $R = \$500$, $n = 10$, find i.

8. Given $S = \$14{,}500$, $R = \$850.62$, $n = 13$, find i.

9. At the end of each quarter for 8 years, a man invests $250 in common stocks. At the end of 8 years he sells out for $12,192.60. If his dividends were paid at the end of each quarter and immediately reinvested in the fund, what quarterly rate was earned? What nominal rate converted quarterly was earned?

10. An annuity with semi-annual payments of $425 for 15 years can be purchased for $8560.29. What semi-annual rate is being paid? What nominal rate converted semi-annually is being paid?

11. At what nominal rate, converted semi-annually, will an annuity of $100 at the end of each 6 months accumulate to $20,000 at the end of 25 years?

12. What is the nominal rate, converted quarterly, if $100 at the end of each 3 months for 12 years has a present value of $2500?

13. Mr. O'Quinn invested $50 per month at the end of each month for 9 years and had $7000 to his credit. What rate, converted monthly, did his investment draw?

14. Mr. Wright borrowed $4200 and paid off his obligation by quarterly payments of $150 at the end of each quarter for 8 years. What rate, converted quarterly, did he pay?

15. If $70.67 at the end of each month for 8 years and 4 months pays off a note of $5654 and interest on it, what is the nominal rate converted monthly?

16. Mr. Scholz bought a house for $9750 and made a cash payment of $1750. He paid off the remainder of the cost by a payment of $437 at the end of each 6 months for 10 years. What rate, converted semi-annually, did he pay?

17. A father deposited $200 at the end of each 6 months for 17 years into an educational fund, and had accumulated $9000 at the end of the time. What rate converted semi-annually did he receive?

18. At what rate, converted semi-annually, will a payment of $250 at the end of each 6 months accumulate to $12,000 in 15 years?

19. What is the rate converted quarterly if 120 quarterly deposits of $50, one at the end of each quarter, are required to pay off a debt of $3475?

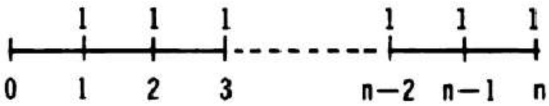
20. At what rate, converted semi-annually, will a payment of $300 at the end of each 6 months for 20 years accumulate to $20,500?

A Dallas bank advertises that it will make home-repair loans according to the following table. Find the nominal rate, converted monthly, in each case.

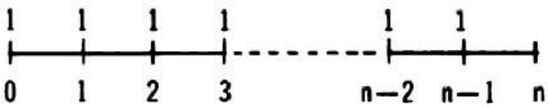
	Loan	*Monthly payment*	*Term*
21.	$ 300	$ 9.58	36 months
22.	$ 400	$12.78	36 months
23.	$ 500	$15.97	36 months
24.	$1000	$31.94	36 months

12-8 ACCUMULATED VALUE OF AN ANNUITY DUE

An *annuity due* was defined in Section 12-1 as an annuity certain in which the periodic payment is made at the beginning of each payment interval. Hence, an annuity due differs from an ordinary annuity only in the time at which each periodic payment is made. The times the payments are made in the two situations are shown in the following diagrams in which n is the number of periodic payments and 1 is the amount of each payment.



Ordinary Annuity



Annuity Due

It follows from the definition and from the diagrams that each payment is made one payment period earlier for an annuity due than for an ordinary annuity. We shall use this fact in deriving a formula for the accumulated value of an annuity due. Since each payment is made one payment period earlier for an annuity due than for an ordinary annuity, each payment draws interest for one more payment period. Consequently, the accumulated value of an annuity due is the value obtained by accumulating the accumulated value of an ordinary annuity for one payment period. Since the interest rate is i per period, the accumulation factor for one period is $1 + i$. Hence, if we use $\ddot{s}_{\overline{n}|i}$ to represent the accumulated value of a simple annuity due of 1 per period for n periods at i per period, we have

(3) $$\ddot{s}_{\overline{n}|i} = (1 + i)\, s_{\overline{n}|i}$$

If the periodic rent is R, then multiplying both sides of (3) by R, and symbolizing the accumulated value by S, we have

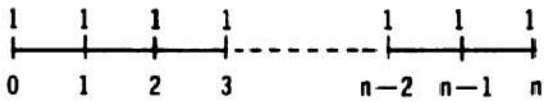
(3′) $$S = R\ddot{s}_{\overline{n}|i} = R(1 + i)\, s_{\overline{n}|i} = R(1 + i)\, \frac{(1 + i)^n - 1}{i}$$

Example 1. Find the accumulated value of an annuity due of \$100, payable semi-annually for 12 years at 6% compounded semi-annually.

Solution. In this example, we have $R = \$100$, $n = 24$, and $i = \frac{j}{2} = \frac{.06}{2} = .03$; hence, substituting in (3′), we have

$$\begin{aligned} S &= \$100\ \ddot{s}_{\overline{24}|.03} \\ &= (\$100)(1.03)\ s_{\overline{24}|.03} \\ &= (\$103)(34.42647022) \\ &= \$3545.93 \end{aligned}$$

A second formula for the accumulated value of an annuity due, which is somewhat simpler than (3′) to use, may be obtained from a consideration of the line diagram of the annuity due with an extra payment of 1 placed at the end of the last period.



With this added payment, the annuity can be considered as an ordinary annuity of $n + 1$ payments, one at the end of each of $n + 1$ intervals. Its accumulated value is $s_{\overline{n+1}|i}$. This symbol includes the extra payment of 1 which we placed at the end of the last interval. If now we subtract this 1 from $s_{\overline{n+1}|i}$, we will obtain the accumulated value of the n payments of the annuity due. Hence, we have

(4) $$\ddot{s}_{\overline{n}|i} = s_{\overline{n+1}|i} - 1$$

If R is the periodic rent, we obtain immediately

(4′) $$S = R\ \ddot{s}_{\overline{n}|i} = R(s_{\overline{n+1}|i} - 1)$$

Example 2. Solving the previous example of this section by this formula, we have

$$\begin{aligned} S &= \$100\ \ddot{s}_{\overline{24}|.03} = \$100(s_{\overline{25}|.03} - 1) \\ &= \$100(36.459264 - 1) \\ &= \$100(35.459264) \\ &= \$3545.93 \end{aligned}$$

It should be noticed that the use of this formula replaces a multiplication by a subtraction which actually can be performed mentally.

12-9 PRESENT VALUE OF AN ANNUITY DUE

To find the present value of an annuity due, symbolized by $\ddot{a}_{\overline{n}|i}$, we will make use of the fact that the present value can be obtained by discounting the accumulated value at the periodic rate for its term. Using (3) and the discount factor at rate i for n periods, we have

$$\begin{aligned}\ddot{a}_{\overline{n}|i} &= (1+i)^{-n}\,\ddot{s}_{\overline{n}|i}\\ &= (1+i)^{-n}\,s_{\overline{n}|i}\,(1+i) \quad \text{substituting for } \ddot{s}_{\overline{n}|i}\end{aligned}$$

Hence

(4) $$\ddot{a}_{\overline{n}|i} = a_{\overline{n}|i}\,(1+i) \quad \text{since } (1+i)^{-n}\,s_{\overline{n}|i} = a_{\overline{n}|i}$$

If R is the periodic rent, we have

(4′) $$\begin{aligned}A &= R\ddot{a}_{\overline{n}|i}\\ &= R(1+i)\,a_{\overline{n}|i}\end{aligned}$$

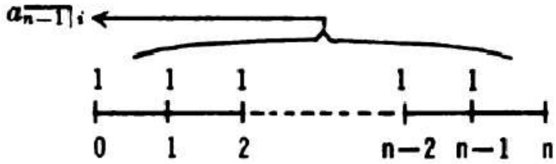
In using formula (4′), it is necessary to find the product of two numbers and then to multiply this by a third number. This can be and often is a tedious operation unless a machine is available.

Example 1. What is the present value of an annuity due of \$250 per quarter for 15 years at 6% compounded quarterly?

Solution. In this problem we have $R = \$250$, $n = (15)(4) = 60$, and $i = \frac{.06}{4} = .015$; hence, by use of (4′), we get

$$\begin{aligned}A &= \$250\ \ddot{a}_{\overline{60}|.015}\\ &= \$250\ (1.015)\ a_{\overline{60}|.015}\\ &= \$253.75(39.380269)\\ &= \$9992.74\end{aligned}$$

Just as in the case of the accumulated value, we can obtain a second formula for the present value of an annuity due by considering a line diagram such as that shown at the top of the following page.



The first payment of 1, due immediately, has a present value of 1. The other $n - 1$ payments can be thought of as being due at the end of each of the first $n - 1$ periods and thus will have a present value of $a_{\overline{n-1}|i}$. Hence, the present value of an annuity due of 1 for n periods at i per period is

(5) $$\ddot{a}_{\overline{n}|i} = 1 + a_{\overline{n-1}|i}$$

and of $\$R$ per period at i per period is

(5′) $$A = R\ddot{a}_{\overline{n}|i} = R(1 + a_{\overline{n-1}|i})$$

Example 2. Solve the first example of this section by formula (5).

$$\begin{aligned} A &= \$250\,\ddot{a}_{\overline{60}|.015} \\ &= \$250(1 + a_{\overline{59}|.015}) \\ &= \$250(1 + 38.970973) \\ &= \$250(39.970973) \\ &= \$9992.74 \end{aligned}$$

It should be noticed that (5′) is easier to use than (4′) because it replaces a multiplication by the addition of 1 to $a_{\overline{n-1}|i}$, which can be done mentally.

Exercise 12-6

Find the present value of each annuity due in Problems 1 through 4.

1. \$200 per year for 10 years at 6%.
2. \$475.50 per year for 13 years at $4\frac{1}{2}$%.
3. \$360 per year for 15 years at $3\frac{1}{2}$%.
4. \$2492.12 per year for 4 years at $2\frac{1}{4}$%.

5. A farmer buys some land and, in addition to a cash payment of \$400, agrees to pay \$400 at the end of each year for 15 years. What cash price would be equivalent to this sequence of payments provided money is worth 4%?

6. A father decided to build up a fund to enable his son to start in business on his 25th birthday. In order to do this, he deposited $250 in a savings bank, which paid 3% converted semi-annually, on the day the child was born, and another $250 at the beginning of each 6 months until and including the day the son was $24\frac{1}{2}$ years old. How much was available on the son's 25th birthday?

7. Find the present value of $200 at the beginning of each quarter for 13 years if money is worth 6% converted quarterly.

8. Find the amount to which $250 at the beginning of each quarter for 17 years will accumulate if money is worth 5% converted quarterly.

9. In order to pay for an addition to its factory, a firm paid $350 down and agreed to pay a like sum at the end of each 6 months for the next $27\frac{1}{2}$ years. What was the cash price of the addition if money is worth 3% converted semi-annually?

10. What quarterly payment made at the beginning of each period for 13 years will accumulate to $9600 if money is worth 6% converted quarterly?

11. A home was listed for $10,200 cash. A buyer wanted the place and was willing to make a very small cash payment and to pay a similar amount at the end of each 6 months from the time of purchase until a total of 30 payments had been made. How much should the cash payment be if money is worth 4% converted semi-annually?

12. A family owes $7478.96 on its home. At what rate converted semi-annually will $250 at the beginning of each 6 months for 20 years pay off the obligation?

13. How many quarterly payments of $100 made at the beginning of the period, and what partial payment at the end of the last period, are equivalent to a cash price of $3311.83 if money is worth $j = 4\%$, $m = 4$?

14. An ex-serviceman bought a house for a cash payment of $840 and a promise to pay $840 on each anniversary of the date of purchase until a total of 21 payments, including the cash one, had been made. What was the equivalent cash price of the house, provided the interest rate was 5% compounded annually?

15. How much will be to one's credit in 10 years if he pays $75 at the beginning of each month into a fund that earns 4% compounded monthly?

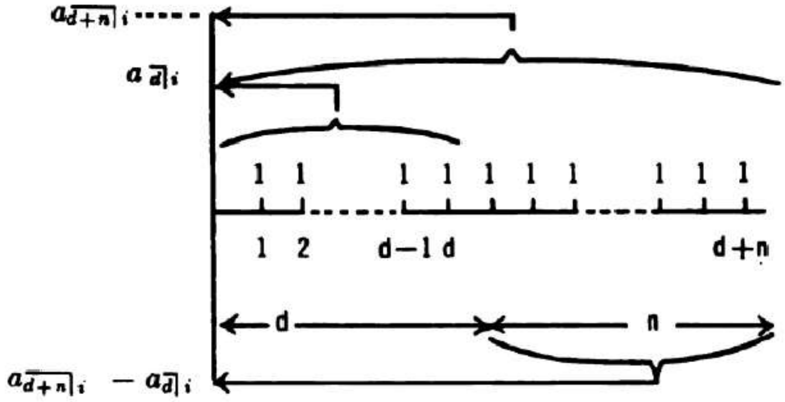
16. Determine the semi-annual payment, made at the beginning of each 6 months, that is required to pay off a loan of $15,000 in 40 payments if $j = .055$, $m = 2$.

12-10 DEFERRED ANNUITIES

A *deferred annuity* is an annuity whose term does not begin until some future date. The time until that future date is called the *term of deferment* and we shall represent it by d. We shall assume that each payment is made at the end of a period after the term of deferment has expired. Thus,

we shall consider only deferred ordinary annuities; if, however, a deferred annuity due should arise, it could be transformed into a deferred ordinary annuity by shortening the term of deferment by one payment period.

We shall now determine the present value of a deferred annuity whose term is n periods after a deferment interval of d periods and with interest rate i per period. If payments had been made during the deferment interval, the term of the annuity would have been $(n + d)$ periods; hence, we can find the present value of the deferred annuity by subtracting the present value of an annuity with d periods as its term for the present value of an annuity whose term is $(n + d)$ periods since this is adding in and subtracting out the present value of an annuity whose term is d periods.



Therefore, using the symbol ${}_{d}|a_{\overline{n}|i}$ to represent the present value of the deferred annuity, we see that

$$(6) \qquad {}_{d}|a_{\overline{n}|i} = a_{\overline{n+d}|i} - a_{\overline{d}|i}$$

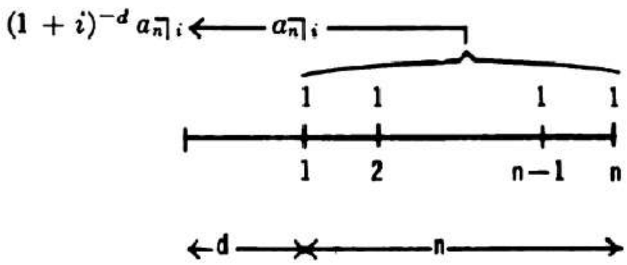
and consequently

$$(6') \qquad A = R\,{}_{d}|a_{\overline{n}|i} = R(a_{\overline{n+d}|i} - a_{\overline{d}|i})$$

where R is the periodic rent.

Formula (6′) is rather simple to use since to do so merely requires a subtraction and a multiplication. An example is given just after (7′) is derived.

An alternate formula may be obtained by finding the value of the annuity at the beginning of the first actual payment interval, and then discounting this for d periods at rate i per period as indicated in the diagram at the top of the following page.



Thus

(7) $$_{d|}a_{\overline{n}|i} = (1+i)^{-d}a_{\overline{n}|i}$$

and if R is the periodic rent

(7′) $$R\,_{d|}a_{\overline{n}|i} = R(1+i)^{-d}a_{\overline{n}|i}$$

Example 1. On January 1, 1958 a philanthropist announced that he was going to give an educational institution \$50,000 on January 1, 1963, and on each January 1 thereafter until 20 such donations had been made. What was the value of the gift on the date of the announcement if money is worth 4% compounded annually?

Solution 1. The payments on this gift form a deferred ordinary annuity of \$50,000 per year with a term of 20 years, deferment period 4 years (January 1, 1958 until January 1, 1962), and $i = 4\%$. Hence, substituting these values in (6′) we have

$$\begin{aligned} A &= \$50{,}000\ _{4|}a_{\overline{20}|.04} \\ &= \$50{,}000(a_{\overline{24}|.04} - a_{\overline{4}|.04}) \\ &= \$50{,}000(15.24696314 - 3.62989522) \\ &= \$50{,}000(11.61706792) \\ &= \$580{,}853.40 \end{aligned}$$

Solution 2. Substituting in (7′) we have

$$\begin{aligned} A &= \$50{,}000\ (1.04)^{-4}\, a_{\overline{20}|.04} \\ &= \$50{,}000(.85480419)(13.59032634) \\ &= \$50{,}000(11.6170679) \\ &= \$580{,}853.40 \end{aligned}$$

Example 2. A man borrowed \$15,000 at 5% compounded semi-annually and agreed to pay it back in 12 semi-annual installments with the first one due $3\frac{1}{2}$ years after he received the money. Find the semi-annual installment.

Solution. In this example $A = \$15{,}000$, $n = 12$, $i = \dfrac{.05}{2} = .025$, the deferment period is 6 semi-annual periods and we want to find R; hence, using (6′), we have

$$
\begin{aligned}
R_6|a_{\overline{12}|.025} &= \$15{,}000 \\
R(a_{\overline{18}|.025} - a_{\overline{6}|.025}) &= \$15{,}000 \\
R &= \frac{\$15{,}000}{a_{\overline{18}|.025} - a_{\overline{6}|.025}} \\
&= \frac{\$15{,}000}{14.35336363 - 5.50812536} \\
&= \frac{\$15{,}000}{8.84523827} \\
&= \$1695.83
\end{aligned}
$$

Exercise 12-7

Determine the value of A in Problems 1 through 8, and the value of R in Problems 9 through 12.

	A	R	i	n	d
1.	?	\$150	5%	10	3
2.	?	200	3%	12	2
3.	?	900	2%	20	4
4.	?	800	$1\frac{1}{2}$%	80	8
5.	?	1000	4%	15	5
6.	?	227	6%	25	10
7.	?	540	3%	5	2
8.	?	875	$5\frac{1}{2}$%	18	4
9.	\$6,000	?	$1\frac{1}{4}$%	32	16
10.	9,800	?	$4\frac{1}{2}$%	16	5
11.	10,120	?	6%	20	3
12.	18,000	?	3%	17	7

13. A farmer borrowed money to buy a farm on May 1, 1958. He did not have to make a payment until November 1, 1963, and then was to continue semi-annual payments of $600 until and including May 1, 1983. How much did he borrow if the interest rate was 5% compounded semi-annually?

14. Find the present value of an ordinary annuity with term 30 years deferred 5 years if the annual rent is $2400 payable quarterly and money is worth 4% converted quarterly.

15. A man who was starting a business borrowed a certain sum on June 8, 1958, on the condition that he was to make the first payment on December 8, 1963, and was to continue quarterly payments of $250 until and including one on June 8, 1973. If money was worth 4% converted quarterly, how much did he borrow?

16. A philanthropist announced that he would give $75,000 to a certain college in 5 years and continue giving that amount annually until 29 years from the time of the announcement. What was the cash value of the gift on the date of his announcement if money was worth 5%?

17. A wealthy member of a congregation promised to give $1500 toward a new building the day it was completed and the same sum semi-annually for 12 years. If the building was completed 2 years after the promise and if money was worth 3½%, compounded semi-annually, what was the cash equivalent of his gift?

18. On the seventeenth birthday of his favorite grandson, a grandfather agreed to give him $150 per month for 12 years beginning one month after he was 21 years of age. What was the value of the gift on the day the promise was made, provided money was worth 4% converted monthly?

19. A man gave a librarian the choice between $3100 cash and $100 per quarter for 15 years for the library. If money was worth 5% converted quarterly and the first payment was to be 3 years and 3 months after the choice was made, which should the librarian have chosen?

20. A medical graduate needed cash for office equipment and borrowed it with the agreement that he would make the first semi-annual repayment of $400 in 3½ years, plus 15 other like sums. How much did he borrow if money was worth 4% converted semi-annually?

SUMMARY

The present and accumulated values of an ordinary annuity with periodic rent R, periodic interest rate i for n periods are found to be

$$A = Ra_{\overline{n}|i} = R\,\frac{1-(1+i)^{-n}}{i}$$

and

$$S = Rs_{\overline{n}|i} = R\,\frac{(1+i)^{n}-1}{i}$$

The present value and the accumulated value of an annuity due are the values of the corresponding ordinary annuity accumulated for one interest period, that is multiplied by the factor $1 + i$. They also may be obtained by using the formulas

$$A = R\ddot{a}_{\overline{n}|i} = R(1 + a_{\overline{n-1}|i})$$

and

$$S = R\ddot{s}_{\overline{n}|i} = R(s_{\overline{n+1}|i} - 1)$$

The present value of an ordinary annuity whose term is n periods, deferred d periods, whose periodic rent is R, and periodic interest rate is i is given by either of the formulas

$$A = R_d|a_{\overline{n}|i} = R(1 + i)^{-d}a_{\overline{n}|i}$$

or

$$A = R_d|a_{\overline{n}|i} = R(a_{\overline{n+d}|i} - a_{\overline{d}|i})$$

Exercise 12-8 (Review)

1. What did a family pay for a home if it was paid for by a cash payment of 10% of the cost and \$300 at the end of each 6 months for 20 years? Assume that money was worth 4% converted semi-annually.

2. A farmer bought a \$12,000 addition to his holdings. How much must he pay at the end of each 6 months in order to pay off the loan in 15 years if money is worth 5% converted semi-annually?

3. In order to accumulate a fund for retiring on his 65th birthday, a man decided on his 40th birthday to deposit \$300 at the end of each year in a fund that pays $3\frac{1}{2}$%. How much was in the fund after the payment on his 65th birthday?

4. In order to build up a fund of \$4000 to be used to send his son to college, a man made a deposit at the end of each year from the time of birth of the child until and including his eighteenth birthday. How much was deposited each year if money was worth 3%?

5. A business firm added a building to its holdings and paid for it by annual payments at the end of each year for 12 years. If each payment was \$5000 and if money was worth 4% converted annually, what was the cost of the building?

6. In order to accumulate a fund to buy houses during an anticipated period of depression, a firm put aside \$15,000 at the end of each year for 7 years. How much was in the fund at the end of the time if it drew interest at $4\frac{1}{2}$%

7. Find the present value of an annuity of \$400 payable at the end of each 3 months for 12 years if $j = .06$, $m = 4$.

8. Will a person be able to buy a \$2300 lot at the end of 5 years if he deposits \$400 at the end of each year in a fund that accumulates at $3\frac{1}{2}\%$? If not, how much will be needed in addition to his savings?

9. Mr. Kahn decided to build up a fund of \$10,000 in 15 years by making annual investments at 5%. He made the deposits for 9 years but could not make the 10th one. How much was then to his credit if the deposits were made at the end of each year?

10. In addition to the down payment, a house costs \$65 per month at the end of each month for 15 years. If the down payment was \$4000 and money is worth 4% compounded monthly, what was the cash price of the house?

11. A friend of mine smokes 100 ten-cent cigars per month and pays the bill at the end of the month. If he started this the day he was 25 years of age and continues it until he is retired at 70, how much could he have in a retirement fund from the money he is now spending on cigars, provided money is worth 3% compounded monthly? *Hint.* $1.0025^{540} = [1.0025^{250}]^2\,(1.0025)^{40}$.

12. An employee of a manufacturing concern has a choice of two retirement plans. Under the first plan, he gets \$125 per month at the end of each month as long as he lives. Under the other plan, he gets \$100 per month as long as he lives, and, if that is less than 10 years, his estate receives \$100 per month for the remainder of the 10 years. If he lives 8 years and money is worth 6% converted monthly, which plan would be the more desirable? On the day of retirement what difference would there be between the values of the plans?

13. A family rented a house for 14 years at \$45 per month in advance. What cash price for a house at the beginning of the time would have cost the same amount provided money is worth 6% converted monthly?

14. Find the present value of an annuity due of \$1000 per year for 20 years at 5%.

15. Find the present value of an annuity due of \$21.32 per period for 120 periods at $\frac{1}{8}\%$ per period.

16. In addition to a down payment of \$3000, the buyer of a house agreed to pay \$53 per month in advance for 12 years. If money is worth 5% converted monthly, what was the cash price of the house?

17. The buyer of a business made a cash payment of \$2700. In addition, he agreed to pay \$1000 quarterly in advance for 5 years. What was the cash price of the business if money was worth 5% converted quarterly?

18. In order to accumulate a retirement fund, a man invested \$400 on his 45th birthday and quarterly thereafter until he was 64 years and 9 months of age. How much was to his credit at 65 if the investments drew 4% converted quarterly?

19. If one deposits \$500 in a savings bank on each January 1 beginning January 1, 1966 and ending on January 1, 1982, how much will be to his credit on January 1, 1983, provided the bank pays 3% converted annually?

20. How much should one pay now for the privilege of receiving $600 cash and $600 at the end of each year for 22 years provided money is worth 5% converted annually?

21. In order to build up a reserve, a firm decided to set aside $900 per year in two equal installments, one at the beginning of the year and the other 6 months later. If this was continued for a total of 12 years, how much reserve was built up at the end of the 12th year, provided money is worth 7% converted semi-annually?

22. Given $A = \$1493.26$, $R = \$530.00$, $n = 3$, find i.

23. Given $S = \$48{,}729.40$, $R = \$1950$, $n = 20$, find i.

24. Given $A = \$17{,}492.16$, $R = \$1925$, $n = 15$, find i.

25. Given $S = \$13{,}548.20$, $R = \$406.50$, $n = 18$, find i.

26. A house sells for $10,000 cash, or $1000 down and $1000 at the end of each year for 11 years. If the buyer can borrow money at 4%, should he do so in order to pay cash?

27. An oil man invests $1,000,000 in an ordinary annuity that will pay him or his heirs $60,000 per year for 25 years. At what rate is interest being paid?

28. At what effective rate will a payment of $500 at the end of each year for 20 years have an accumulated value of $14,500?

29. What is the effective rate if $200 at the end of each year for 15 years accumulates to $4073?

30. What is the cash equivalent of an annuity of $500 at the end of each year with a term of 10 years deferred 4 years if money is worth 5%?

31. A grandfather made an agreement with his grandson on his 21st birthday. The agreement was that the grandson was to receive $1000 at the end of each 6 months until 14 payments were received and that the first was to be given when the boy was $25\frac{1}{2}$ years of age. If money was worth $4\frac{1}{2}\%$ converted semi-annually, what was the value of the gift on the day it was promised?

32. A farmer bought a $15,000 tract of land and signed a 4% note for it. The note stated that the interest was to be added to the principal for the first 5 years, and at the end of the fifth year and annually thereafter until a total of 25 payments were made, equal payments were to be made until the debt was paid off. How much was each payment if money was worth $3\frac{1}{2}\%$ after the fifth year?

33. How many payments of $300 are required to save $1600 if invested at the end of each 6 months at 5% converted semi-annually? What additional partial payment is needed at the time of the last full payment to have exactly $1600?

34. A man wants to accumulate $23,000 by making deposits of $900 each at the end of each year as long as necessary. What partial payment, in addition

to the usual $900, must be made at the time of the last full payment in order to have exactly $23,000 if money is worth $3\frac{1}{2}\%$?

35. At what rate will $200 at the end of each year accumulate to $2700 in 11 years?

36. What is the rate if the present value of $500 at the end of each year for 12 years is $4500?

37. In order to accumulate $3200 to help pay their son's college expenses, a couple started depositing $50 per quarter in a fund. If the first deposit was made on the son's fifth birthday, and the last was made when he was $17\frac{3}{4}$ years old, what rate converted quarterly was earned on the investment?

38. A couple decided to accumulate $20,000 toward retiring on the husband's 65th birthday. They made a semi-annual deposit of $200 at the end of each 6 months for 30 years. What rate converted semi-annually did they receive?

39. In order to accumulate $12,200 in 20 years by quarterly payments of $100 at the end of each quarter, what rate converted quarterly must be earned?

40. Mr. Thomas owes $6300 on his house and can pay it off by payments of $100 at the end of each quarter for 25 years. What rate converted quarterly does he pay?

41. At what effective rate will $600 at the beginning of each year for 15 years accumulate to $12,014.16?

42. How many semi-annual payments of $100 made at the beginning of each period are required in order to accumulate $4,129.86 provided money is worth 3% converted semi-annually?

43. Determine the annual payment made at the beginning of each year that is required in order to have $8,976.11 at the end of 22 years. Assume that money is worth 5% compounded annually.

44. A buyer agreed to pay $60 per month in advance for 14 years for a house in addition to a down payment of $2000. What was the equivalent cash price if money is worth 5% converted monthly?

13

Amortization and sinking funds

13-1 AMORTIZATION

People in general say that a debt is amortized if it is paid off by any set of payments. In this chapter, however, we shall say that a debt is *amortized* if it and interest on it are paid off by a sequence of equal payments at equal intervals. If the payments are made at the ends of the intervals, they form an ordinary annuity whose present value is the original debt.

Example. Mr. Jones bought a lot for \$4500 and made a cash payment of \$500. What semi-annual payment

must be made at the end of each 6 months for 3 years to finish paying for the lot if money is worth 5% converted semi-annually?

Solution. Since the payments are to be equal and are to be made at equal intervals, they form an ordinary annuity. We have $A = \$4500 - \$500 = \$4000$, $n = 2(3) = 6$ and $i = \frac{1}{2}(5\%) = 2.5\%$; hence, the formula for the present value of an annuity becomes

$$\$4000 = Ra_{\overline{6}|.025}$$

Therefore, solving for R, we see that

$$\begin{aligned} R &= \frac{\$4000}{a_{\overline{6}|.025}} \\ &= \$4000\,(.18154997) \\ &= \$726.20 \end{aligned}$$

is the amount of each semi-annual payment.

It is customary for lending agencies to use any fractional part of a cent as a whole cent in determining the periodic payment. The reason for doing this is purely psychological since most people are happy if the final payment is a few cents less than the others but are unhappy if it is larger than others.

13-2 OUTSTANDING PRINCIPAL

The debtor or creditor may want to liquidate the debt by making a cash payment at some time; hence, it is desirable to be able to determine the outstanding principal or liability at a specified time. The *outstanding principal* or *liability* at any time is the value at that time of all future payments. If the time is immediately after a periodic payment has been made and if there are still k payments to be made, then the outstanding principal is the present value of an ordinary annuity of R per period for k periods at rate i per period. Therefore

$$\textit{Outstanding principal} = A = Ra_{\overline{k}|i}$$

Example 1. Find the outstanding liability of the debt in the example of Section 13-1 just after the second periodic payment is made.

Solution. In that problem, there are still four payments to be made after the second one since there are six in all. We found that each should be $726.20; hence, the outstanding principal just after the second periodic payment is

$$\begin{aligned} A &= \$726.20\, a_{\overline{4}|.025} \\ &= \$726.20\,(3.76197421) \\ &= \$2731.95 \end{aligned}$$

There is a second method for determining the outstanding liability just after a payment has been made. It consists of finding how much the debt would have accumulated to if no payments had been made, and then subtracting from that the accumulated value of the payments that have been made.

Example 2. Use the method just described to find the outstanding liability of the debt of the example of Section 13-1 just after the second periodic payment.

Solution. The second periodic payment is made at the end of a year after the debt of $4000 is contracted; hence, at that time it has accumulated to

$$\begin{aligned} \$4000\,(1.025)^2 &= \$4000\,(1.050625) \\ &= \$4202.50 \end{aligned}$$

Furthermore, the value of the two payments at the time of the second one is

$$\begin{aligned} \$726.20\, s_{\overline{2}|.025} &= \$726.20\,(2.025) \\ &= \$1470.56 \end{aligned}$$

Therefore, the outstanding liability is

$$\$4202.50 - \$1470.56 = \$2731.94$$

This differs from the outstanding liability as determined by Example 1 but that should not be surprising since the periodic payment used in the two calculations is correct only to the nearest cent.

Exercise 13-1

Find the periodic payment that is necessary to amortize each of the following debts by means of a simple annuity where j is the nominal rate that is converted m times per year for t years.

	Debt	*j*	*m*	*t*		*Debt*	*j*	*m*	*t*
1.	$ 5,000	4%	2	8	**2.**	$ 3,000	5%	2	5
3.	$ 2,000	5%	4	3	**4.**	$ 3,500	4%	4	4
5.	$12,000	6%	12	16	**6.**	$ 8,000	7%	12	15
7.	$10,000	6%	12	15	**8.**	$11,000	5%	12	16

Determine the outstanding principal at the indicated time in each of Problems 9 through 12 by finding the value of all future payments, and in each of Problems 13 through 16 by the second method.

9. Just after the 10th payment in Problem 1.
10. Just after the 6th payment in Problem 2.
11. Just after the 8th payment in Problem 3.
12. Just after the 12th payment in Problem 4.
13. Just after the 144th payment in Problem 5.
14. Just after the 100th payment in Problem 6.
15. Just after the 150th payment in Problem 7.
16. Just after the 180th payment in Problem 8.
17. Mr. Gault bought a house for $18,200 and made a cash payment of $3200. He agreed to pay off the remainder in monthly installments for the next 15 years. If money is worth 6% converted monthly, what single payment just after the 96th monthly payment would equitably pay off the remaining liability?
18. What single payment just after the 6th semi-annual payment will finish paying for a house that was bought for $9600 and was to have been paid for by equal semi-annual payments at the ends of the periods for 5 years? Assume money is worth 5% compounded semi-annually.
19. A debt of $42,000 is being amortized by quarterly payments for 4 years with money worth 6% converted quarterly. Just after the 10th payment, an agreement is reached that calls for the remaining liability to be repaid in two equal payments due in 6 months and a year. How much should each be if the new interest rate is 7% converted semi-annually?
20. Mr. Miller owes $128,000 on his ranch and is repaying it and interest at 4% by a series of eight equal annual payments with the first due one year after incurring the debt. Just after the third payment, a new agreement is reached, calling for the remaining liability to be paid off by four equal semi-annual payments. How much is each if the first is due 6 months after the agreement and if the new interest rate is 5% converted semi-annually?

13-3 AMORTIZATION IN TERMS OF SIMPLE INTEREST

We shall assume that we owe a debt of A and that it and interest on the outstanding principal at the rate i per period are to be repaid by a sequence

of n equal payments of R at the ends of the periods; furthermore, we shall determine R without using compound interest and shall see that it is given by the equation

$$A = Ra_{\overline{n}|i}$$

Since the debt is A and the interest rate per period is i, it follows that the interest for that period is Ai and the accumulated value at the end of the first period is $A(1 + i)$. There is, however, a payment of R made at this time; hence, the outstanding principal at the end of the first period is

$$(1) \qquad A_1 = A(1 + i) - R$$

Similarly, the accumulated liability at the end of the second period is $A_1(1 + i)$ and the outstanding principal just after the payment of R is

$$A_2 = A_1(1 + i) - R$$

$$(2) \qquad A_2 = A(1 + i)^2 - R[1 + (1 + i)] \quad \text{using (1)}$$

Now, the accumulated liability at the end of the third period is $A_2(1 + i)$ and the outstanding principal just after the payment of R is

$$A_3 = A_2(1 + i) - R$$

$$A_3 = A(1 + i)^3 - R[1 + (1 + i) + (1 + i)^2] \quad \text{using (2)}$$

If this process is continued, we find that the outstanding principal just after the payment of R at the end of the nth period is

$$\begin{aligned} A_n &= A(1 + i)^n - R[1 + (1 + i) + (1 + i)^2 + \cdots + (1 + i)^{n-1}] \\ &= A(1 + i)^n - R\frac{1 - (1 + i)^n}{-i} \end{aligned}$$

since the terms in the bracket form a geometric progression with $a = 1$, $r = 1 + i$, and $l = (1 + i)^{n-1}$; hence, multiplying the numerator and denominator of the coefficient of R by -1, we have

$$\begin{aligned} A_n &= A(1 + i)^n - R\,\frac{(1 + i)^n - 1}{i} \\ &= 0 \end{aligned}$$

since the debt is repaid in n payments. Therefore

$$A(1 + i)^n = R\,\frac{(1 + i)^n - 1}{i}$$

$$\begin{aligned} A &= R\,\frac{(1 + i)^n - 1}{i(1 + i)^n} && \text{dividing through by } (1 + i)^n \\ &= R\,\frac{1 - (1 + i)^{-n}}{i} && \text{performing the indicated division by } (1 + i)^n \\ &= Ra_{\overline{n}|i} \end{aligned}$$

Thus, we see that amortization can be thought of in terms of simple interest.

13-4 AN AMORTIZATION SCHEDULE

It is often desirable to show the progress of the amortization of a debt by making a table showing the part of each payment that is used for interest and the part that goes toward reducing the debt. Such a table is called an *amortization schedule* and usually also shows the outstanding principal at the end of each period.

Example 1. A debt of \$5000 bears interest at 4% and is to be repaid by six equal annual payments at the ends of the years. Make an amortization schedule for the debt.

Solution. We must first find the annual rent. Since $A = \$5000$, $i = 4\%$ and $n = 6$ the annual payment is R as determined by

$$\$5000 = Ra_{\overline{6}|.04}$$

Hence

$$\begin{aligned} R &= \frac{\$5000}{a_{\overline{6}|.04}} \\ &= \$5000\ (.19076190) \\ &= \$953.81 \end{aligned}$$

The arrangement of the following table enables us to make the necessary calculations rather readily.

Period	*Principal at beginning*	*Interest at 4%*	*Total payment*	*Payment on principal*	*Principal at end*
1	\$5000.00	\$200.00	\$953.81	\$753.81	\$4246.19
2	4246.19	169.85	953.81	783.96	3462.23
3	3462.23	138.49	953.81	815.32	2646.91
4	2646.91	105.88	953.81	847.93	1798.98
5	1798.98	71.96	953.81	881.85	917.13
6	917.13	36.69	953.82	917.13	(none)

The first entry in the Principal at beginning column is the original debt. Each entry in the Interest column is obtained by multiplying the interest

rate by the entry in the Principal at beginning column. Each entry in the Payment on principal column is obtained by subtracting the interest for the period from the total payment. If the Payment on principal entry is subtracted from the corresponding entry in the Principal at beginning column, we obtain the entry in the Principal at end column and this is the principal at the beginning of the next period.

NOTE. If necessary, the final total payment is adjusted so that it exactly takes care of the outstanding principal and interest on it. This can become necessary even though the total payment is to the "over cent" when a fraction is involved since the entries in the interest column are correct to only the nearest cent. This situation did arise in making the above table and one cent was added to the final total payment.

Example 2. Page 192 is part of a table used by the Federal Housing Administration to give the borrower details on the distribution of each monthly payment on a \$4000, 20-year loan that bears interest at 4% converted monthly. Quite often the taxes and insurance are included in the monthly payment but they cannot be included in the table since they vary from year to year.

The monthly payment of \$24.24 on principal and interest was found by solving

$$\$4000 = Ra_{\overline{240}|}\,{}_{\frac{1}{3}}$$

for R. The \$19.70 entry in the Mortgage insurance column is $\frac{1}{2}\%$ of the average Balance due for the first year and is paid at the time the loan is made. The \$18.96 entry in that column is $\frac{1}{2}\%$ of the average Balance due for the second year and is collected monthly during the first year of the loan so as to be on hand at the beginning of the second year. Each Payment to interest entry is $\frac{1}{12}$ (.04) of the Balance due at the end of the previous month. Each Payment to principal entry is \$24.24 minus the Payment to interest. Each Balance due entry, after the \$4000, is the previous one minus the Payment to principal.

FEDERAL HOUSING ADMINISTRATION AMORTIZATION SCHEDULE

Monthly Payment to Principal and Interest, $24.24

Payment Number	Mortgage Insurance Premium ½ Per Cent	Payment to Interest 4 Per Cent	Payment to Principal	Total Monthly Payment	Balance Due
	$19.70				$4,000.00
1	1.58	$13.33	$10.91	$25.82	3,989.09
2	1.58	13.30	10.94	25.82	3,978.15
3	1.58	13.26	10.98	25.82	3,967.17
4	1.58	13.22	11.02	25.82	3,956.15
5	1.58	13.19	11.05	25.82	3,945.10
6	1.58	13.15	11.09	25.82	3,934.01
7	1.58	13.11	11.13	25.82	3,922.88
8	1.58	13.08	11.16	25.82	3,911.72
9	1.58	13.04	11.20	25.82	3,900.52
10	1.58	13.00	11.24	25.82	3,889.28
11	1.58	12.96	11.28	25.82	3,878.00
12	1.58	12.93	11.31	25.82	3,866.69
	18.96	157.57	133.31	309.84	
13	1.53	12.89	11.35	25.77	3,855.34
14	1.53	12.85	11.39	25.77	3,843.95
15	1.53	12.81	11.43	25.77	3,832.52
16	1.53	12.78	11.46	25.77	3,821.06
17	1.53	12.74	11.50	25.77	3,809.56
18	1.53	12.70	11.54	25.77	3,798.02
19	1.53	12.66	11.58	25.77	3,786.44
20	1.53	12.62	11.62	25.77	3,774.82
21	1.53	12.58	11.66	25.77	3,763.16
22	1.53	12.54	11.70	25.77	3,751.46
23	1.53	12.50	11.74	25.77	3,739.72
24	1.53	12.47	11.77	25.77	3,727.95
	18.36	152.14	138.74	309.24	
. . .	. . .	. . .	. . .	. . .	. . .
229		.95	23.29	24.24	261.11
230		.87	23.37	24.24	237.74
231		.79	23.45	24.24	214.29
232		.71	23.53	24.24	190.76
233		.64	23.60	24.24	167.16
234		.56	23.68	24.24	143.48
235		.48	23.76	24.24	119.72
236		.40	23.84	24.24	95.88
237		.32	23.92	24.24	71.96
238		.24	24.00	24.24	47.96
239		.16	24.08	24.24	23.88
240		.08	23.88	23.96	
		6.20	284.40	290.60	

Exercise 13-2

Form an amortization schedule for each of the following debts where the rate j is converted m times per year for t years. Assume that the payment period and interest period coincide and that the first payment is made one period after the debt is incurred. If the amortization requires more than 8 periods, make the table for only 8 periods.

	Debt	j	m	t		*Debt*	j	m	t
1.	\$ 8,000	4%	2	4	2.	\$ 9,000	6%	2	3
3.	\$11,000	5%	1	7	4.	\$21,000	8%	4	2
5.	\$ 7,200	6%	2	3	6.	\$ 9,600	4%	1	5
7.	\$13,400	8%	4	2	8.	\$ 6,400	5%	2	4
9.	\$ 7,600	6%	2	7	10.	\$33,000	6%	12	15
11.	\$21,700	4%	12	10	12.	\$16,800	5%	12	16

13-5 SINKING FUNDS

Another method of repayment of a debt is known as *the sinking fund method* and consists of paying the interest on the debt periodically as it comes due and of accumulating a sum equal to the debt by means of an annuity. The annuity that accumulates to the amount of the debt is referred to as a *sinking fund*.

The rate of interest earned by the sinking fund may be the same as that paid on the debt but the two often are different.

Example. A debt of \$8000 bears interest at 4% converted semi-annually and the principal is to be repaid in one installment after 4 years. If the debtor creates a sinking fund by making equal payments at the end of each 6 months into a fund that accumulates at 3% converted semi-annually, what is the total semi-annual expense of the debt? Construct a table showing the condition of the sinking fund at the end of each half year.

Solution. The semi-annual interest that must be paid as due is $\$160 = \$8000\ (.04)\ (\frac{1}{2})$ and the other part of the semi-annual cost of the debt is the periodic rent on the annuity which must accumulate to \$8000 in four years at 3% converted semi-annually. Thus, for the annuity, the number of periods is $2(4) = 8$ and the periodic rate is $\frac{1}{2}(3\%) = 1.5\%$.

Hence

$$\$8000 = Rs_{\overline{8}|.015}$$

and

$$R = \$8000 \frac{1}{s_{\overline{8}|.015}}$$

$$= \$8000 \left(\frac{1}{a_{\overline{8}|.015}} - .015\right)$$

$$= \$8000(.11858402)$$

$$= \$948.67$$

Therefore, the total semi-annual expense of the debt is $160 + $948.67 = $1108.67.

The following table is self-explanatory and shows the condition of the sinking fund at the end of each half year.

SINKING FUND SCHEDULE

End of period	*Payment*	*Interest at 1.5%*	*Accumulation*
1	$948.67		$ 948.67
2	948.67	$ 14.23	1911.57
3	948.67	28.67	2888.91
4	948.67	43.33	3880.91
5	948.67	58.21	4887.79
6	948.67	73.32	5909.78
7	948.67	88.65	6947.10
8	948.67	104.21	7999.98

In practice the borrower would make a payment of $948.69 at the end of the last period so as to have the required $8000 on hand. Each payment and interest credit is correct to the nearest cent.

13-6 BOOK VALUE OF A DEBT

If a debt is being retired by the sinking fund method, the difference between the debt and the accumulation in the sinking fund is called the *book value* of the debt. This can be found readily since the amount of the debt is known and the accumulation in the sinking fund can be determined as the accumulated value of an annuity.

Example. Find the book value of the debt in the example of Section 13-5 just after the third payment is made.

Solution. We found that the semi-annual rent is $R = \$948.67$ and must find the accumulated value of annuity of R per period for three periods with money worth 3% converted semi-annually. Thus, we have $R = \$948.67$, $i = 1.5\%$ and $n = 3$; hence

$$\begin{aligned} S &= Rs_{\overline{n}|i} \\ &= \$948.67\, s_{\overline{3}|.015} \\ &= \$948.67\,(3.045225) \\ &= \$2888.91 \end{aligned}$$

is the amount accumulated in the sinking fund after three payments. Consequently, the book value of the \$8000 debt is

$$\$8000 - \$2888.91 = \$5111.09$$

Therefore, just after the third payment, the debtor needs \$5111.09 in addition to the amount in the sinking fund in order to be able to pay off the debt.

Exercise 13-3

Find the total periodic expense required to create a sinking fund to care for each of the following debts provided the debt bears interest at rate j converted m times per year, has a term of t years, and m payments per year are made into the sinking fund. Make a sinking fund schedule for Problems 1, 2, and 5 and for the first eight periods in 3, 4, 6, 7, and 8.

	Debt	j	m	t	*Interest rate on the sinking fund*
1.	\$ 1,000	4%	1	5	5%, $m = 1$
2.	\$ 4,000	5%	2	3	4%, $m = 2$
3.	\$ 3,000	6%	12	1	7%, $m = 12$
4.	\$ 7,000	4%	4	3	6%, $m = 4$
5.	\$ 5,000	4%	4	2	6%, $m = 4$
6.	\$ 6,000	6%	12	5	4%, $m = 12$
7.	\$ 9,000	6%	2	10	5%, $m = 2$
8.	\$13,000	4%	12	15	6%, $m = 12$

Find the book value of the debt at the indicated time in each of Problems 9 through 16.

9. Just after the 3rd payment in Problem 1.
10. Just after the 4th payment in Problem 2.
11. Just after the 7th payment in Problem 3.
12. Just after the 10th payment in Problem 4.
13. Just after the 3rd payment in Problem 5.
14. Just after the 2nd payment in Problem 6.
15. Just after the 11th payment in Problem 7.
16. Just after the 70th payment in Problem 8.

17. A corporation borrowed \$50,000 at 5% converted semi-annually and agreed to pay the interest as it became due and the principal in a lump sum at the end of 4 years. What is the semi-annual expense of the debt if the sinking fund is invested at 4% converted semi-annually? Construct a table.

18. Find the semi-annual expense of the debt in Problem 17 if the two rates of interest are interchanged.

19. The owner of a machine shop wants to accumulate \$42,000 by making equal quarterly payments at the ends of the quarters for 3 years into a fund that bears interest at 4% converted quarterly. If the interest rate is changed to 5% converted semi-annually just after the 4th payment, what 5 equal semi-annual payments will complete the accumulation? Assume the first semi-annual payment is made 6 months after the last quarterly payment.

20. Rework Problem 19 with 4% replaced by 6%.

13-7 COMPARISON OF AMORTIZATION AND SINKING FUND METHODS

We shall compare the two methods of extinguishing a debt by considering a debt of A that bears interest at rate i per period and determining the periodic expense if the debt is amortized in n periods and if it paid off by means of a sinking fund that accumulates at rate r per period for n periods.

If the debt is amortized, the periodic cost is the periodic payment on the annuity; hence, it is R as determined by $A = Ra_{\overline{n}|i}$ and is

$$R = A\frac{1}{a_{\overline{n}|i}} = A\left(i + \frac{1}{s_{\overline{n}|i}}\right) \tag{1}$$

If the debt is paid off by the sinking fund method, the periodic cost is made up of the periodic interest on the debt at rate i per period and the payment into the sinking fund that is to accumulate to A in n periods and

which draws interest at rate r per period. The periodic interest on the debt is

$$I = Ai$$

and the payment into the sinking fund is

$$R = A\frac{1}{s_{\overline{n}|r}}$$

as determined by $A = Rs_{\overline{n}|r}$. Consequently, the total periodic cost if the sinking fund method is used is

$$A\left(i + \frac{1}{s_{\overline{n}|r}}\right) \tag{2}$$

The difference between the two periodic costs is amortization $-$ sinking fund $= A\left[i + \frac{1}{s_{\overline{n}|i}} - i - \frac{1}{s_{\overline{n}|r}}\right]$

$$\begin{aligned} &= A\left[\frac{1}{s_{\overline{n}|i}} - \frac{1}{s_{\overline{n}|r}}\right] \\ &= A\,\frac{s_{\overline{n}|r} - s_{\overline{n}|i}}{s_{\overline{n}|r}\,s_{\overline{n}|i}} \\ &> 0 \text{ for } r > i \\ &= 0 \text{ for } r = i \\ &< 0 \text{ for } r < i \end{aligned}$$

Consequently, the amortization method costs more if $r > i$, costs less if $r < i$, and the two cost the same if $r = i$.

13-8 RETIRING A BONDED DEBT

In retiring or paying off a debt that consists of bonds of given face value, it is common practice to pay the interest periodically on all outstanding bonds and to retire as many bonds as the cash on hand will allow. Many concerns follow the practice of paying off the number of bonds that comes more nearly than any other number to using the periodic payment as indicated by applying an annuity to the debt, and we shall do this. It is impossible to pay the same amount each period since the interest charge varies with the period and all of the bonds have the same face value.

Example. A school district borrows \$200,000 to erect a building and finances it by issuing \$1000 bonds that bear 4% interest. Form a schedule for retiring the bonds if the debt is to be paid off in 8 years.

Solution. If it were possible to make equal payments, each would be R as determined from the annuity with $A = \$200{,}000$, $n = 8$, and $i = 4\%$; hence

$$\$200{,}000 = Ra_{\overline{8}|.04}$$

$$R = (\$200{,}000)\left(\frac{1}{a_{\overline{8}|.04}}\right)$$

$$= \$200{,}000\ (.14852783)$$

$$= \$29{,}705.57$$

Consequently, in making the schedule, we shall come as nearly as possible to using \$29,705.57 each period; hence, each periodic payment must be between \$29,205.57 = \$29,705.57 − \$500 and \$30,205.57 = \$29,705.57 + \$500 since each bond is worth \$1000.

SCHEDULE FOR RETIRING BONDS

Year	*Value of outstanding bonds*	*Interest for the year*	*Payment*	*Value of bonds retired*
1	\$200,000	\$ 8,000	\$ 30,000	\$ 22,000
2	\$178,000	\$ 7,120	\$ 30,120	\$ 23,000
3	\$155,000	\$ 6,200	\$ 30,200	\$ 24,000
4	\$131,000	\$ 5,240	\$ 29,240	\$ 24,000
5	\$107,000	\$ 4,280	\$ 29,280	\$ 25,000
6	\$ 82,000	\$ 3,280	\$ 29,280	\$ 26,000
7	\$ 56,000	\$ 2,240	\$ 29,240	\$ 27,000
8	\$ 29,000	\$ 1,160	\$ 30,160	\$ 29,000
Total	\$938,000	\$37,520	\$237,520	\$200,000

The total in the Interest column is 4% of the total of the column which gives the value of the outstanding bond as it should be; furthermore, the total in the Interest column plus the total in the Value of bonds retired column should be and is the total in the Payment column.

Exercise 13-4

1. Mr. Brewer contracts a debt of \$10,000 with interest at 3.5% payable annually. He is given the choice between amortizing the debt and paying it off by means of a sinking fund. Which method should he choose if he can get 3% on the contributions to the sinking fund?

2. Which method of payment should the Mr. Brewer of Problem 1 choose if he can invest the sinking fund at 4%? At 3.5%?

3. A debtor wants to pay off a note of \$5000 with interest at 4% converted semi-annually in 12 years by equal semi-annual payments. Should he amortize the debt or create a sinking fund to care for it provided the sinking fund accumulates at 3% converted semi-annually? If his savings by the proper choice can be invested at 3% with $m = 2$, how much will be to his credit at the end of the 12 years?

4. If the sinking fund in Problem 3 can be invested at 5% converted semi-annually, which choice should the debtor make? What periodic gain would be made by the choice?

Make a table that shows the schedule for retiring each set of bonds described in Problems 5 through 8. Bonds are retired annually in each case.

	Total issue	*Face of bond*	*Interest rate*	*Term*
5.	\$ 50,000	\$1000	4%	8 years
6.	\$ 40,000	\$ 500	5%	6 years
7.	\$ 85,000	\$1000	4%	5 years
8.	\$100,000	\$ 500	3%	6 years

9. A city issues \$180,000 worth of \$1000 bonds that bear interest at $3\frac{1}{2}\%$. Construct a table for the retirement of the bonds in 7 annual payments that are as nearly equal as possible.

10. A county was enabled to build a new court house by selling 250 bonds. Construct a table for their retirement if each had a face value of \$500 and drew interest at 4% and the debt was to be retired in 6 years by annual payments as nearly equal as possible.

11. Draw up a schedule for retiring a debt of 400 \$1000 bonds in 5 annual payments as nearly equal as possible, if the bonds bear interest at 5%.

12. A church paid for its building by issuing \$160,000 worth of \$500 bonds with interest at 3%. Draw up a schedule for retiring the debt by means of 7 annual payments that are as nearly equal as possible.

SUMMARY

The amortization and sinking fund methods of extinguishing a debt are described and compared. The periodic expenses of the two methods are shown to be

$$(\mathrm{A}) = A\left(i + \frac{1}{s_{\overline{n}|i}}\right)$$

and

$$(\mathrm{SF}) = A\left(i + \frac{1}{s_{\overline{n}|r}}\right)$$

where

(A) = the periodic expense of amortization
(SF) = the periodic expense of the sinking fund method
A = the debt
n = the number of periods in the term
i = the periodic interest rate on the debt
r = the periodic interest rate earned on the sinking fund

It is also shown that the sinking fund method is better for the debtor if $r > i$, the amortization method is better for him if $r < i$, and the two methods cost the same if $r = i$.

Finally, there is a discussion of retiring a bonded debt.

Exercise 13-5 (Review)

Find the periodic expense of amortizing each of the following debts. Make a schedule in each case.

	A	j	m	*Term*
1.	\$12,000	5%	1	7 years
2.	\$39,000	6%	1	6 years
3.	\$75,000	6%	2	4 years
4.	\$66,000	5%	2	3 years

Find the periodic expense of extinguishing each of the following debts by the sinking fund method. Make a schedule for each.

	A	j	m	*Term*	r
5.	\$40,000	4%	1	8 years	5%
6.	\$75,000	5%	1	7 years	4%
7.	\$60,000	6%	2	4 years	2.5%
8.	\$36,000	5%	2	3 years	3%

9. Find the book value of the debt in Problem 5 just after the third payment into the sinking fund.

10. Find the book value of the debt in Problem 6 just after the fourth payment into the sinking fund.

11. Draw up a schedule for retiring a debt of \$40,000 in \$500 bonds in 6 as nearly equal annual payments as possible if the bonds bear interest at 4%.

12. A school district paid for its building by issuing \$140,000 worth of \$1000 bonds with interest at 4%. Draw up a schedule for retiring the debt by means of 5 annual payments that are as nearly equal as possible.

14

Depreciation and capitalized cost

14-1 TERMS AND SYMBOLS

Property in which capital has been invested is called an *asset*; some such property decreases in value due to age or use. Any decrease in value that cannot be cared for by current repairs is called *depreciation* and a fund should be created to offset this loss of value. Such a fund is called a *depreciation fund* or *replacement fund* and the sum in it at any time is called the *accrued depreciation*. We shall refer to the difference between the original cost and the accrued depreciation as the *book value* of the asset.

Even though an asset, such as a machine, has reached the state when it should be replaced, it may have some value; any value that it retains after it has passed its age of usefulness is called *scrap value*. The difference between the cost and the scrap value is called the *wearing value* or *total depreciation*.

We shall use the following symbols in working with depreciation:

C = original cost
S = estimated scrap value
W = total depreciation or wearing value
B = book value
R = periodic contribution to the depreciation fund
n = estimated number of periods in the life of the asset

There are several methods used in caring for the depreciation and we shall discuss four of them. All of them have the common feature that *the amount in the depreciation fund at the end of the useful life of the asset is equal to the total depreciation.*

14-2 THE STRAIGHT LINE METHOD

The assumptions used in the straight line method are that the depreciation contributions draw no interest and that equal contributions are made at the ends of equal periods. We shall consider an asset with an original cost of C, an estimated scrap value of S, and a probable useful life of n periods. Consequently, the wearing value, W, must be accumulated by n equal non-interest-bearing contributions. If each contribution is represented by R, then

$$nR = W = C - S$$

Hence, the periodic contribution should be

$$R = \frac{W}{n} = \frac{C - S}{n}$$

Example. A machine cost \$4000, has a probable life of 6 years and an estimated scrap value of \$800. Use the straight line method to compute the annual contribution to the depreciation fund and then construct a schedule.

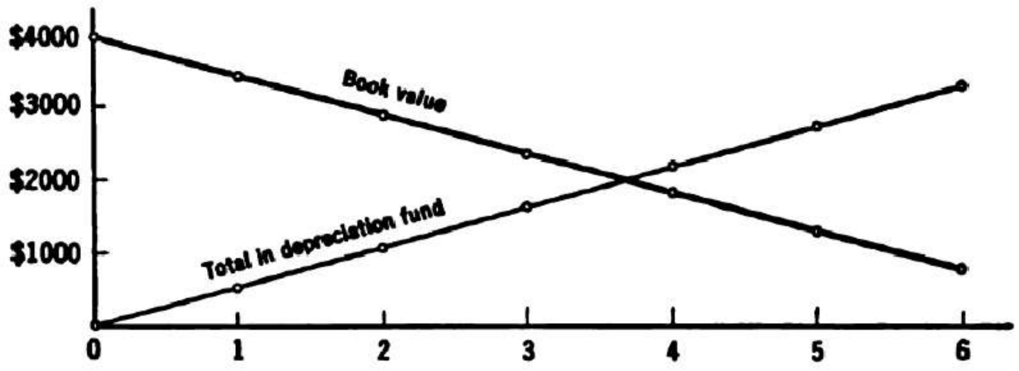
Solution. We know that the wearing value is \$3200 since that is the difference between the cost and scrap value; furthermore, this is to be distributed equally in 6 annual payments. Consequently, the annual contribution to the depreciation fund is

$$\begin{aligned} R &= \frac{W}{n} \\ &= \frac{\$3200}{6} \\ &= \$533.33\tfrac{1}{3} \\ &= \$533.33 \quad \text{to the nearest cent} \end{aligned}$$

DEPRECIATION SCHEDULE—STRAIGHT LINE METHOD

At end of period	*Periodic contribution*	*Total in depreciation fund*	*Book value*
0			\$4000
1	\$533.33	\$ 533.33	\$3466.67
2	\$533.33	\$1066.66	\$2933.34
3	\$533.33	\$1599.99	\$2400.01
4	\$533.33	\$2133.32	\$1866.68
5	\$533.33	\$2666.65	\$1333.35
6	\$533.33	\$3199.98	\$ 800.02

Each entry in the second column is the periodic payment of \$533.33. Each entry in the third column is \$533.33 more than the previous one and each entry in the fourth column can be obtained by subtracting the entry in the third column from the cost. The book value at the end of 6 years is \$800.02 instead of the scrap value of \$800, since each periodic contribution is \$533.33 instead of the exact value of \$533.33$\frac{1}{3}$. The chart given on the following page shows a pictorial form of the information given in the schedule.



14-3 FIXED PERCENTAGE OF BOOK VALUE METHOD

During the useful life of an asset the book value decreases from the original cost to the scrap value. In using the fixed percentage of book value method, we assume that each periodic contribution to the depreciation fund is a fixed percentage of the book value at the end of the preceding period and that the contributions draw no interest.

To derive a formula for use with this method, we shall consider an asset that cost C, has a probable life of n periods, and a scrap value of S; the depreciation factor will be represented by x. Consequently, the depreciation and book value at various times are as shown in the following table.

Period	*Depreciation during*	*Book value at the end*
1	Cx	$C - Cx = C(1 - x)$
2	$C(1 - x)x$	$C(1 - x) - C(1 - x)x = C(1 - x)^2$
3	$C(1 - x)^2x$	$C(1 - x)^2 - C(1 - x)^2x = C(1 - x)^3$
$n - 1$		$C(1 - x)^{n-1}$
n	$C(1 - x)^{n-1}x$	$C(1 - x)^n$

Since the book value at the end of the nth period and the scrap value are equal we see that

$$C(1 - x)^n = S \tag{1}$$

is the equation that determines the depreciation factor x if the fixed percentage of book value method is used. This equation can be solved for x by use of logarithms.

From (1) it is readily seen that $x = 1$ for $S = 0$ regardless of the value of n; furthermore, if S/C is small the value of x is near one and a large part of the wearing is charged to the first few years or even to the first year. One should realize this fact before deciding to use this method.

If we represent the book value at the end of k periods by B_k, then in keeping with the discussion used in obtaining (1), we have

$$B_k = C(1 - x)^k \tag{2}$$

The value of B_k can be found if we know the values of C, x, and k. In any problem, the cost C is known and x can be determined by use of (1); furthermore, k takes on each integral value from one to n. Incidentally, if $k = n$, the book value is equal to the scrap value.

The Federal Revenue Act of 1954 made it legal to use $x = \dfrac{2}{n}$ in connection with the fixed percentage of book value method. This is twice the rate used with the straight line method; furthermore, the method does not use a predetermined scrap value.

Example 1. If a machine cost \$4000, and has a probable useful life of 6 years and an estimated scrap value of \$800, find the depreciation factor.

Solution. In this problem, $C = \$4000$, $n = 6$, $S = \$800$, and we want to determine x; hence, by use of

$$\begin{aligned} C(1 - x)^n &= S \\ \$4000(1 - x)^6 &= \$800 \\ (1 - x)^6 &= .2 \quad \text{dividing by } \$4000 \end{aligned}$$

Consequently

$$\begin{aligned} \log(1 - x)^6 &= \log .2 \\ 6 \log(1 - x) &= \log .2 \\ \log(1 - x) &= \tfrac{1}{6} \log .2 \\ &= \tfrac{1}{6} (9.30103 - 10) \\ &= \tfrac{1}{6} (59.30103 - 60) \\ &= 9.88350 - 10 \end{aligned}$$

Therefore

$$1 - x = .76472$$
$$x = .23528$$

Example 2. Make a depreciation schedule for a \$6000 asset which has a probable useful life of 5 years by using the rate $\frac{2}{n} = \frac{2}{5} = .4$ as legalized by the Federal Revenue Act of 1954.

Solution. We shall solve

$$B_k = C(1 - x)^k$$

for B_1, B_2, B_3, B_4, and B_5 by use of logarithms with $C = \$6000$ and $x = .4$. Hence

$$B_k = 6000\ (.6)^k$$

and, using logarithms, we have

$$\begin{aligned} \log B_k &= \log 6000 + k \log .6 \\ &= 3.77815 + k(9.77815 - 10) \end{aligned}$$

Therefore

$$\log B_1 = 3.77815 + 1(9.77815 - 10) = 3.55630,\ B_1 = \$3600$$
$$\log B_2 = 3.77815 + 2(9.77815 - 10) = 3.33445,\ B_2 = \$2160$$
$$\log B_3 = 3.77815 + 3(9.77815 - 10) = 3.11260,\ B_3 = \$1295.97$$
$$\log B_4 = 3.77815 + 4(9.77815 - 10) = 2.89075,\ B_4 = \$777.58$$
$$\log B_5 = 3.77815 + 5(9.77815 - 10) = 2.66890,\ B_5 = \$466.56$$

The following table shows the annual depreciation and the total in the depreciation fund at the end of each year in addition to the annual book value. The depreciation for any year is the difference between the book values at the beginning and the end of the year. The book values as determined above were used in constructing the table. The entries are not exactly as they would have been had each depreciation been calculated as .4 of the book value at the beginning of the year without logarithms.

DEPRECIATION SCHEDULE—FIXED PERCENTAGE OF BOOK VALUE

Age in years	*Annual depreciation*	*Total in depreciation fund*	*Book value*
0			\$6000
1	\$2400	\$2400	\$3600
2	\$1440	\$3840	\$2160

Age in years	*Annual depreciation*	*Total in depreciation fund*	*Book value*
3	\$ 864.03	\$4704.03	\$1295.97
4	\$ 518.39	\$5222.42	\$ 777.58
5	\$ 311.02	\$5533.44	\$ 466.56

14-4 SUM OF THE YEARS DIGITS METHOD

If this method for computing the periodic depreciation charge is employed, we use a different determinable fractional part of $C - S$ as the depreciation for each period. If the asset has a probable life of n years, the denominator of each of these fractions is

$$n + (n - 1) + (n - 2) + \cdots + 2 + 1 = \frac{n}{2}(n + 1)$$

and the numerators are n for the first period, $n - 1$ for the second, $n - 2$ for the third, up to 2 for the $(n - 1)$st and 1 for the last or nth period. Therefore, the total depreciation is $C - S$ as it must be.

Example. If a machine cost \$4000, has an estimated useful life of 6 years and a scrap value of \$800, find the depreciation for each year.

Solution. Since the probable life is 6 years the denominator for each fraction is $6 + 5 + 4 + 3 + 2 + 1 = 21$, and the numerators are 6, 5, 4, 3, 2, and 1; furthermore, the wearing value is \$4000 − \$800 = \$3200. Therefore, the annual depreciation charges are

$$\begin{aligned}
\tfrac{6}{21}(\$3200) &= \$914.29 \text{ at the end of the first year,}\\
\tfrac{5}{21}(\$3200) &= \$761.90 \text{ at the end of the second year,}\\
\tfrac{4}{21}(\$3200) &= \$609.52 \text{ at the end of the third year,}\\
\tfrac{3}{21}(\$3200) &= \$457.14 \text{ at the end of the fourth year,}\\
\tfrac{2}{21}(\$3200) &= \$304.76 \text{ at the end of the fifth year,}\\
\tfrac{1}{21}(\$3200) &= \$152.38 \text{ at the end of the sixth year.}
\end{aligned}$$

The total of the depreciation lacks one cent of being \$3200 but that is satisfactory since each depreciation is to the nearest cent.

Exercise 14-1

1. A family pays $2800 for a car that has a probable life of 5 years and a scrap (trade-in) value of $600. Find the annual depreciation charge by the straight line method and construct a depreciation schedule.

2. An air-conditioning unit for a house cost $1300, has an estimated usable life of 12 years and a scrap value of $100. Find the annual depreciation charge by the straight line method and make a depreciation schedule.

3. A kitchen stove costs $386, has an estimated life of 14 years and a scrap value of $50. Find the annual depreciation charge by the straight line method and make a depreciation schedule.

4. A tractor and equipment costs $1690, has an estimated life of 6 years and a scrap value of $250. Find the annual depreciation by the straight line method and make a depreciation schedule.

5. Using the data of Problem 1, find the fixed percentage of the book value and construct a depreciation schedule.

6. Find the fixed percentage of the book value from the data given in Problem 2 and construct a depreciation schedule.

7. Use the cost, scrap value, and life as given in Problem 3 to determine the fixed percentage of the book value. Construct a depreciation schedule.

8. Find the fixed percentage of book value as determined by the data of Problem 4. Construct a depreciation schedule.

9. Use the cost and probable life as given in Problem 1 to make a depreciation schedule with the rate $\frac{2}{n}$ as made legal by the Federal Revenue Act of 1954.

10. Use the cost and life of Problem 2 and the legal rate of $\frac{2}{n}$ to make a depreciation schedule.

11. Make a depreciation schedule for the stove described in Problem 3 by using the rate $\frac{2}{n}$.

12. Make a depreciation schedule for the tractor of Problem 4 by using the legal rate $\frac{2}{n}$.

13. Use the data of Problem 1 and the sum of the digits method to find the annual depreciation charge.

14. Construct a table for the annual depreciation charge by use of the data of Problem 2 and the sum of the digits method.

15. Construct a table that shows the annual depreciation charge by using the data of Problem 3 and the sum of the digits method.

16. Use the sum of the digits method and the data of Problem 4 to construct a table showing the annual depreciation charge.

14-5 THE SINKING FUND METHOD

If the sinking fund method of caring for depreciation is used, we make a deposit of R into the depreciation fund at the end of each period. These deposits are invested at a known periodic rate and accumulated to the wearing value at the end of the useful life of the asset. If the original cost is C and the scrap value is S, then the wearing value is $W = C - S$; hence, *an annuity of R per period for the n period probable useful life of the asset at rate r per period must accumulate to* W. Therefore

$$W = Rs_{\overline{n}|r}$$

and

$$R = \frac{W}{s_{\overline{n}|r}}$$

is the annual payment to the depreciation fund. Consequently, the accumulation in the sinking fund at the end of k periods just after the payment has been made is

$$S_k = Rs_{\overline{k}|r} = \frac{W}{s_{\overline{n}|r}} s_{\overline{k}|r}$$

Furthermore, the book value B_k at that time is

$$B_k = C - \frac{W}{s_{\overline{n}|r}} s_{\overline{k}|r}$$

since the book value is defined as the difference between the cost and the accumulation in the sinking fund.

Example. If a machine cost \$4000, has a probable useful life of 6 years and an estimated scrap value of \$800, determine the annual payment into the sinking fund that is created to take care of the depreciation. Assume that this fund draws interest at 4%. Make a depreciation schedule and find the book value just after the fourth payment, independently of the schedule.

Solution. We must accumulate \$3200 = \$4000 − \$800 in 6 years by equal annual payments at the end of each year into a fund that draws interest at 4%; hence, the annual deposit is determined by

$$\$3200 = Rs_{\overline{6}|.04}$$

$$R = \frac{\$3200}{s_{\overline{6}|.04}}$$
$$= \$3200\ (.15076190)$$
$$= \$482.44$$

To find the book value B_4 just after the fourth payment, we shall first find the amount in the sinking fund at that time. It is

$$S = \$482.44\ s_{\overline{4}|.04}$$
$$= \$482.44\ (4.246464)$$
$$= \$2048.66$$

Consequently

$$B_4 = \$4000 - \$2048.66$$
$$= \$1951.34$$

The depreciation schedule that follows contains an interest column since the deposits draw interest if the sinking fund method is used.

DEPRECIATION SCHEDULE—SINKING FUND METHOD

End of year	*Interest*	*Payment*	*Accumulation*	*Book value*
1	none	\$482.44	\$ 482.44	\$3517.56
2	\$ 19.30	\$482.44	\$ 984.18	\$3015.82
3	\$ 39.37	\$482.44	\$1505.99	\$2494.01
4	\$ 60.24	\$482.44	\$2048.67	\$1951.33
5	\$ 81.95	\$482.44	\$2613.06	\$1386.94
6	\$104.52	\$482.44	\$3200.02	\$ 799.98

The book value at the end of the useful life is found to be \$799.98 instead of the \$800 scrap value. This is within allowable limits since the payment and interest entries are correct only to the nearest cent.

14-6 COMPOSITE LIFE OF A PLANT

Most shops, manufacturing plants, and some other types of property consist of several parts, which have different probable useful lives. The

composite life of a plant is the time needed for the total periodic contribution to accumulate to the total wearing value. Any one of the methods of caring for depreciation can be used in connection with composite life but we shall use only the sinking fund method.

Example. A plant consists of part A with a probable life of 7 years and a wearing value of \$37,000, part B with a life of 11 years and a wearing value of \$29,000, and part C with an estimated life of 25 years and a wearing value of \$71,000. Find the composite life of the plant if a sinking fund can be accumulated at 4%.

Solution. We shall begin by finding the periodic payment toward accumulating each wearing value since we need the sum of these payments. If we make use of each wearing value and corresponding life, we see that the payments are determined by

$$\$37000 = R_1 s_{\overline{7}|.04}, \quad \$29000 = R_2 s_{\overline{11}|.04}$$
$$\text{and } \$71000 = R_3 s_{\overline{25}|.04}$$

They are

$$R_1 = \frac{\$37000}{s_{\overline{7}|.04}}, \; R_2 = \frac{\$29000}{s_{\overline{11}|.04}}, \text{ and } R_3 = \frac{\$71000}{s_{\overline{25}|.04}}$$

Hence

$$R_1 = \$37000\,(.12660961) = \$4684.56$$
$$R_2 = \$29000\,(.07414904) = \$2150.32$$
$$R_3 = \$71000\,(.02401196) = \$1704.85$$

Consequently, the total periodic payment $R_1 + R_2 + R_3 = \$8539.73$; the total wearing value is \$37,000 + \$29,000 + \$71,000 = \$137,000. Therefore, the composite life is the time required for an annual payment of \$8539.73 to accumulate to \$137,000 at 4% and is determined by

$$137000 = 8539.73 s_{\overline{n}|.04}$$

Solving for $s_{\overline{n}|.04}$ gives

$$s_{\overline{n}|.04} = \frac{137000}{8539.73}$$
$$= 16.0425$$

Therefore, n is between 12 and 13; finally, by interpolation, we see that $n = 12.635$ years.

Exercise 14-2

Use the sinking fund method to determine the annual depreciation, make a depreciation schedule for the life or first 8 years thereof, and find the book value at the indicated time independently of the schedule in Problems 1 through 8.

	Cost	*Scrap value*	*Useful life*	*r*	*Book value after*
1.	$ 8,000	$1200	9 years	4%	5 years
2.	$ 500	$ 40	7 years	3%	2 years
3.	$ 6,700	$ 900	20 years	3.5%	12 years
4.	$ 4,600	$ 600	12 years	3%	10 years
5.	$ 7,200	$ 700	8 years	4%	3 years
6.	$23,000	$2500	11 years	4%	7 years
7.	$17,500	$1800	17 years	3%	14 years
8.	$27,600	$2300	16 years	5%	6 years

9. An automobile costs $2800, has a probable useful life of 5 years and an estimated trade-in value of $500. Find the book value after 3 years if money is worth 5%.

10. A house cost $15,000 and has a probable salvage value of $4000 at the end of its 35-year life. What would be a fair price for it when it is 9 years old if money is worth 5.5%?

11. A tractor and equipment costs $5200, has a probable useful life of 10 years and a scrap value of $600. If money is worth 5%, find the book value at the end of 4 years.

12. An air-conditioning unit costs $1100, has a probable useful life of 12 years and a scrap value of $100. Find the book value at the end of 5 years if money is worth 4.5%.

Find the composite life in each of Problems 13 through 16. Use $i = 4\%$.

	Part	*Life*	*Wearing value*
13.	*A*	7 years	$19,000
	B	11 years	$42,000
14.	*A*	9 years	$27,000
	B	14 years	$59,000
15.	*A*	9 years	$ 700
	B	11 years	$ 450
	C	8 years	$ 970
16.	*A*	5 years	$18,000
	B	9 years	$13,000
	C	14 years	$38,000

14-7 MINING AND OIL PROPERTY

Some types of property produce an income for a limited time and are then essentially worthless. Mining and oil property have this characteristic. Since the productive life of property of this type is limited, the owner should set aside enough of the income periodically to create a sinking fund that will be equal to the original cost at the end of the productive life. After this payment into the sinking fund is made, the remainder of the net income (total income minus operating costs) is used to pay interest on the investment.

From the point of view of the mathematics of finance, the essential quantities connected with such property are:

V = the value of the property less any salvage value there may be at the end of the useful life

N = the periodic net income

i = the periodic interest rate on the investment

n = the number of periods of productive life

R = the periodic payment into the sinking fund

r = the periodic rate earned by the sinking fund

We shall now derive a formula for N in terms of V, i, n, R, and r. The basis for this will be the fact that the net periodic income N must provide for interest at rate i per period on the value V of the property and for the periodic payment of R into a sinking fund that accumulates to V in n periods at rate r per period. Consequently, $N = Vi + R$ where R is determined by $V = Rs_{\overline{n}|r}$ and is $\frac{V}{s_{\overline{n}|r}}$.

Therefore

$$N = Vi + \frac{V}{s_{\overline{n}|r}}$$

or

$$N = V\left(i + \frac{1}{s_{\overline{n}|r}}\right)$$

The reader should realize that this equation contains several quantities and that, at times, it may be desirable to solve it for some letter other than N.

Example. A mine is estimated to have a productive life of 14 years and to bring in a net income of $25,000 per year. Find the value of the mine if the buyer wants to make 6% on his investment and can get 4% on the sinking fund deposits.

Solution. We are told that $N = \$25{,}000$, $i = 6\%$, $n = 14$, and $r = 4\%$, and we want to determine V. Therefore,

$$\$25{,}000 = V\left(.06 + \frac{1}{s_{\overline{14}|.04}}\right)$$

since the $25,000 income must provide interest on V at 6% and a payment of $R = \dfrac{V}{s_{\overline{14}|.04}}$.

Solving for V gives

$$\begin{aligned} V &= \frac{\$25{,}000}{.06 + \dfrac{1}{s_{\overline{14}|.04}}} \\ &= \frac{\$25{,}000}{.11466897} \text{ since } \frac{1}{s_{\overline{14}|.04}} = .05466897 \\ &= \$218{,}018.88 \end{aligned}$$

Exercise 14-3

Determine the missing quantities in Problems 1 through 8.

	N	V	i	R	n	r
1.	?	$100,000	5%	?	17	3%
2.	?	$425,000	6%	?	9	4%
3.	?	?	4%	$2400	12	4%
4.	?	?	7%	$7300	8	3.5%
5.	$2431.31	?	?	$1198.31	17	3%
6.	$8400.27	?	?	$3207.89	10	4%
7.	$9300.04	$129,000	?	?	13	4.5%
8.	$4734.07	$278,500	?	?	8	3.5%

9. The annual net income from an oil well is estimated at $45,000 for its 15-year life. How much should an investor pay for it in order to make 6% if he can accumulate a replacement fund at 4%?

10. It is estimated that a piece of oil property will yield an income of $150,000 per year for 9 years. If a buyer wishes to earn 8% on his investment and can accumulate a sinking fund at 3.5%, what should he be willing to pay for the property?

11. An investor pays $170,000 for property that will be worthless after 15 years. What annual net income must it provide in order for the investor to make 6%. Assume he can invest his sinking fund deposits at 3.5%.

12. A piece of property cost $2,160,000 and will yield an income of $600,000 annually for 9 years. What rate does the investor make if he can accumulate a sinking fund at 4%?

14-8 UNIT COST OF PRODUCTION

The unit cost of production takes into account the operating cost, the cost of repairs, the depreciation charge, the interest on the investment, and the number of units of output of the machine. We are then able to compute the total cost of producing one unit; hence, we can compare two machines and can find how much can be spent profitably to increase the output of a machine by a certain number of units.

Since we know how to compute the interest on the investment and the depreciation charge and since we can estimate the number of units of output, the operating cost, and the cost of repairs for a given machine, we shall not develop a formula but shall consider the factors that enter into this method.

Example 1. A shop owns a machine that costs $500 per year to operate, needs $100 worth of repairs per year, has an estimated life of 4 years and an annual output of 80 units. For $2000, the owner can buy a new machine that has an output of 110 units per year and an estimated life of 9 years. If the cost of operating the new machine is $600 per year and the estimated annual repair bill is $80, find the value of the old machine if money is worth 3%.

Solution. We shall first find the unit cost of operating the new machine and then determine x so that the unit cost of production with the old machine is that same amount.

Four costs enter into the unit cost; we shall calculate the depreciation since we know the other three. It is the value of R determined by

$$\$2000 = Rs_{\overline{9}|.03}$$

since we must accumulate the cost of \$2000 in 9 years at 3%. Therefore

$$\begin{aligned} R &= \frac{\$2000}{s_{\overline{9}|.03}} \\ &= \$2000\ (.09843386) \\ &= \$196.87 \end{aligned}$$

Thus, the annual cost of using the new machine is \$196.87 for depreciation, \$600 for operation, \$80 for repairs, and \$2000 (.03) = \$60 for interest; hence, a total cost of \$936.87 for the 110-unit output. Consequently, the unit cost is \$936.87 ÷ 110 = \$8.517.

If the old machine is worth x, the annual cost of using it is $.03x$ for interest and R determined by

$$x = Rs_{\overline{4}|.03}$$

for depreciation besides the \$100 for repairs and \$500 for operation; hence, the unit cost is

$$\frac{\$100 + \$500 + .03x + \dfrac{x}{s_{\overline{4}|.03}}}{80} = \$8.517$$

since it produces 80 units that are to cost \$8.517 each. Now, multiplying by 80 and finding the value of $\dfrac{1}{s_{\overline{4}|.03}}$, we obtain

$$\begin{aligned} \$600 + .03x + .23902705x &= \$681.36 \\ .26902705x &= \$81.36 \end{aligned}$$

Hence $$x = \$302.42$$

is the value of the old machine.

Example 2. How much can one afford to spend for renovation on a \$1600 machine that has an output of 40 units per year, an operating cost of \$300, and a life of 3 years in order to get an output of 55 units with an operating cost of \$450? Assume that money is worth 3%, that repairs will extend the life of the machine to 5 years, and that the annual cost of repairs is cut from \$100 to \$60.

Solution. If we represent the cost of repairs by x, we can make the following table:

	Not renovated	*Renovated*
Annual repairs	\$100	\$ 60
Operation	\$300	\$450
Interest	$\$48 = \$1600(.03)$	$(\$1600 + x)(.03)$
Depreciation	$\dfrac{\$1600}{s_{\overline{3}\rceil .03}}$	$\dfrac{\$1600 + x}{s_{\overline{5}\rceil .03}}$

Unit cost of production for the machine not renovated is

$$\frac{\$448 + \dfrac{\$1600}{s_{\overline{3}\rceil .03}}}{40}$$

and for the renovated machine is

$$\frac{\$510 + (\$1600 + x)(.03) + \dfrac{\$1600 + x}{s_{\overline{5}\rceil .03}}}{55}$$

Consequently, if the unit costs are to be equal, we must have

$$\frac{\$510 + (\$1600 + x)(.03) + \dfrac{\$1600 + x}{s_{\overline{5}\rceil .03}}}{55} = \frac{\$448 + \dfrac{\$1600}{s_{\overline{3}\rceil .03}}}{40}$$

Now, multiplying by the common denominator and using the tables, we have

$$\$4080 + (\$1600 + x)(.24) + 8(\$1600 + x)(.18835457)$$
$$= \$4928 + \$17600(.32353036)$$
$$\$4080 + \$384 + .24x + \$2410.94 + 1.50684x$$
$$= \$4928 + \$5694.13$$
$$\$6874.94 + 1.74684x = \$10,622.13$$
$$1.74684x = \$3747.19$$

Hence

$$x = \frac{\$3747.19}{1.74684}$$
$$= \$2145.12$$

is the amount that can be spent on renovation under the conditions of the problem.

Exercise 14-4

1. A machine costs \$150 per year to operate and requires annual repairs of \$200 in order to extend its life 5 years and produce 90 units per year. A new machine can be bought for \$2000 that will produce 125 units per year for 8 years and require \$120 per year to operate and \$100 per year for repairs. What is the value of the old machine if money is worth 4%?

2. An old machine requires \$300 per year for operation and repairs. It will last 4 years and produce 200 units per year or can be sold for \$1200 and be replaced by a new machine. The new machine will require \$200 annually for operation and repairs and will produce 260 units per year for 9 years. What can the owner of the old machine afford to pay for the new one if money is worth $3\frac{1}{2}$%?

3. A buyer can get either of two machines. One costs \$2000, has a life of 8 years, an annual output of 105 units, and requires \$700 annually for repairs and operation. The other costs \$2500, has a life of 9 years, an annual output of 120 units, and requires \$600 per year for operation and repairs. Which should he buy if money is worth 5%?

4. A shop owner has a machine that can be sold for \$1000. He can make it have an output of 250 units per year for 5 years by having it remodeled and then spending \$400 per year for operation and repairs. Instead of doing this, the owner can buy a new machine for \$3000 that will have an annual output of 350 units for 10 years and will cost \$450 per year for repairs and operation. How much can the owner afford to pay for remodeling the old machine if money is worth 5%?

5. A new machine with a life of 10 years and an annual output of 500 units can be bought for \$6000 and will require \$1000 per year for repairs and operation. How much can the owner of an old machine that can be sold for \$1200 afford to pay for a complete repair job in order to have the remodeled machine last 4 years with an annual output of 400 units and a cost of \$800 per year for repairs and operation? Assume that money is worth $3\frac{1}{2}$%.

6. An operator of a machine shop can buy either of two machines. The first costs \$4000, has a life of 10 years, produces 500 units per year, and costs \$800 for repairs and operation. The second costs \$3000, has a life of 9 years, produces 450 units per year, and costs \$700 per year for repairs and operation. Which machine should he buy if money is worth 4%?

7. An old machine is worth \$1200 and can be made to produce 170 units per year for 6 years with an outlay of \$500 annually for operation and repairs, provided it is remodeled for \$2000. A new machine that will produce 220 units per year for 9 years with an outlay of \$400 annually for operation and repairs is available. How much is the new machine worth if money produces $3\frac{1}{2}$%?

8. Two machines are available. One costs \$4800, will produce 180 units yearly for 8 years with an annual expense of \$1000 for operation and repairs. The other costs \$6000, has a life of 10 years, and will cost \$800 annually for repairs

and operation. How many units must it produce annually in order to make it more desirable than the first if money is worth 4%?

14-9 CAPITALIZED COST

Many people are misled as to the expense of an article by not taking into consideration the replacement cost and length of time the article will be useful. The original cost plus the present value of the perpetuity necessary for all future replacements is called the *capitalized cost*.

If the original cost of an article is C and the scrap value is S, then the replacement cost is $C - S$; furthermore, if the expected life is k periods and if the buyer can accumulate a sinking fund at r per period, then the periodic deposit R necessary to have the replacement cost on hand when needed is determined by

$$C - S = Rs_{\overline{k}|r}$$

Therefore

$$R = (C - S)\frac{1}{s_{\overline{k}|r}}$$

and the present value of all future replacements is

$$\frac{R}{r} = \frac{C - S}{r}\frac{1}{s_{\overline{k}|r}}$$

Consequently, *the capitalized cost is*

$$\boldsymbol{K = C + \frac{C - S}{r}\frac{1}{s_{\overline{k}|r}}}$$

for an article that costs C, has a scrap value of S, and a useful life of k periods, provided money is worth r per period.

Since the useful life and scrap value are estimates the assumption is often made that S is zero. In that case, we have

$$\begin{aligned}
K &= C + \frac{C}{r}\frac{1}{s_{\overline{k}|r}} \\
&= \frac{Cr}{r} + \frac{C}{r}\frac{1}{s_{\overline{k}|r}} && \text{multiplying } C \text{ by } \frac{r}{r} \\
&= \frac{C}{r}\left(r + \frac{1}{s_{\overline{k}|r}}\right) && \text{factoring out } \frac{C}{r} \\
&= \frac{C}{r}\frac{1}{a_{\overline{k}|r}}
\end{aligned}$$

Therefore, *the capitalized cost of an article that costs C originally and for each replacement and has a life of k periods is*

$$K = \frac{C}{r}\,\frac{1}{a_{\overline{k}|r}}$$

provided money is worth r per period.

Example 1. Find the capitalized cost of a machine that sells for \$2500, must be replaced every 10 years, and has a scrap value of \$300. Assume that the sinking fund accumulates at 4% converted semi-annually.

Solution. Instead of using the formula for capitalized cost, we shall note that the semi-annual contribution needed to have the required \$2200 = \$2500 − \$300 on hand at the end of each 10 years = 20 periods is determined by

$$\$2200 = Rs_{\overline{20}|.02}$$

since the rate is $\frac{1}{2}(4\%) = 2\%$ per period and is $R = \dfrac{\$2200}{s_{\overline{20}|.02}}$. Therefore, the present value of the perpetuity necessary for all future replacements is

$$\frac{R}{.02} = \frac{\$2200}{.02s_{\overline{20}|.02}} = \$4527.24$$

and the capitalized cost is

$$K = \$2500 + \frac{\$2200}{.02s_{\overline{20}|.02}}$$
$$= \$7027.24$$

There are several grades and prices of most products and they last for different lengths of time. In deciding which grade to buy, we should find the capitalized cost of each.

Example 2. A housewife can get a vacuum cleaner for \$64.50 that will last 7 years and another for \$79 that will last 9 years. Which is the more economical if money is worth 5% and each replacement cost is the same as the original?

Solution. We shall decide which to purchase by comparing the capitalized costs. The capitalized cost for the lower-priced machine is

$$K = \frac{\$64.50}{.05}\,\frac{1}{a_{\overline{7}|.05}} = \$1290(.17281982) = \$222.94$$

and that for the higher-priced one is

$$K = \frac{\$79}{.05}\,\frac{1}{a_{\overline{5}|.05}} = \$1580(.14069008) = \$222.29$$

Hence, it is slightly more economical to buy the higher-priced machine.

Exercise 14-5

Find the capitalized cost in Problems 1 through 8.

	C	S	*Term*	j	m
1.	\$ 1000	\$ 300	3 years	3%	1
2.	\$48000	\$3000	23 years	3.5%	1
3.	\$ 7200	\$1200	15 years	3.5%	1
4.	\$ 3800	\$ 800	6 years	5%	1
5.	\$ 5000	\$ 400	8 years	4%	2
6.	\$10000	\$3000	25 years	4%	2
7.	\$ 5000	\$1200	8 years	6%	4
8.	\$ 7800	\$1300	9 years	5%	4

9. A refrigerator costs \$475 and must be replaced every 12 years at a cost of \$400. What is the capitalized cost if money is worth 5%?

10. Find the capitalized cost of a floor furnace that costs \$375 and must be replaced every 15 years at a cost of \$325. Assume money is worth 3.5%.

11. If the family car costs \$2800 and it and replacements are expected to have a trade-in value of \$700 after 4 years' use, find the capitalized cost provided money is worth 4.5%.

12. The architect for a proposed church building estimated the cost to be \$300,000; furthermore, he estimated that repairs of \$5000 at the end of each 3 years would keep the building in good condition. Find the capitalized cost if money is worth 4%.

In Problems 13 through 20, the replacement cost and original cost are equal.

13. One refrigerator can be bought for \$200 and will last for 9 years. Another costs \$230 and will last 11 years. Which is the more desirable one? Assume that money is worth 4%.

14. A building that will last 20 years can be built for \$46,000, and one for the same purpose that will last 27 years can be erected for \$62,000. Which should the owner build if money is worth $3\frac{1}{2}$%?

15. A machine to do a certain job can be bought for \$2000 and will last 7 years. What can the owner afford to pay for a machine to do the same job provided it will last 9 years? Assume that money is worth 5%.

16. A lawnmower that will last 8 years can be bought for \$190. How much can the buyer afford to pay for one that will last for 5 years if money is worth 4% converted semi-annually?

17. An untreated post will last 15 years and cost \$1.75. A treated post will last 25 years. If money is worth 4% converted semi-annually, how much can one afford to pay for the treated post?

18. A shingle roof will last 10 years and cost \$400. What can one afford to pay for a slate roof that will last 30 years if money is worth 3%?

19. A house can be painted with a grade of paint that will last 2 years or one that will last 5 years. If the latter costs \$6 per pail and if the cost of labor for applying either is \$3 per pail, what can one afford to pay per pail for the poorer grade? Assume that money is worth $3\frac{1}{2}$%.

20. A bridge that will last 20 years can be built for \$38,000. If money is worth 3%, how much could be spent economically for one that will last 30 years?

SUMMARY

Four different methods for determining the depreciation of an asset have been discussed. With their appropriate formulas, they are:

1. Straight line method:

$$R = \frac{W}{n} = \frac{C - S}{n}$$

2. Fixed percentage of book value method:

$$C(1 - x)^n = S$$

$$B_k = C(1 - x)^k$$

3. Sum of the years digits method:

$$R = \frac{k}{\frac{n}{2}(n + 1)} \qquad k = n, n - 1, \cdots, 2, 1$$

4. Sinking fund method:

$$R = \frac{W}{s_{\overline{n}|i}}$$

The estimated value of mining and oil property is determined by the formula

$$N = V\left(i + \frac{1}{s_{\overline{n}|r}}\right)$$

The unit cost of production, composite life, and capitalized cost have also been discussed and illustrated.

Exercise 14-6 (Review)

1. A delivery truck costs $3000, has an estimated life of 5 years and a trade-in value of $500. Find the annual depreciation charge by the straight line method and make a depreciation schedule.

2. If the truck in Problem 1 is depreciated by the fixed percentage of book value method, find its book value at the end of each year and make a depreciation schedule.

3. A machine costs $50,000, has an estimated life of 20 years and a scrap value of $2000. Determine its book value at the end of the 10th year by the sinking fund method if the sinking fund is invested at $2\frac{1}{2}\%$.

4. If the machine in Problem 3 is depreciated by the fixed percentage of book value method, find its book value (a) at the end of the 10th year; (b) at the end of the 15th year.

5. A piece of fire-fighting equipment costs a city $40,000. If its estimated life is 15 years and its scrap value $2500, determine the depreciation charge for the first 2 and the last 2 years of its life by the sum of the years digits method.

6. If the fire engine in Problem 5 is depreciated by setting up a sinking fund at 3%, determine the book value at the end of (a) the 8th year; (b) the 12th year.

7. A man builds a brick apartment costing $100,000. For income-tax purposes he is allowed to depreciate the entire cost in 30 years by the straight line method. What is his annual depreciation allowance?

8. A bus company has to replace its buses every 10 years. If a bus costs $25,000 and has a scrap value of $2500, determine the depreciation charge for each of the first 3 years by the sum of the years digits method.

9. A piece of oil property costs $1,000,000 and has an estimated income of $250,000 per year for 9 years. What rate does the purchaser make on his investment if he can accumulate a sinking fund at 2%?

10. What should be paid for a piece of timber property that has an annual net income of $10,000 for 20 years, after which time it is worthless, if the buyer wants to earn 8% on his money and can invest his sinking fund at 2%?

11. A steel company pays $200,000 for a mine from which it expects to receive an annual income of $55,000 for 10 years, after which time it will be worthless. If a replacement fund can be invested at $2\frac{1}{2}\%$, what rate of interest does the company earn on its investment?

12. A manufacturing concern has a choice of buying either of two machines. The first costs $18,000, has a life of 9 years, an annual output of 1000 units, and requires $5200 per year for repairs and operation. The second costs $20,000, has a life of 8 years, an annual output of 1300 units, and requires $5000 per year for repairs and operation. Which machine should be purchased if money is worth $3\frac{1}{2}\%$?

13. What is the value of an old machine that will last 4 years, has an output of 350 units per year, and costs $1750 per year for operation and repairs if a

new machine, which will last 8 years, has an output of 400 units per year, and costs \$1600 per year for operation and repairs can be purchased for \$8500? Assume $i = .045$.

14. Find the composite life of a plant that consists of: Part A with a life of 7 years and a cost of \$19,000, part B with a life of 9 years and a cost of \$28,000, and part C with a life of 10 years and a cost of \$16,000. Assume that money is worth 4%.

15. Find the capitalized cost of a stove that cost \$450 and will last 13 years. Assume that each replacement will also cost \$450 and that money is worth 4%.

16. A machine to do a given job can be bought for \$13,000 and will last 8 years. If money is worth 5%, how much is a machine worth that will do the job for 5 years? Assume that replacement for each costs the same as the machine.

15

Bonds

15-1 TERMINOLOGY

A *bond* is a written agreement to pay a specified sum on a given or determinable date and to pay another specified sum periodically during the term of the agreement. The sum printed on the bond is called the *face value*. The sum that is paid periodically is called the *interest* or *coupon*. The interest is ordinarily determined as a per cent of the face value and this fraction is called the *interest rate*, *coupon rate*, or *bond rate*. The period between interest payments and the times of paying interest are stated in

the bond. The given or determinable date on which a specified sum is paid is called the *redemption date*. The sum, exclusive of interest, received by the holder of the bond on the redemption date is called the *redemption price;* it is expressed as a per cent and means that per cent of the face value.

If a bond is sold on an interest date, the seller takes the interest that is due that day and the buyer is entitled to all future interest.

Example. "District 4 Drainage Association acknowledges itself to owe and for value received promises to pay to bearer ONE THOUSAND DOLLARS on April 1, 1979 with interest at 4% per annum payable semi-annually on October 1 and April 1 from April 1, 1959 until the principal sum is paid."

Discussion. The statement in the example contains all the essentials of a bond contract. The bond may be described as a 20-year, 4%, \$1000 bond with interest payable O-A* since the redemption date is 20 years after the date of issue, the interest rate is 4%, the face value is \$1000, and the semi-annual interest payments are payable on October 1 and April 1. Each interest payment is $\$1000(.04)(\frac{1}{2}) = \$20$.

15-2 PURCHASE PRICE ON AN INTEREST DATE

Bonds are ordinarily bought at a price different from the redemption price since the price a buyer is willing to pay depends on the redemption price, redemption date, the periodic interest, and the desired rate of income. This rate of income is called the *yield* or *investment rate*. The price paid by the purchaser of a bond consists of the sum of the present value of the redemption price and the present value of the annuity formed by the interest payments, provided both are computed at the investor's yield rate.

To express a formula for the purchase price of a bond, we shall employ the following symbols:

C = the redemption price

i = periodic yield rate desired by the buyer

*It is customary to indicate the interest months by giving the first letter of each.

n = number of periods until the redemption date

R = periodic interest or coupon

V = purchase price

Consequently, if we use the formula for the present value of a single future payment at compound interest and the one for the present value of an ordinary annuity, we find that

$$V = C(1 + i)^{-n} + Ra_{\overline{n}|}. \tag{1}$$

is the purchase price of a bond with a redemption price of C due in n periods and a periodic interest of R at the end of each period provided the buyer wants to make i per period.

Example. In order to make 6% converted semi-annually what should be paid 5 years before redemption date for a \$1000, 4% bond with interest paid semi-annually? Assume the bond is to be redeemed at face value.

Solution. We are given that $C = \$1000$, $i = \frac{1}{2}(6\%) = 3\%$, $n = 10 = 5(2)$, and $R = \$20 = \$1000(.04)(\frac{1}{2})$, and we want to find V. We can use the formula of this section or can make use of the way that formula is made up.

The present value of the redemption price is $\$1000(1.03)^{-10} = \$1000\,(.74409391) = \$744.094$ and the present value of the annuity formed by the dividends is $\$20\, a_{\overline{10}|.03} = \$20(8.53020284) = \$170.604$; consequently, the purchase price should be

$$\begin{aligned} V &= \$744.094 + \$170.604 \\ &= \$914.70 \end{aligned}$$

NOTE. If the bond had been redeemable at 107%, the redemption price would have been $\$1000(1.07) = \$1070$ and its value at the time the bond was purchased would have been

$$\begin{aligned} \$1070(1.03)^{-10} &= \$1070(.74409391) \\ &= \$796.18 \end{aligned}$$

Exercise 15-1

Find the price that should be paid for each bond described below provided the investor wants to make the rate j converted m times per year and the bond is bought t years before redemption date.

	Face	C	*Interest rate*	*Interest payable*	j	m	t
1.	$1000	103%	3%	F	4%	1	20
2.	$2000	100%	4%	N	5%	1	14
3.	$ 500	102%	5%	Aug.	6%	1	9
4.	$1000	103%	6%	May	5%	1	17
5.	$1000	100%	4%	J-J	4%	2	10
6.	$1000	100%	4%	F-A	5%	2	10
7.	$ 500	100%	5%	M-S	6%	2	15
8.	$2000	103%	6%	M-N	5%	2	12
9.	$ 500	100%	6%	JAJO	4%	4	$8\frac{1}{2}$
10.	$1000	103%	5%	FMAN	6%	4	13
11.	$1000	106%	5%	MJSD	8%	4	$7\frac{1}{2}$
12.	$2000	100%	4%	JAJO	6%	4	15

13. A 20-year, $1000, 4% bond with interest payable J-J was issued on July 1, 1958. What price should be paid on January 1, 1965 in order to make 5% converted semi-annually if the bond is redeemable at face value?

14. What price should be paid on July 1, 1972 for the bond described in Problem 13 in order to make 5% converted semi-annually provided the bond is to be redeemed at 103%?

15. A 30-year, 5%, $2000 bond with interest payable semi-annually was dated March 1, 1958. What price should be paid March 1, 1970 if the purchaser wants to make 6% converted semi-annually? Assume the bond will be redeemed at 104%.

16. A 15-year, 4%, $1000 bond with interest payable quarterly was issued May 1, 1958 and is to be redeemed at 102%. What price should be paid for it on November 1, 1967 in order to make 6% converted quarterly?

17. A 25-year, 5%, $2000 bond with interest payable semi-annually was issued on May 3, 1957. What price should be paid on November 3, 1968 in order to make 6% if the bond is to be redeemed at face value?

18. What price should be paid on November 3, 1971 for the bond of Problem 17 if the buyer wants to make 5% converted semi-annually and if the bond is to be redeemed at face value?

19. What price should be paid on May 3, 1969 for the bond of Problem 17 provided the buyer wants to make 4% converted semi-annually? Assume the redemption price is 103%.

20. If the redemption price of the bond of Problem 17 is 102% and if the buyer wants to make 7% compounded semi-annually, what price should he pay on May 3, 1972?

15-3 PREMIUM, PAR, AND DISCOUNT

The term *excess* is often used in connection with bonds and is defined as *the purchase price minus the redemption price*. This can be stated symbolically as

$$E = V - C \tag{1}$$

and E can be positive, zero, or negative since a bond may be bought for more than the redemption price, for the redemption price, or for less than the redemption price. We say that a bond is bought at a *premium*, *par*, or a *discount* according as the excess is positive, zero, or negative.

We shall change the form of (1) by using the expression for V as given by (1) of Section 15-2. Thus

$$\begin{aligned} E &= V - C \\ &= C(1+i)^{-n} + Ra_{\overline{n}|i} - C \\ &= Ra_{\overline{n}|i} - C[1 - (1+i)^{-n}] \quad \text{factoring } -C \text{ from two terms} \\ &= Ra_{\overline{n}|i} - Ci\frac{1-(1+i)^{-n}}{i} \quad \text{multiplying and dividing by } i \\ &= Ra_{\overline{n}|i} - Cia_{\overline{n}|i} \end{aligned}$$

Now, factoring out $a_{\overline{n}|i}$, we see that

$$E = (R - Ci)a_{\overline{n}|i} \tag{2}$$

and can say that *the bond is bought at:*

$$\begin{aligned} &a \textit{ premium if } R > Ci \\ &\textit{par if } R = Ci \\ &a \textit{ discount if } R < Ci \end{aligned}$$

where R is the periodic interest or coupon, C is the redemption price, and i is the periodic yield desired by the investor for the n periods until maturity.

Example 1. An investor bought a 10-year, \$500, 4% bond at the time of issue so as to make 5% converted semi-annually. Find the excess and purchase price if the bond is redeemable at 103% and if the interest is payable semi-annually.

Solution. In this problem, we have $C = \$500(1.03) = \$515$, $R = \$500\,(.04)(\frac{1}{2}) = \$10$, $n = 10(2)$, and $i = \frac{1}{2}(.05) = 2.5\%$; hence, the excess is

$$\begin{aligned} E &= (R - Ci)a_{\overline{n}|i} \\ &= [\$10 - \$515(.025)]a_{\overline{20}|.025} \\ &= (\$10 - \$12.875)(15.58916229) \\ &= -\$44.82 \end{aligned}$$

Since E is negative, the bond was bought at a discount, i.e., for less than the redemption price. By use of (1), we have

$$-\$44.82 = V - \$515$$

Therefore

$$V = \$470.18$$

is the purchase price.

Example 2. Find the excess and purchase price if a \$1000, 5% bond is bought 7 years before it is redeemed at 101%. Assume the buyer wants to make 4.5%.

Solution. In this problem we have $C = \$1000(1.01) = \$1010$, $R = \$1000\ (.05) = \$50$, $n = 7$, and $i = 4.5\%$; hence

$$\begin{aligned} E &= (R - Ci)a_{\overline{n}|i} \\ &= [\$50 - \$1010(.045)]a_{\overline{7}|.045} \\ &= (\$50 - \$45.45)(5.89270094) \\ &= \$26.81 \end{aligned}$$

Consequently, the bond is bought at a premium and, by use of (1), we have

$$\$26.81 = V - \$1010$$

therefore

$$V = \$1036.81$$

Exercise 15-2

Determine the excess and price for each bond in Problems 1 through 8, where the investor wants to make the rate j converted m times per year and the bond is bought t years before redemption date.

	Face	C	*Interest rate*	*Interest payable*	j	m	t
1.	\$2000	103%	4%	F	3.5%	1	12
2.	\$ 100	100%	5%	Aug.	4.5%	1	11

	Face	C	Interest rate	Interest payable	j	m	t
3.	\$5000	101%	3.5%	Jan.	4.5%	1	13
4.	\$2000	103%	4.5%	O	5%	1	17
5.	\$1000	100%	4%	J-J	3%	2	7
6.	\$1000	102%	3%	M-S	5%	2	8
7.	\$2500	104%	4%	JAJO	8%	4	9.25
8.	\$ 750	100%	4%	FMAN	6%	4	6.75

9. A 4%, 20-year, \$1000 bond with interest payable F-A is dated August 1, 1958. By finding the excess determine the price that should be paid on February 1, 1969 for the purchaser to make 5% converted semi-annually if the bond is redeemed at face value.

10. Find the excess and purchase price of the bond described in Problem 9 if it is bought on August 1, 1968 so as to yield the buyer 4% converted semi-annually.

11. Determine the excess and purchase price of the bond described in Problem 9 if it is bought on February 1, 1967 so as to produce 3.5% converted semi-annually.

12. A \$1000, 5% bond is bought 9 years before redemption date so as to yield the purchaser 6%. Find the price if the bond is to be redeemed at 102%.

15-4 AMORTIZATION OF A PREMIUM

The price that should be paid for a bond on a specified date in order to yield a desired rate for the buyer is often referred to as the *book value.* If the bond is bought at a premium, then the book value is greater than the redemption price, as pointed out in Section 15-3; furthermore, the periodic interest R on the bond is greater than the interest on the book value at investor's rate and this extra income is offset by decreasing the book value toward the redemption price.

Example. Make a schedule for the amortization of the premium on the bond described in Example 2 of Section 15-3.

Solution. In the problem we had $C = \$1010$, $R = \$50$, $n = 7$, and $i = 4.5\%$. We found the premium to be \$26.81. We can now construct the following table.

AMORTIZATION OF A PREMIUM

End of period	*Bond interest*	*Interest on book value*	*Amortization of premium*	*Book value*
0				\$1036.81
1	\$50	\$46.66	\$3.34	\$1033.47
2	\$50	\$46.51	\$3.49	\$1029.98
3	\$50	\$46.35	\$3.65	\$1026.33
4	\$50	\$46.18	\$3.82	\$1022.51
5	\$50	\$46.01	\$3.99	\$1018.52
6	\$50	\$45.83	\$4.17	\$1014.35
7	\$50	\$45.65	\$4.35	\$1010.00

The first entry in the book value column is the price of the bond. Each entry in the bond interest column is \$50 since that is the periodic interest. Each entry in the third column is 4.5% of the book value at the end of the previous period. If the entry in column 3 is subtracted from \$50, we obtain the entry in column 4. If this is subtracted from the book value for the previous period, we obtain the new book value.

15-5 ACCUMULATION OF A DISCOUNT

If a bond is bought at a discount, the book value is less than the redemption price, as pointed out in Section 15-3; furthermore, the periodic interest R on the bond is less than the interest on the book value at the investor's rate and this deficit of income is made up by increasing the book value toward the redemption price.

Example. Find the excess and purchase price of a \$1000, 4% bond with interest payable semi-annually if bought 3 years before redemption at face value so as to yield 5% converted semi-annually. Make a schedule for accumulation of the discount on the bond.

Solution. In this problem $C = \$1000$, $i = \frac{1}{2}(5\%) = 2.5\%$, $n = 2(3) = 6$, $R = \$1000(.04)(\frac{1}{2}) = \$20$, and we want to find E and V before making a schedule for accumulating the discount. Therefore

$$\begin{aligned} E &= (R - Ci)a_{\overline{n}|i} \\ &= [\$20 - \$1000(.025)]a_{\overline{6}|.025} \\ &= -\ \$5(5.50812536) \\ &= -\ \$27.54 \end{aligned}$$

Hence, the bond is bought at a discount of \$27.54 and the book value at the time of purchase is \$1000 − \$27.54 = \$972.46. We can now construct the following table.

ACCUMULATION OF A DISCOUNT

End of period	*Bond interest*	*Interest on book value*	*Accumulation of discount*	*Book value*
0				\$ 972.46
1	\$20	\$24.31	\$4.31	\$ 976.77
2	\$20	\$24.42	\$4.42	\$ 981.19
3	\$20	\$24.53	\$4.53	\$ 985.72
4	\$20	\$24.64	\$4.64	\$ 990.36
5	\$20	\$24.76	\$4.76	\$ 995.12
6	\$20	\$24.88	\$4.88	\$1000.00

The first entry in the book value column is the price of the bond. Each entry in the bond interest column is \$20 since that is the periodic interest. Each entry in the next column is 2.5% of the book value at the end of the previous period. If \$20 is subtracted from the entry in column 3, we obtain the entry in column 4. If this is added to the book value for the previous period, we obtain the new book value.

Exercise 15-3

Make a schedule for amortization of the premium or accumulation of the discount on each bond described below. The buyer wants to make the rate j converted m times per year and the bond is bought t years before redemption date. If the table calls for more than eight periods, make it for only the first eight.

	Face	C	*Interest rate*	*Interest payable*	j	m	t
1.	\$1000	100%	4%	Jan.	5%	1	5
2.	\$2000	103%	5%	F	4%	1	12

	Face	*C*	*Interest rate*	*Interest payable*	*j*	*m*	*t*
3.	\$5000	100%	5%	O	5%	1	10
4.	\$2000	101%	4%	Aug.	4%	1	9
5.	\$1500	102%	4%	J-J	6%	2	4
6.	\$ 500	102%	4%	F-A	4%	2	6
7.	\$ 750	100%	3%	M-S	4%	2	6
8.	\$ 500	104%	5%	A-O	4%	2	5
9.	\$ 750	100%	5%	JAJO	4%	4	3
10.	\$1000	101%	4%	FMAN	6%	4	3
11.	\$2000	103%	5%	MJSD	6%	4	2
12.	\$5000	100%	6%	FMAN	8%	4	3

15-6 THE STRAIGHT LINE METHOD FOR HANDLING EXCESS

Instead of using annuities for handling a premium or a discount, many banks use the straight line method. If this is done, the excess is divided by the number of periods between purchase date and redemption date in order to determine the periodic amount used in amortization of the premium or in accumulation of the discount.

Example 1. Find the periodic payment toward amortization of the premium in the example of Section 15-4 if the straight line method is used.

Solution. We found the premium to be \$26.81 in that problem; hence, the periodic payment toward amortization of the premium should be $\$26.81 \div 7 = \$3.83$ since the bond was bought seven periods before maturity date.

Example 2. Find the periodic payment toward accumulation of the discount in the example of Section 15-5 if the straight line method is used.

Solution. Since the bond was bought six periods before maturity and at a discount of \$27.54, the periodic cost of accumulating the discount should be $\$27.54 \div 6 = \$4.59$. Consequently, the book value at the time of purchase was $\$972.46 = \$1000 - \$27.54$ and it increases by \$4.59 at the end of each period.

Exercise 15-4

Find, to the nearest cent, the periodic payment toward caring for the excess on each bond described below by use of the straight line method. The buyer wants to make the rate j converted m times per year and the bond is bought t years before redemption date.

	Face	C	*Interest rate*	*Interest payable*	j	m	t
1.	$1000	100%	4%	July	6%	1	4
2.	$2000	100%	4%	Oct.	5%	1	5
3.	$5000	100%	5%	J-J	4%	2	4
4.	$3000	100%	6%	M-S	5%	2	6
5.	$ 500	102%	4%	A-O	4%	2	4
6.	$1000	101%	6%	F-A	4%	2	5
7.	$3000	103%	5%	M-N	6%	2	3
8.	$2000	101%	3%	J-D	4%	2	6
9.	$5000	102%	6%	JAJO	4%	4	3
10.	$3000	101%	4%	FMAN	4%	4	4
11.	$1000	103%	4%	JAJO	6%	4	3
12.	$2000	102%	3%	MJSD	4%	4	5

15-7 PRICE OF A BOND BETWEEN INTEREST DATES

We have seen how to find the price of a bond on an interest date, and we shall now determine the price to be paid between interest dates. The price actually paid on any day is called the *flat price*. The simplest means of determining the flat price is by use of the equation:

$$\textit{flat price} = \textit{quoted price} + \textit{accrued interest}$$

where the quoted price (sometimes called the *and-interest price*) is a per cent of the face value and the *accrued interest* is computed as ordinary simple interest at approximate time on the face of the bond at the bond rate.

Example 1. Find the and-interest price and the flat price of a $1000, 6% bond if bought at 103 two months after an interest payment.

Solution. The and-interest price \$1030 since that is 103% of \$1000. The accrued interest is $\$1000(.06)(\frac{2}{12}) = \$10$, hence

$$\text{flat price} = \$1030 + \$10 = \$1040$$

A second method that is sometimes used in determining the flat price of a bond between interest dates takes into consideration the yield rate desired by the buyer. If this method is used, *the flat price consists of the sum of the purchase price on the immediately preceding interest date and simple interest on that price until the date of purchase computed at the investor's desired yield rate.* This simple interest is ordinary interest at approximate time.

A formula for this can be derived but it need not be since we know how to compute simple interest and know that the value of a bond on an interest date is the sum of the values on that date of the redemption price and the annuity formed by the future interest payments.

Example 2. A \$5000, 4%, 10-year bond is dated August 1, 1959 and bought on December 23, 1961 so as to yield 6%. What was the flat price if the bond is to be redeemed at 104% of face value?

Solution. The redemption price is \$5200 since that is 104% of \$5000; the interest date just before December 23, 1961 is August 1, 1961 and is 8 years before redemption date; hence, the value of the redemption price on the interest date just before purchase is $\$5200(1.06)^{-8}$. Furthermore, the value on August 1, 1961 of the annuity formed by the future interest payment is $\$200a_{\overline{8}|.06}$ since interest is at 4% on \$5000. Consequently, the purchase price on August 1, 1961 is

$$\begin{aligned} V &= \$5200(1.06)^{-8} + \$200a_{\overline{8}|.06} \\ &= \$5200(.62741237) + \$200(6.20979381) \\ &= \$4504.503 \end{aligned}$$

We must now accumulate this at simple interest at 6%, the investor's rate, for the 142 days from August 1, 1961 until December 23, 1961. Thus, we have

$$\begin{aligned} I &= \$4504.503(.06)(142/360) \\ &= \$106.607 \end{aligned}$$

and, finally, the flat price on December 23, 1961 should be

$$\$4504.503 + \$106.607 = \$4611.11$$

Exercise 15-5

Find the flat price on the given purchase date of each bond described in the following table if:

(a) It is bought at a quoted price of 100% plus accrued interest at the bond rate, and

(b) It is bought to yield the rate j converted m times per year.

Use approximate time and ordinary interest for the time from the immediately preceding interest date until the date of purchase. The rate j and redemption price C are not needed in (a).

	Face	*Interest rate paid*		*Issued*	*Redemption date*	*Purchased*	C	j	m
1.	**$1000**	3.5%	O	9-1-57	9-1-72	12-1-59	100%	4%	1
2.	**$5000**	2.5%	June	6-1-48	6-1-73	10-1-66	100%	3.5%	1
3.	**$ 500**	3%	F	2-1-59	2-1-74	4-1-68	103%	3.5%	1
4.	**$1000**	4%	D	12-12-58	12-12-78	2-24-70	102%	4%	1
5.	**$2000**	3%	M-S	3-1-58	3-1-68	4-16-62	101%	5%	2
6.	**$3000**	5%	J-J	7-1-58	7-1-78	8-16-64	100%	4%	2
7.	**$ 500**	7%	F-A	8-1-58	8-1-73	10-1-67	102%	5%	2
8.	**$2000**	6%	A-O	4-15-58	4-15-78	12-15-69	100%	7%	2
9.	**$ 500**	5%	MANF	5-1-56	5-1-76	1-16-66	103%	6%	4
10.	**$ 500**	4%	JAJO	1-1-59	1-1-79	8-1-67	102%	6%	4
11.	**$ 700**	6%	SDMJ	9-1-58	9-1-68	11-1-64	104%	8%	4
12.	**$5000**	4%	NFMA	11-11-57	11-11-67	8-26-64	103%	6%	4

15-8 PURCHASE PRICE AND YIELD BY USE OF BOND TABLES

There are tables giving the price that should be paid for a bond to make a desired yield rate if the purchase is made a specified length of time before the redemption date. The table given at the top of page 239 is an extract from one such table. It is for a 4% bond redeemed at 100% of face value and with interest converted semi-annually and shows the purchase price as a per cent of the face value in order to yield rates varying from 3% to 7% converted semi-annually if bought at the indicated lengths of time before redemption date.

PRICE OF A 4% BOND WITH $m = 2$

Years until redemption	*Yield rate converted semi-annually* 3%	3.5%	4.5%	5%	5.5%	6%	7%
1	100.98	100.49	99.52	99.04	98.56	98.09	97.15
2	101.93	100.96	99.05	98.12	97.20	96.28	94.40
3	102.85	101.41	98.61	97.25	95.90	94.58	92.01
4	103.74	101.85	98.19	96.41	94.68	92.98	89.69
5	104.61	102.28	97.78	95.62	93.52	91.47	87.53
10	108.58	104.19	96.01	92.21	88.58	85.12	78.68
15	112.01	105.80	94.59	89.53	84.81	80.40	72.41
20	114.96	107.15	93.45	87.45	81.94	76.89	67.97

Example 1. What should be the purchase price of a \$500, 4% bond 15 years before redemption date in order to yield 5%? Assume both rates are compounded semi-annually.

Solution. We can use the table of this section since the bond rate is 4% with $m = 2$ and the desired yield rate is given in the table. We look under 5% and across from 15 since the desired yield rate is 5% and the bond is bought 15 years before maturity. Thus, we find that the price is 89.53% of maturity value; hence, one should pay (.8953)\$500 = \$447.65.

Example 2. What yield converted semi-annually is produced if one buys a \$500, 4% bond with interest compounded semi-annually for \$511.40 5 years before maturity?

Solution. The buyer paid \$511.40 for a \$500 bond; hence, he paid $\frac{511.40}{500}$ = 102.28% of the face value. Since he bought 5 years before maturity, we must locate 102.28 across from 5. It is under $3\frac{1}{2}\%$; consequently, that is the rate he made.

15-9 APPROXIMATE YIELD BY INTERPOLATION

Brokers and bond houses quote the price of a bond but do not always give the corresponding yield rate. It is often a matter of practical impor-

tance to know the approximate yield rate and we shall now consider that problem by means of three examples.

Example 1. Use the bond table given in Section 15-8 along with interpolation to find an approximation to the nominal rate converted semi-annually produced by buying a \$500, 4%, $m = 2$ bond for \$531.15 10 years before maturity date.

Solution. This \$500 bond is bought at \$531.15; hence, at $\frac{531.15}{500} =$ 106.23% of the face value 10 years before maturity. Consequently, we try to find 106.23 in the row with 10 but discover that it is not there. We must interpolate between 108.58 and 104.19 since they are nearer the desired value than any others. We shall represent the rate by y and use the usual table for interpolation:

$$4.39 \left\{ \begin{array}{l} 2.04 \left\{ \begin{array}{l} 104.19 \\ 106.23 \end{array} \right. \\ \\ 108.58 \end{array} \right. \qquad \left. \begin{array}{l} 3\tfrac{1}{2}\% \\ y \\ \\ 3\% \end{array} \right\} .5\%$$

Now, we see that 106.23 is $\frac{2.04}{4.39}$ of the way from 104.19 toward 108.58 and we assume that y is that same part of the way from $3\frac{1}{2}\%$ toward 3%. Consequently, we start at 3.5% and go $\left(\frac{2.04}{4.39}\right)(.5\%) = .0023$ toward 3% and find that

$$\begin{aligned} y &= .035 - .0023 \\ &= .0327 \\ &= 3.27\% \end{aligned}$$

Example 2. By use of annuity tables and linear interpolation, find the approximate yield if a \$100, 5% bond is bought for \$96 seven years before being redeemed at face value.

Solution. Since the bond is to be redeemed at face value and is bought at less than that, the buyer will make more than the 5% bond rate. We must find two rates such that the price of the bond is less than \$96 using

one of them and more than \$96 for the other and then interpolate between the two. If we use 5.5% and the fact that the interest is \$5, then the price is

$$\begin{aligned} V &= C(1+i)^{-n} + Ra_{\overline{n}|i} \\ &= \$100(1.055)^{-7} + \$5a_{\overline{7}|.055} \\ &= \$100(.68743681) + \$5(5.68296712) \\ &= \$68.744 + \$28.415 \\ &= \$97.16 \end{aligned}$$

This is greater than the cost of \$96; hence, the yield rate must be more than 5.5%. If we use 6%, we find the cost to be

$$\begin{aligned} V &= \$100(1.06)^{-7} + \$5a_{\overline{7}|.06} \\ &= \$100(.66505711) + \$5(5.58238144) \\ &= \$66.506 + \$27.912 \\ &= \$94.42 \end{aligned}$$

Therefore, the yield rate is less than 6%.

We shall now interpolate between 5.5% and 6%. Thus, if y is the rate, we form the table:

$$2.74 \left\{ \begin{array}{l} 1.16 \left\{ \begin{array}{lcl} 97.16 & & 5.5\% \\ 96 & & y \end{array} \right. \\ \quad 94.42 \qquad\qquad 6\% \end{array} \right\} .5\%$$

for interpolating. Since 96 is $\frac{1.16}{2.74}$ of the way from 97.17 toward 94.42 we assume that y is that same part of the way from 5.5% toward 6%. Thus

$$\frac{1.16}{2.74}(.5\%) = .21\%$$

and the yield is

$$\begin{aligned} y &= 5.5\% + .21\% \\ &= 5.71\% \end{aligned}$$

Example 3. Use annuity tables and linear interpolation to find the approximate yield rate if a \$1000, 4% bond is bought for \$933.70 four years before being redeemed at 102% of the face value.

Solution. Since the bond is to be redeemed for \$1020, 102% of \$1000, and is bought for less than that, the buyer will make more than the 4% bond rate. We must find two rates such that the price of the bond is less than \$933.70 for one of them and more than that for the other, so that we can interpolate between them. If we use 6% and the fact that the periodic interest is \$40, then the price is

$$\begin{aligned} V &= C(1+i)^{-n} + Ra_{\overline{n}|i} \\ &= \$1020(1.06)^{-4} + \$40a_{\overline{4}|.06} \\ &= \$1020(.79209366) + \$40(3.46510561) \\ &= \$807.936 + \$138.604 \\ &= \$946.54 \end{aligned}$$

This is more than the cost of \$933.70; hence, the yield rate is more than 6%. If we use 6.5%, we find that the cost is

$$\begin{aligned} V &= \$1020(1.065)^{-4} + \$40a_{\overline{4}|.065} \\ &= \$1020(.77732309) + \$40(3.42579860) \\ &= \$792.870 + \$137.032 \\ &= \$929.90 \end{aligned}$$

This is less than the cost of \$933.70; hence, the yield rate is less than 6.5%. Consequently, we shall determine the yield by interpolating between 6% and 6.5%. Using the usual tabular scheme, we have

$$\left.16.64\left\{\begin{array}{l} 12.84\left\{\begin{array}{ll} 946.54 & 6\% \\ 933.70 & y \end{array}\right. \\ \qquad\quad\;\; 929.90 \qquad 6.5\% \end{array}\right.\right\}.5\%$$

Since 933.70 is $\frac{12.84}{16.64}$ of the way from 946.54 toward 929.90 we assume that y is that same part of the way from 6% toward 6.5%. Thus

$$\frac{12.84}{16.64}(.5\%) = .39\%$$

and the yield is

$$\begin{aligned} y &= 6\% + .39\% \\ &= 6.39\% \end{aligned}$$

Exercise 15-6

What should be paid for a $500, 4% bond with interest payable semi-annually to yield the rate given in each of Problems 1 through 8 if purchase is made the given number of years before redemption at face value? Use the bond table.

1. 6%, $m = 2$, 10 years.
2. 3.5%, $m = 2$, 5 years.
3. 5%, $m = 2$, 4 years.
4. 4%, $m = 2$, n years.
5. 3%, $m = 2$, 2 years.
6. 7%, $m = 2$, 15 years.
7. 4.5%, $m = 2$, 20 years.
8. 5.5%, $m = 2$, 3 years.

What nominal yield rate converted semi-annually is produced if a $500, 4% bond with $m = 2$ is bought at the given price or per cent of face value and the given number of years before redemption date in each case listed in Problems 9 through 20? Use the bond table and interpolate if necessary.

9. 104.61%, 5 years.
10. 108.58%, 10 years.
11. 105.80%, 15 years.
12. 87.45%, 20 years.
13. $467.25, 20 years.
14. $529, 15 years.
15. $402, 15 years.
16. $511.40, 5 years.
17. 90%, 4 years.
18. 96%, 3 years.
19. $512, 3 years.
20. $450, 10 years.
21. Solve Problem 17 by use of annuity tables.
22. Solve Problem 18 by use of annuity tables.
23. Solve Problem 19 by use of annuity tables.
24. Solve Problem 20 by use of annuity tables.

What yield rate converted annually is produced if a $500, 4% bond is bought at the stated price the given number of years before redemption at the specified per cent of face value? Use annuity tables and interpolation.

	Price	*Years before redemption*	*Redeemed at*
25.	$527.43	7	102%
26.	$487.12	12	102%
27.	$412.73	18	101%
28.	$525.64	8	103%

15-10 SERIAL BONDS

If the bonds of an issue are to be redeemed in installments instead of all at one time, we say the bonds are *serial bonds*. Interest is paid periodically

on the outstanding bonds. If the face value of a bond is to be redeemed in installments and interest paid periodically as due, we say the bond is a *serial bond*. Such a bond is in reality several bonds combined in one contract and can be thought of as an issue of several bonds. The price that should be paid for the entire issue is the sum of the prices of the various installments figured at the buyer's rate.

Example. A \$50,000 issue of 4% bonds with interest payable semi-annually is to be redeemed by payments of \$10,000 in 1 year, \$20,000 in 3 years, and \$20,000 in 5 years. What should a buyer pay for them on the day of issue in order to make 5% converted semi-annually?

Solution. The total price must be the sum of the prices of the three parts figured at the buyer's rate. They are

$$\begin{aligned}
V(\$10{,}000 \text{ due in 1 year}) &= \$10{,}000(1.025)^{-2} + \$200a_{\overline{2}|.025}\\
&= \$10{,}000(.95181440) + \$200(1.92742415)\\
&= \$9{,}518.144 + \$385.485\\
&= \$9{,}903.629\\
V(\$20{,}000 \text{ due in 3 years}) &= \$20{,}000(1.025)^{-6} + \$400a_{\overline{6}|.025}\\
&= \$20{,}000(.86229687) + \$400(5.50812536)\\
&= \$17{,}245.937 + \$2{,}203.250\\
&= \$19{,}449.187\\
V(\$20{,}000 \text{ due in 5 years}) &= \$20{,}000(1.025)^{-10} + \$400a_{\overline{10}|.025}\\
&= \$20{,}000(.78119840 + \$400(8.75206393)\\
&= \$15{,}623.968 + \$3500.826\\
&= \$19{,}124.794
\end{aligned}$$

Consequently, the purchase price of the entire issue is

$$\begin{aligned}
V(\text{entire issue}) &= [9{,}903.629 + \$19{,}449.187 + \$19{,}124.794]\\
&= \$48{,}477.61
\end{aligned}$$

15-11 ANNUITY BONDS

If a bond is to be repaid by equal periodic payments including face and interest, it is called an *annuity bond*. Since these equal periodic payments form an annuity whose present value at the bond interest rate is the face

of the bond we can find the periodic payment if the face, term, and bond rate are known. The purchase price at any time is the value at that time of the annuity formed by future periodic payments figured at the buyer's desired yield rate.

Example. A \$7500, 4% annuity bond is to be repaid in 10 equal annual installments. What should be paid for it just after the third payment if the buyer wants to make 6%?

Solution. We must first find the annual payment that will be made by the concern that issued the bond. It is R determined by the annuity in which $A = \$7500$, $i = 4\%$, and $n = 10$; hence

$$\$7500 = Ra_{\overline{10}|.04}$$

$$R = \frac{\$7500}{a_{\overline{10}|.04}}$$

$$= \$7500(.12329094)$$

$$= \$924.68$$

is the periodic payment to be made by the issuer of the bonds.

If the bond is bought just after the third payment, there are still 7 payments to be made; hence, the price that should be paid is the present value of $R = \$924.68$ at the end of each year for 7 years at the buyer's rate of 6%. Thus

$$A = \$924.68\ a_{\overline{7}|.06}$$

$$= \$924.68(5.58238144)$$

$$= \$5161.92$$

is the price that should be paid.

Exercise 15-7

1. Find the price that should be paid on the date of issue for \$50,000 worth of 3.5% bonds that are to be redeemed by payments of \$15,000 in 10 years, \$15,000 in 15 years, and \$20,000 in 20 years. Assume that the buyer wants to make 4%.

2. What should be paid on the date of issue for the bonds described in Problem 1 if the buyer wants to make 5%?

3. What should be paid 5 years from the date of issue just after the interest has been paid for the bonds of Problem 1 in order to make 6%?

4. What should a buyer pay for the bonds of Problem 1 if he buys just after the interest has been paid 7 years from date of issue and wants to make 3.5%?

5. An issue of \$100,000 worth of 4% bonds is to be redeemed by payments of \$50,000 in 5 years and in 10 years. What should a buyer pay on the date of issue in order to make 6% on his investment?

6. What should a buyer pay on the date of issue for the bonds of Problem 5 in order to make 5% on his investment?

7. What should be paid 2 years after issue for the bonds of Problem 5 in order to make 3.5% on the investment provided the purchase is made after the interest has been paid?

8. What should be paid for the bonds described in Problem 5 on the date of issue in order to make 4%?

9. A \$20,000 annuity bond pays 4% converted semi-annually and is to be redeemed by 5 equal semi-annual payments beginning 6 months from date of issue. What should a buyer pay on date of issue in order to make 5% converted semi-annually?

10. What should be paid on date of issue for the bond of Problem 9 if the buyer wants to make 6% converted semi-annually?

11. What should be paid one year after date of issue for the bond of Problem 9 in order to make 6% compounded semi-annually?

12. What should be paid one year after date of issue for the bond of Problem 9 in order to make 5% converted semi-annually?

13. A 3% annuity bond with face value \$9,500 is to be repaid in 20 equal annual installments. What price should be paid for it just after the sixth installment in order to earn 4%?

14. What should be paid for the bond of Problem 13 just after the fourth installment to earn 5%?

15. If a buyer wants to make 7%, what should he pay for the bond of Problem 13 on the date of issue?

16. In order to make 6%, what should a buyer pay for the bond of Problem 13 if bought just after the seventh installment?

17. A \$20,000, 4% annuity bond with interest payable semi-annually is to be redeemed in 12 equal semi-annual payments beginning 6 months after date of issue. What should the price be on the issue date to earn 3% converted semi-annually?

18. What should a buyer pay for the bond of Problem 17 just after the third installment to earn 5% compounded semi-annually?

19. If he wants to make 4% converted semi-annually, what should a buyer pay for the bond of Problem 17 just after the fourth installment?

20. In order to make 6% converted semi-annually what should be paid for the bond of Problem 17 just after the fifth installment?

SUMMARY

The formula

$$V = C(1 + i)^{-n} + Ra_{\overline{n}|i}$$

for the price on an interest date of a bond with a redemption price of C due in n periods and with interest of R per period is derived under the assumption that the buyer wants to make i per period. Furthermore, the formula

$$E = (R - Ci)a_{\overline{n}|i}$$

for the excess is obtained.

Schedules for amortization of the premium when the excess is positive and for accumulation of the discount when the excess is negative are discussed.

The purchase price between dividend dates is defined and discussed. The use of bond tables is then discussed and is followed by methods for obtaining the approximate yield. Finally, serial and annuity bonds are discussed.

Exercise 15-8 (Review)

1. Find the purchase price of a \$1000, 4% bond with interest payable semi-annually if bought 6 years before redemption at face value and if the buyer wants to make 3.5%.

2. Make a schedule showing the amortization of the premium of the bond of Problem 1.

3. Find the purchase price of the bond of Problem 1 if it is to be redeemed at 102 instead of at face value.

4. Use the bond of Problem 3 and make a schedule showing amortization of the premium or accumulation of the discount.

5. A stadium association issued \$2,600,000 worth of 30-year bonds that pay 4%. They were sold to a bond house so that it would make 3%. How much did the association receive?

6. An investor bought a 4% \$1000 bond for \$970. Determine the approximate yield by use of an annuity table under the assumption that the bond was bought 27 years before redemption at face value.

7. If the investor of Problem 6 had paid \$1020 for the bond, what yield would he have received?

8. A 15-year, 5%, \$500 bond with interest payable semi-annually was dated July 1, 1959. What price should be paid for it on January 1, 1962 if the investor

wants to make 4.5% converted semi-annually? Assume the bond is to be redeemed at 102.

9. What price should be paid for the bond described in Problem 8 if it is purchased July 1, 1962 so as to yield 4.5% converted semi-annually?

10. What price should be paid for the bond described in Problem 8 if bought October 1, 1962 at 102 plus accrued interest?

11. A 3% bond paying interest semi-annually was issued February 1, 1957 and is to be redeemed on February 1, 1977. If on August 1, 1959 it was quoted at 104.25, find the approximate yield by use of an annuity table.

12. If the bond of Problem 11 had been quoted at 98.70, what would the yield rate have been?

13. The Park Cities Fresh Water District issued $3,000,000 worth of 4%, 25-year bonds on May 1, 1959. The bonds were purchased on that date by a syndicate at a price that would give them a yield rate of 3.5%. Find the price paid if the bonds are to be redeemed at face value.

14. An oil company agrees to pay off $250,000 worth of 5% bonds by payments of $125,000 at the end of 3 years and of 5 years. The bonds were purchased on the day of issue by a bank so as to yield 4%. Find the price paid.

15. What price should an investor pay for the bonds of Problem 14 if bought two years after issue so as to yield 6%?

16. A $17,500 annuity bond bears interest at 4.5% and is to be repaid by 10 equal annual payments at the ends of the years beginning one year from issue. What price should be paid just after the fifth payment by an investor who wants to make 6%?

16

General annuities

16-1 INTRODUCTION

Up to this point in our discussion of annuities, we have considered only the simple case, that is those in which the payment interval and the interest period coincide. Although most of the problems that arise in business are covered by these circumstances, there are situations in which the interest period does not coincide with the payment interval. For instance, before the advent of the H.O.L.C. and the F.H.A., there were a great many mortgage contracts in which the interest rate was effective

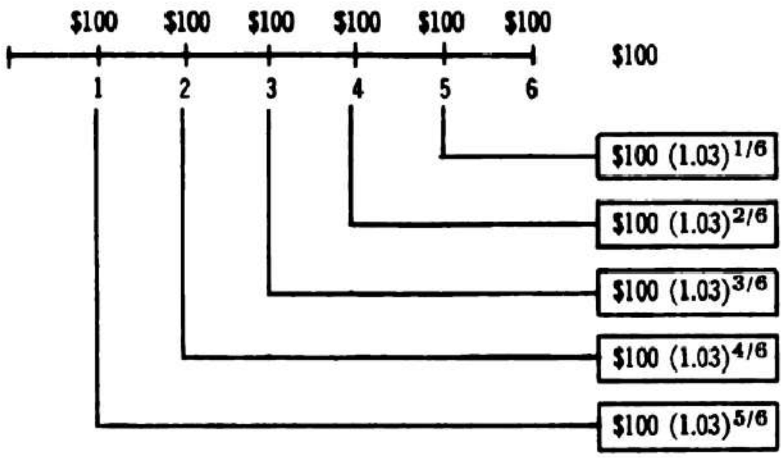
and payments were made more often than once a year. A *general annuity* is an annuity in which the payment period and the conversion interval do not coincide. We shall consider the two cases of the general annuity most often encountered: (a) when the interest period is an integral multiple of the payment interval and (b) when the payment interval is an integral multiple of the interest period. In each case our procedure will be to make the solution of the problem depend on the value of an equivalent simple annuity.

16-2 ANNUITIES WITH AN INTEGRAL NUMBER OF PAYMENTS PER INTEREST PERIOD

To derive formulas for the present and accumulated value of an annuity of p payments of $\frac{1}{p}$ per interest period, we shall assume that the formula for compound interest is true for fractional periods.

Example. If \$600 is paid in installments of \$100 at the end of each month for 6 months and if the interest rate is 3% per 6 months, find the accumulated value of the payments at the end of the 6-month period.

The first payment draws interest for 5 months or $\frac{5}{6}$ of the interest period at rate 3%. Its amount is $\$100(1.03)^{5/6}$. The second payment draws interest for 4 months or $\frac{4}{6}$ of the interest period at rate 3%. Its amount is $\$100(1.03)^{4/6}$ · · · The last payment of \$100 draws no interest. The ac-



cumulated value, illustrated by the accompanying diagram, is the sum of the geometric progression:

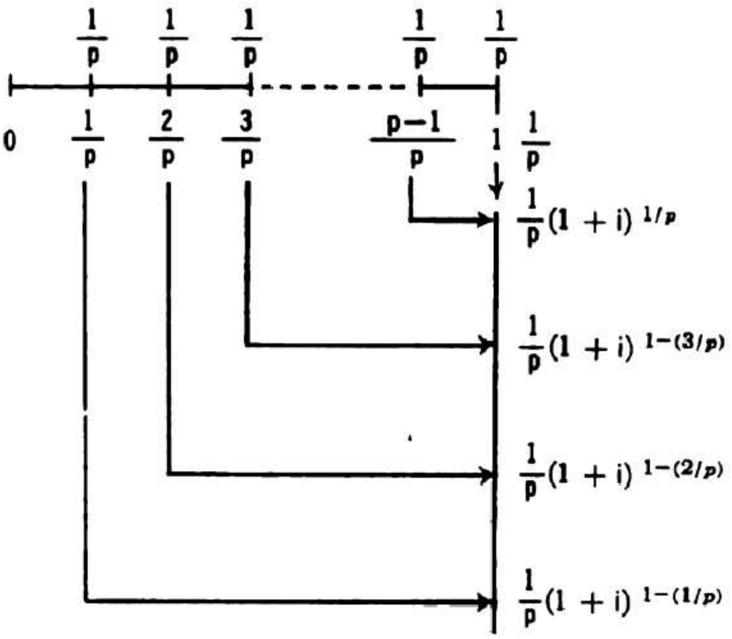
$$\$100[1 + (1.03)^{1/6} + (1.03)^{2/6} + \cdots + (1.03)^{5/6}]$$

This is
$$\$100 \frac{1 - (1.03)^{1/6}\,(1.03)^{5/6}}{1 - (1.03)^{1/6}}$$
$$= \$100 \frac{1 - 1.03}{1 - (1.03)^{1/6}}$$
$$= \$100 \frac{-.03}{1 - (1.03)^{1/6}}$$
$$= \$100 \frac{.03}{(1.03)^{1/6} - 1}$$
$$= \$100 \frac{.03}{1.00493862 - 1} \qquad \text{by Table VIII}$$
$$= \$100 \frac{.03}{.00493862}$$
$$= \$100\ (6.07457)$$
$$= \$607.46$$

Thus the accumulated value of the six payments of \$100 is \$607.46. This payment, which could be made at the end of 6 months, is the equivalent of the six payments at a periodic rate of 3%, and hence can be exchanged for them.

Payment number	*Paid after*	*Accumulates for*	*Accumulates to*
1	$\frac{1}{p}$ of a period	$\left(1 - \frac{1}{p}\right)$ of a period	$\frac{1}{p}(1+i)^{1-1/p}$
2	$\frac{2}{p}$ of a period	$\left(1 - \frac{2}{p}\right)$ of a period	$\frac{1}{p}(1+i)^{1-2/p}$
3	$\frac{3}{p}$ of a period	$\left(1 - \frac{3}{p}\right)$ of a period	$\frac{1}{p}(1+i)^{1-3/p}$
. . .	. . .	. . .	. . .
$p-1$	$\frac{p-1}{p}$ of a period	$\frac{1}{p}$ of a period	$\frac{1}{p}(1+i)^{1/p}$
p	1 period	no period	$\frac{1}{p}$

We shall continue the derivations by determining the amount to which p payments of $\frac{1}{p}$ each will accumulate in one interest period at rate i per period. The accompanying table and diagram will help.



Hence, these payments accumulate to

$$[1 + (1+i)^{1/p} + \cdots + (1+i)^{1-2/p} + (1+i)^{1-1/p}]\frac{1}{p}$$

in one interest period. These terms form a geometric progression of p terms, first term 1, common ratio $(1+i)^{1/p}$, and last term $(1+i)^{1-1/p}$; hence, since the sum of a geometric progression is the quotient obtained by using the first term minus the common ratio times the last term as numerator and one minus the common ratio as denominator, their sum is

$$\frac{1}{p}\left[\frac{1-(1+i)^{1/p}(1+i)^{1-1/p}}{1-(1+i)^{1/p}}\right] = \frac{1}{p}\left[\frac{1-(1+i)}{1-(1+i)^{1/p}}\right] =$$

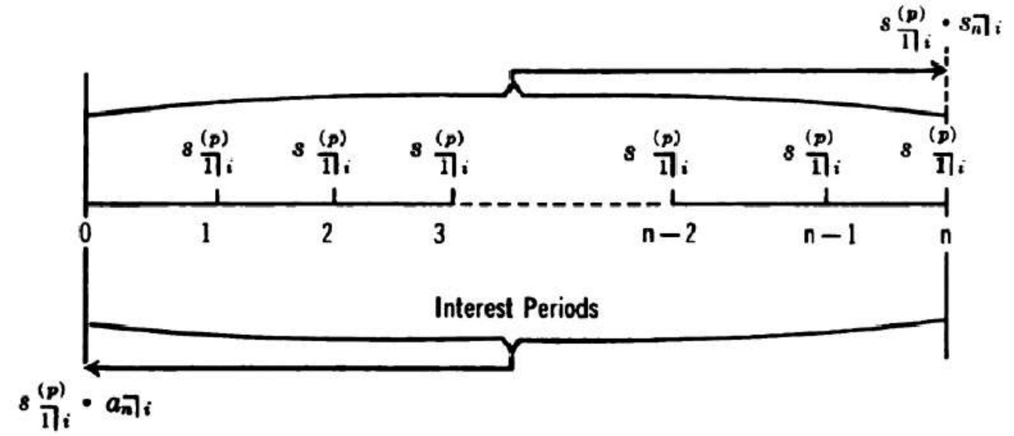
$$\frac{1}{p}\left[\frac{i}{(1+i)^{1/p}-1}\right]$$

Consequently, the p payments of $\frac{1}{p}$ accumulate to

$$\frac{1}{p}\left[\frac{i}{(1+i)^{1/p}-1}\right] = s^{(p)}_{\overline{1}|i}$$

in one interest period. This amount is symbolized by $s^{(p)}_{\overline{1}|i}$, as indicated.

We are now ready to find the present value, $a^{(p)}_{\overline{n}|i}$, and the accumulated value, $s^{(p)}_{\overline{n}|i}$, of an ordinary annuity, with p payments of $\frac{1}{p}$ each per interest period for n periods at a periodic interest rate of i. Consider the following diagram in which, instead of indicating separate payments of $\frac{1}{p}$, we indicate the equivalent amount of each group of p payments at the end of each period by $s^{(p)}_{\overline{1}|i}$.



$s^{(p)}_{\overline{1}|i}$ is now the periodic rent on a simple annuity, and we may write immediately

(1) $$s^{(p)}_{\overline{n}|i} = s^{(p)}_{\overline{1}|i} \cdot s_{\overline{n}|i}$$

and

(2) $$a^{(p)}_{\overline{n}|i} = s^{(p)}_{\overline{1}|i} \cdot a_{\overline{n}|i}$$

If the total periodic rent is R, these equations become

(1′) $$S = Rs^{(p)}_{\overline{n}|i} = Rs^{(p)}_{\overline{1}|i}\, s_{\overline{n}|i}$$

and

(2′) $$A = Ra^{(p)}_{\overline{n}|i} = Rs^{(p)}_{\overline{1}|i}\, a_{\overline{n}|i}$$

Some values of $s^{(p)}_{\overline{1}|i}$ are found in Table IX.

Example 2. Find the accumulated value of an annuity of \$1200 per year, paid in monthly installments of \$100 each, for 10 years at a nominal rate of 6% compounded semi-annually.

Solution. In this problem the periodic interest rate $i = \frac{j}{m} = \frac{.06}{2} = .03$, the number of interest periods is $(2)(10) = 20$, the number of payments, p, per interest period is six since that is equal to the number of payments per year divided by the number of interest periods per year, and the rent per interest period is \$600. Hence

$$\begin{aligned} S &= \$600\, s^{(6)}_{\overline{20}|.03} \\ &= \$600\, s^{(6)}_{\overline{1}|.03} \cdot s_{\overline{20}|.03} \\ &= \$600(1.01242816)(26.87037449) \\ &= \$607.456896(26.87037449) \\ &= \$16{,}322.59 \end{aligned}$$

Let us point out that the figure \$607.456896, which appears in the next to the last line, rounds off to the \$607.46 that was obtained in Example 1 earlier in this section.

Example 3. What would it cost to buy an annuity of \$2000 per year, paid in semi-annual installments of \$1000 each for 20 years, if the effective rate is 4%?

Solution. In this problem, the periodic interest rate is $i = .04$, the number of interest periods is $n = 20$, the number of payments per interest period is $p = 2$, and the rent per period is \$2000. Hence

$$\begin{aligned} A &= \$2000\, a^{(2)}_{\overline{20}|.04} \\ &= \$2000\, s^{(2)}_{\overline{1}|.04}\, a_{\overline{20}|.04} \\ &= \$2000(1.00990195)(13.59032634) \\ &= \$27{,}449.79 \end{aligned}$$

Example 4. Find the monthly payment required to pay off a debt of \$10,000 in 15 years if $j = .05$, $m = 4$.

Solution. In this problem, $A = \$10{,}000$, the periodic interest rate is $i = \frac{.05}{4} = .0125$, the number of interest periods is $(4)(15) = 60$, the number of payments per period is three since that is the number of payments per year divided by the number of interest periods per year, and the quarterly rent is R. Hence

$$\begin{aligned} Ra^{(3)}_{\overline{60}|.0125} &= \$10{,}000 \\ Rs^{(3)}_{\overline{1}|.0125}\, a_{\overline{60}|.0125} &= \$10{,}000 \\ R(1.00415516)(42.03459179) &= \$10{,}000 \\ R = \frac{\$10{,}000}{42.209254} &= \$236.91 \end{aligned}$$

Thus \$236.91 is the total quarterly payment. The monthly payment, therefore, is $\frac{\$236.91}{3} = \$78.97$.

Exercise 16-1

1. What amount paid at the end of a year is equivalent to \$10 paid at the end of each month for a year if money is worth 5%?

2. If money is worth 7%, what amount paid at the end of a year would be equivalent to payments of \$50 at the end of each month for a year?

3. What amount paid at the end of 6 months is equivalent to \$300 paid at the end of each quarter if the semi-annual rate is 2%?

4. What amount paid at the end of each quarter is equivalent to \$150 paid at the end of each month if $j = .06$, $m = 4$?

Find (a) the present value and (b) the accumulated value of the ordinary annuities in Problems 5 through 16.

	Rent per interest period	*Payments per interest period*	*Time in years*	j	m
5.	\$1200	12	10	.04	1
6.	\$ 600	6	8	.05	1
7.	\$ 900	4	12	.06	1
8.	\$ 600	2	10	.04	1
9.	\$ 600	6	10	.03	2

10.	\$ 600	2	20	.045	2
11.	\$1500	3	15	.08	4
12.	\$ 750	3	20	.06	4
13.	\$ 150	6	7	.07	2
14.	\$ 500	2	10	.03	2
15.	\$ 150	3	12	.04	4
16.	\$ 300	3	5	.06	4

17. A man purchases a house for \$5000 cash and \$60 per month for the next 20 years. What is the equivalent cash price if money is worth 6% compounded semi-annually and the first monthly payment is made one month after the cash payment?

18. What payment must be made at the end of each quarter for 15 years in order to accumulate \$12,000 if money is worth 5% compounded semi-annually?

19. In order to accumulate a retirement fund at age 68, a man begins at age 40 to deposit \$50 at the end of each month. If money is worth 4% compounded quarterly, how much will he have in the fund if he survives to age 68?

20. Determine the payment that must be made at the end of each 6 months for 20 years in order to accumulate \$18,000 if money is worth 6% compounded annually.

16-3 ANNUITIES WITH AN INTEGRAL NUMBER OF INTEREST CONVERSION PERIODS PER PAYMENT INTERVAL

We shall find formulas for the present value and accumulated value of an annuity in which the interest rate is compounded an integral number of times per payment interval, but shall work two numerical examples before doing that.

Example 1. What payment made at the end of each quarter is equivalent to \$1000 at the end of each year if $j = .06$, $m = 4$?

Solution. The desired periodic payment is R, the periodic interest rate is $i = \frac{j}{m} = \frac{.06}{4} = .015$, the number of quarterly payments is $n = 4$, and $S = \$1000$. Setting the accumulated value of the payments equal to \$1000, we have

$$R s_{\overline{4}|.015} = \$1000$$

Hence
$$R = \frac{\$1000}{s_{\overline{4}|.015}}$$
$$= \$1000(.24444478) \qquad \text{by Table VII}$$
$$= \$244.44$$

Thus, a payment of \$244.44 per quarter at $i = .015$ per quarter is equivalent to \$1000 at the end of a year.

Example 2. What payment at the end of each month is equivalent to a payment of \$600 at the end of 6 months if $j = .06$, $m = 12$?

Solution. We are given $i = \frac{.06}{12} = .005$, $n = 6$, $S = \$600$, and we want to find R, the monthly payment. Setting the accumulated value of the monthly payments equal to the desired semi-annual payment, we have

$$R s_{\overline{6}|.005} = \$600$$
$$R = \frac{\$600}{s_{\overline{6}|.005}}$$
$$= \$600(.16459546)$$
$$= \$98.76$$

This is the amount which, if paid at the end of each month, is equivalent to paying \$600 at the end of 6 months at 6% compounded monthly.

Returning now to the general problem, we want to find the payment that must be made at the end of each conversion period in order to be equivalent to the actual payment, P, which is made at the end of the payment interval. If R is the payment per conversion period, i is the rate per conversion period and c is the number of conversion periods per payment interval, we have

$$R s_{\overline{c}|i} = P$$

and solving for R gives

(3)
$$R = \frac{P}{s_{\overline{c}|i}}$$

This is the payment per interest period that is equivalent to P at the end of each payment interval.

In order, now, to find the present value and the accumulated value of an ordinary annuity with c conversion periods per payment interval, we

multiply the payment R per interest period by $a_{\overline{n}|i}$ and $s_{\overline{n}|i}$, respectively, where n is the number of interest periods in the term and i is the periodic interest rate. Thus

$$A = Ra_{\overline{n}|i} \tag{4}$$

and, using (3)

$$A = \frac{P}{s_{\overline{c}|i}} a_{\overline{n}|i} \tag{4$'$}$$

Similarly

$$S = Rs_{\overline{n}|i} \tag{5}$$

and

$$S = \frac{P}{s_{\overline{c}|i}} \cdot s_{\overline{n}|i} \tag{5$'$}$$

Example 3. Find the present value of an ordinary annuity of \$600 payable at the end of each 6 months for 15 years if $j = .06$, $m = 12$.

Solution. In this problem, $P = \$600$, $i = \frac{j}{m} = \frac{.06}{12} = .005$, $n = (12)(15) = 180$, the number of interest periods per payment interval is $c = 6$. This can always be found by dividing the number of conversion periods per year by the number of payments per year. Hence

$$\begin{aligned}
R &= \frac{P}{s_{\overline{c}|i}} \\
R &= \frac{\$600}{s_{\overline{6}|.005}} \\
&= \$600(.16459546) \\
&= \$98.757276 \quad \text{as in Example 2}
\end{aligned}$$

Therefore

$$\begin{aligned}
A &= Ra_{\overline{n}|i} \\
&= (\$98.757276)\, a_{\overline{180}|.005} \\
&= \$98.757276\ (118.503515) \\
&= \$11{,}703.08
\end{aligned}$$

Example 4. Find the amount, payable at the end of each quarter for 12 years, that is necessary to accumulate \$12,500 if $j = .05$, $m = 12$.

Solution. In this problem $i = \frac{.05}{12} = \frac{5}{12}\%$, $n = (12)(12) = 144$, $c = \frac{12}{4} = 3$, $S = \$12{,}500$, and we want to find P. Substituting in (5′), we have

$$\$12{,}500 = \frac{P}{s_{\overline{3}|.05/12}} s_{\overline{144}|.05/12}$$

Solving for P, we have

$$\begin{aligned} P &= \frac{\$12{,}500\, s_{\overline{3}|.05/12}}{s_{\overline{144}|.05/12}} \\ &= \frac{\$12{,}500(3.0125174)}{196.76373} \\ &= \frac{\$37{,}656.468}{196.76373} \\ &= \$191.38 \end{aligned}$$

Example 5. A man is paying off a mortgage of \$18,500 by making payments at the end of each year for 20 years. (a) If the interest rate is $4\frac{1}{2}\%$ compounded quarterly, find the annual payment. (b) What does he still owe just after the 13th payment has been made?

Solution. In this problem $i = \frac{.045}{4} = .01125 = 1\frac{1}{8}\%$, $n = (4)(20) = 80$, $c = \frac{4}{1} = 4$, $A = \$18{,}500$, and we want to find P. Substituting in (4′), we have

$$\begin{aligned} \$18{,}500 &= \frac{P}{s_{\overline{4}|.01125}} a_{\overline{80}|.01125} \\ P &= \frac{\$18{,}500\, s_{\overline{4}|.01125}}{a_{\overline{80}|.01125}} \\ &= \frac{(\$18{,}500)(4.06800767)}{52.567311} \\ &= \frac{\$75{,}258.142}{52.567311} \\ &= \$1431.65 \end{aligned}$$

Solution of (b). The remaining liability just after the 13th payment is the value at that time of the seven payments still to be made. We know

from (a) that $P = \$1431.65$, $i = 1\frac{1}{8}\%$, $c = 4$, and we now have $n = (4)(7) = 28$. Hence, substituting again in (4′), we have

$$
\begin{aligned}
A &= \frac{\$1431.65}{s_{\overline{4}|.01125}} a_{\overline{28}|.01125} \\
&= \frac{(\$1431.65)(23.904579)}{4.06800767} \\
&= \$8412.72
\end{aligned}
$$

Exercise 16-2

1. What amount paid at the end of each quarter is equivalent to \$800 paid at the end of one year if $j = .05$, $m = 4$?

2. What amount paid at the end of each 6 months is equivalent to \$1250 paid at the end of one year if $j = .045$, $m = 2$?

3. What amount paid at the end of each month is equivalent to \$5000 paid at the end of 6 months if $j = .04$, $m = 12$?

4. What amount paid at the end of each month is equivalent to \$950 paid at the end of one year if $j = .08$, $m = 12$?

Find (a) the present value and (b) the accumulated value of the ordinary annuities in Problems 5 through 20.

	Rent per payment interval	*Payments per year*	*Time in years*	j	m
5.	\$2000	1	15	.04	12
6.	\$2000	1	15	.04	4
7.	\$2000	1	15	.04	2
8.	\$1000	2	15	.04	12
9.	\$1000	2	15	.04	4
10.	\$ 500	4	15	.04	12
11.	\$1600	1	20	.05	12
12.	\$1600	1	20	.05	4
13.	\$1600	1	20	.05	2
14.	\$ 800	2	20	.05	12
15.	\$ 800	2	20	.05	4
16.	\$ 400	4	20	.05	12
17.	\$5280	1	10	.07	12
18.	\$2640	2	10	.07	12
19.	\$2640	2	10	.07	4
20.	\$1320	4	10	.07	12

21. The cash price of a house is $18,260. After a down payment of $5000, a man makes semi-annual payments at the end of each 6 months for 18 years at 5% compounded monthly. (a) Find the semi-annual payment. (b) How much does he still owe just after making the 24th semi-annual payment?

22. In order to make a down payment on a home at the end of 5 years, a couple deposit $1200 at the end of each year in a savings bank that pays $2\frac{1}{2}\%$ compounded semi-annually. How much do they have in the fund at the end of the time?

23. An endowment policy for $25,000 matures at age 65. Instead of taking the cash, the policy-holder elects to take an ordinary annuity certain for 20 years, payable to him or his estate every 6 months. If the insurance company pays interest at the rate of 3% compounded quarterly, find the semi-annual payment.

24. Find the periodic payment of an annuity, payable quarterly, whose accumulated value in 8 years is $7250, if $j = .06$, $m = 12$.

SUMMARY

Formulas for the present value and the accumulated value of general ordinary annuities have been obtained in two cases. If there is an integral number of payments per interest period, the formulas are

$$S = Rs^{(p)}_{\overline{1}|i}\, s_{\overline{n}|i}$$

and

$$A = Rs^{(p)}_{\overline{1}|i}\, a_{\overline{n}|i}$$

where R = the total periodic rent

p = the number of payments per interest period

i = the rate per interest period

n = the number of interest periods

If there is an integral number of conversion periods per payment interval, the formulas are

$$A = \frac{P}{s_{\overline{c}|i}}\, a_{\overline{n}|i}$$

and

$$S = \frac{P}{s_{\overline{c}|i}}\, s_{\overline{n}|i}$$

where P = payment per payment interval

c = number of conversion periods per payment interval

i = rate per conversion period

n = number of interest periods in the term

Exercise 16-3 (Review)

1. Find the present value of an ordinary annuity of \$900 quarterly for 7 years if $j = .03$, $m = 12$.

2. Find the accumulated value of an ordinary annuity of \$900 quarterly for 7 years if $j = .03$, $m = 12$.

3. Find the present value of an ordinary annuity of \$300 monthly for 7 years if $j = .03$, $m = 4$.

4. Find the accumulated value of an ordinary annuity of \$300 monthly for 7 years if $j = .03$, $m = 4$.

5. What payment must be made at the end of each month for 9 years to pay off a debt of \$10,120 if $j = .05$, $m = 2$?

6. What payment must be made at the end of each month for 9 years in order to accumulate a fund of \$10,120 if $j = .05$, $m = 2$?

7. What payment must be made at the end of each 6 months for 9 years to pay off a debt of \$10,120 if $j = .05$, $m = 4$?

8. What payment must be made at the end of each 6 months for 9 years in order to accumulate \$10,120 if $j = .05$, $m = 4$?

9. To pay off a mortgage of \$15,200 a man agrees to make payments of \$120 per month as long as necessary with interest at $j = .05$, $m = 2$. Just after the monthly payment at the end of 8 years, he refinances the balance at $j = .03$, $m = 12$, and agrees to make payments at the end of each year for 10 years. Find the annual payment for these 10 years.

10. A debt of \$100,000 is to be paid off by making payments at the end of each 6 months for 10 years into a sinking fund that bears interest at 2% compounded quarterly. (a) Find the semi-annual payment. (b) How much is in the fund just after the payment at the end of the 6th year?

11. An insurance company sells a house for \$2000 cash and twenty annual payments of \$500. If the interest rate charged is $4\frac{1}{2}$% compounded semi-annually, what is the equivalent cash price of the house? How much is the unpaid balance immediately after the 13th payment?

12. At the end of each year for 12 years, \$1500 is invested with the Ajax Building and Loan Association, which pays 3% compounded monthly. What is the amount of the investment at the end of the 12th year?

13. Mr. Carver needs \$10,000 cash. He is offered \$4000 cash and five annual payments of \$1350. If money is worth 6% compounded quarterly, how much would he gain or lose by accepting the proposition?

14. A man buys a farm for \$2500 cash and ten equal annual payments of \$1000. If money is worth 5% compounded quarterly, what is the equivalent cash price of the farm? What amount is still outstanding just after the 7th annual payment?

15. In order to pay off a loan, a man makes payments of \$600 per year for 4 years. If interest is at 5% compounded monthly, what sum was borrowed?

16. An ex-serviceman bought a house for a cash payment of \$840 and a promise to pay \$840 on each anniversary of the date of purchase until a total of 12 payments, including the cash one, had been made. What was the cash price of the house provided money is worth $3\frac{1}{2}\%$ converted semi-annually?

17. How much will be to one's credit in 10 years if he pays \$450 at the end of each 6 months into a fund that earns 4% converted quarterly?

18. Determine the semi-annual payment made at the end of each 6 months that is required to pay off a loan of \$13,842.28 and interest on the outstanding principal in 40 payments. Assume that money is worth $4\frac{1}{2}\%$ compounded annually.

19. An employee of a manufacturing concern has a choice of two retirement plans. Under the first plan, he gets \$125 per month at the end of each month as long as he lives. Under the other plan, he gets \$100 per month as long as he lives, and, if that is less than 10 years, his estate receives \$100 per month for the remainder of the 10 years. If he lives 8 years and money is worth 6% converted quarterly, which plan would be the more desirable? On the day of retirement what difference would there be between the values of the plans?

20. Mr. Walker was paying for his car, which cost \$1827, by monthly payments of \$55 at the end of each month. He decided to finish paying for it at the time of the 15th payment. If money is worth 4% converted quarterly, how much, in addition to the regular amount, did he have to pay?

17

Life annuities and life insurance

17-1 INTRODUCTION

The study of the mathematics of life annuities and life insurance is based on the following three general concepts:
(1) Compound interest.
(2) The probability of death of a particular person within some particular period of time.
(3) Equations of value.
The student is familiar with the first and third of these and it is the second that must now be discussed. It is not possible to tell just when any given individual will

die. If it were possible, we should probably have no need for life insurance companies. It is the uncertainty of the time of death that makes the protection that life insurance gives an individual not only desirable but often necessary.

Although it is not possible to know just when a given individual will die, it is possible to ascertain from records kept by life insurance companies just how many individuals out of a large number will die within a given period of time. If such records are listed in tabular form indicating the number of individuals out of the group under consideration who die at each age, then the ratio of the number dying at any age to the number who start that age can be computed with sufficient accuracy to enable a life insurance company to operate safely. Such a tabular form is called a *mortality table*. The Commissioners Standard Ordinary Mortality Table (C.S.O.) is used by most insurance companies. It is given in Table X in the back of the book. It can be seen from this table that 2312 out of 951,483 alive at age 20 die before reaching age 21. Thus the relative frequency of death at age 20 is .00243 or 2.43 per thousand.

17-2 PROBABILITY

If the total number of ways in which an event can happen is n, these n ways being equally likely, and if s of these ways are successes and f are failures, then the probability that the event will succeed in any one trial is defined as

$$p = \frac{s}{n} = \frac{s}{s+f}$$

and the probability that it will fail to happen in any one trial is defined as

$$q = \frac{f}{n} = \frac{f}{s+f}$$

Example. If, out of an ordinary deck of 52 cards, 1 card is drawn, the probability that this card will be a spade is $\frac{13}{52}$, or $\frac{1}{4}$, and the probability that it will not be a spade is $\frac{39}{52}$, or $\frac{3}{4}$, since there are 13 different ways in which a spade can be drawn, and 39 different ways in which other cards can be drawn. Similarly, the probability that the card drawn will be a king is $\frac{4}{52}$, or $\frac{1}{13}$, whereas the probability that the card will be the king of spades is $\frac{1}{52}$.

It will be noticed from this definition and example that it is possible to determine theoretically the probability of success or failure of a given event before the event is ever tried. This is called *a priori* probability.

We are particularly interested in the probability of life or death of an individual during a given period of time. Here it is impossible to determine, *a priori*, all the ways in which the event can occur. In such instances it is necessary, therefore, that we observe the event in question, keeping a record of the number of times the event happens or fails to happen out of a given number of trials. If s is the number of successes and f the number of failures out of n trials, the ratios

$$\frac{s}{s+f} = \frac{s}{n} \text{ and } \frac{f}{s+f} = \frac{f}{n}$$

are the relative frequencies of success and failure, respectively, out of n trials. Now if we consider an event such as the drawing of one card from a deck of 52, and we desire the probability that this card be a spade, we have determined, *a priori*, that $p = \frac{1}{4}$. Suppose, however, instead of determining the probability *a priori*, we had tried drawing a card from the deck, replacing the card after it had been drawn, reshuffling the deck, and drawing again, and so forth. If this experiment is performed a large number of times and a record is kept of the number of spades drawn, the relative frequency $\frac{s}{n}$ will not in general be equal to $p = \frac{1}{4}$, but we will find that, if the number of times the experiment is tried is continually increased, the relative frequency approaches closer and closer to the *a priori* probability $p = \frac{1}{4}$. Hence, as n becomes larger and larger, the relative frequency becomes a better and better estimate of the value of the probability. In fact, if a limit exists we define the probability that an event will happen in a single trial as the limit of the relative frequency as n becomes infinite, that is

(1) $$p = \lim_{n \to \infty} \frac{s}{n}$$

This is called *a posteriori* probability.

This is the definition of probability that we are going to use in our discussion of the mortality table. Although in this instance it is not possible to determine the value of this limit exactly, it is possible, as has been stated in Section 17-1, to determine it with a sufficient degree of accuracy to enable life insurance companies to operate safely. Referring again to

Table X we thus determine that the probability that a person aged 20 will die before reaching age 21 is

$$p = \frac{d_{20}}{l_{20}} = \frac{2312}{951,483} = .00243$$

and that the probability that a person aged 20 will live to be 21 is $p = \frac{949171}{951483} = .99757$. It may be noticed that the sum of the probabilities of a person aged 20 living to age 21 and of dying prior to age 21 is .00243 + .99757 = 1. This means that one or the other of the events is sure to happen.

Similarly, the probability that a person aged 20 will survive to age 30 is

$$\frac{924609}{951483} = .97176$$

17-3 THE MORTALITY TABLE

The Commissioners Standard Ordinary (1941) Mortality Table, to which reference has already been made, was set up by the committee appointed by the National Association of Insurance Commissioners. Mortality statistics collected from life insurance companies were used in its preparation. We will not discuss here the methods employed by them, but we will use the table exclusively in our future calculations.

It consists primarily of four columns, the first two of which give the essential information. The first column lists the ages from 0 up to and including 99. There are 1,023,102 persons considered in this table and this number is called the *radix* of the table. The second column lists the number of people alive at each age with 1,023,102 at age 0 and finishing with 125 out of these 1,023,102 who are still alive at age 99. Each intervening figure represents the number of people who are alive at each listed age out of those who started at age 0. Thus $l_{40} = 883,342$. The third column lists the number of individuals who die between ages x and $x + 1$ out of the 1,023,102 who were alive at age 0. Thus the number dying between ages 40 and 41 is $d_{40} = 5459$. The d_x column is really obtained by differencing the l_x column. Thus $d_1 = l_0 - l_1 = 1,023,102 - 1,000,000 = 23,102$ and $d_{40} = l_{40} - l_{41} = 883,342 - 877,883 = 5459$. In general, $d_x = l_x - l_{x+1}$. The fourth column is headed 1000 q_x, and q_x is the symbol used to represent the probability that a person whose age is x will die before reaching age $x + 1$. From our definition, $q_x = \frac{d_x}{l_x}$. So this column is

obtained by dividing each entry in the third column by the corresponding entry in the second column, and multiplying by 1000, thus giving the rate of mortality per 1000 of population of age x.

The other columns in the table are called *commutation* columns. Their development and use will be explained in later sections.

17-4 SYMBOLS

In addition to the symbols l_x, d_x, and q_x, which have already been defined, many others will be useful. Some of these, and relations existing between them, follow:

(x) = a person whose age is x

p_x = the probability that (x) will live one year

Hence

$$p_x = \frac{l_{x+1}}{l_x} \tag{2}$$

Since $d_x = l_x - l_{x+1}$ and $q_x = \dfrac{d_x}{l_x}$

$$\begin{aligned} p_x + q_x &= \frac{l_{x+1}}{l_x} + \frac{d_x}{l_x} \\ &= \frac{l_{x+1} + d_x}{l_x} \\ &= \frac{l_x}{l_x} = 1 \end{aligned}$$

${}_np_x$ = the probability that (x) will survive n years

Thus

$$ {}_np_x = \frac{l_{x+n}}{l_x} \tag{2'}$$

${}_nq_x$ = the probability that (x) will die within n years

Hence

$$ {}_nq_x = \frac{l_x - l_{x+n}}{l_x} = 1 - \frac{l_{x+n}}{l_x} = 1 - {}_np_x \tag{3}$$

since $l_x - l_{x+n}$ is the number of people who die within n years out of the l_x at age x, and ${}_np_x$ is the probability that (x) will live n years.

${}_{n|}q_x$ = the probability that (x) will survive n years and then die in the $(n+1)$st year

Out of the l_x people at age x, d_{x+n} of them die between ages $x+n$ and $x+n+1$. Hence

(4) $$_{n|}q_x = \frac{d_{x+n}}{l_x} = \frac{l_{x+n}}{l_x} \cdot \frac{d_{x+n}}{l_{x+n}} = {}_np_x \cdot q_{x+n}$$

This states that $_{n|}q_x$ is equal to the product of the probability that (x) will survive n years and the probability that $(x+n)$ will die within one year.

We might also have written

(4′) $$_{n|}q_x = \frac{d_{x+n}}{l_x} = \frac{l_{x+n} - l_{x+n+1}}{l_x} = {}_np_x - {}_{n+1}p_x \,.$$

$_{m|n}q_x$ = the probability that (x) will survive to age $x+m$ and then die before reaching age $x+m+n$

Therefore

(5) $$_{m|n}q_x = \frac{l_{x+m} - l_{x+m+n}}{l_x} = {}_mp_x - {}_{m+n}p_x$$

ω = first age at which all persons in the table are dead

In the C.S.O. Table, $\omega = 100$. Hence

(6) $$l_\omega = l_{100} = 0$$

Example 1. Find the probability that a person aged 50 will die between ages 70 and 71.

Solution. This is an example of equation (4) or (4′) with $x = 50$ and $n = 20$. Using (4′)

$$\begin{aligned} {}_{20|}q_{50} &= {}_{20}p_{50} - {}_{21}p_{50} \\ &= \frac{454{,}548 - 427{,}593}{810{,}900} \\ &= \frac{26950}{810{,}900} = .03324 \end{aligned}$$

Example 2. Find the probability that (50) will die between ages 70 and 80.

Solution. This is an example of equation (5) where $x = 50$, $m = 20$, and $n = 10$. Hence

$$
\begin{aligned}
{}_{20|10}q_{50} &= {}_{20}p_{50} - {}_{30}p_{50} \\
&= \frac{454{,}548 - 181{,}765}{810{,}900} \\
&= \frac{272{,}783}{810{,}900} = .33640
\end{aligned}
$$

Exercise 17-1

Make all computations on the basis of $2\frac{1}{2}\%$ C.S.O. Table.

1. Find the probability that a person aged 40:
   (a) Will live 10 years.
   (b) Will die before reaching age 50.
   (c) Will live to age 60 and die before reaching age 61.
2. Find the probability that a person aged 20:
   (a) Will live one year.
   (b) Will live 50 years.
   (c) Will die before reaching age 60.
3. State the meaning of the following symbols:
   (a) ${}_{10}p_{20}$ (c) q_{50} (e) ${}_{10|20}q_{30}$
   (b) ${}_{20|}q_{30}$ (d) p_{50}
4. Find the value of each symbol in Problem 3.
5. State the meaning of the following symbols:
   (a) ${}_{15}p_{25}$ (c) ${}_{15|15}q_{15}$ (e) p_{35}
   (b) ${}_{10}q_{35}$ (d) q_{35}
6. Find the value of each symbol in Problem 5.
7. Show that ${}_np_x = p_x \cdot p_{x+1} \cdot p_{x+2} \cdot \cdots \cdot p_{x+n-2} \cdot p_{x+n-1}$.
   Give a verbal interpretation of the meaning of this identity.
8. Show that: (a) ${}_{11}p_{30} = {}_{10}p_{30} - {}_{10|}q_{30}$; (b) ${}_np_x = {}_{n-1}p_x - {}_{n-1|}q_x$.
9. Prove that ${}_{m+n}p_x = {}_mp_x \cdot {}_np_{x+m} = {}_np_x \cdot {}_mp_{x+n}$.

17-5 PRESENT VALUE OF A PURE ENDOWMENT

By the present value of a pure endowment we mean the amount (x) will have to pay now in order to receive \$1.00 at age $x + n$ if he is then alive. This present value is symbolized by ${}_nE_x$.

If each of the l_x people alive at age x contributed \$${}_nE_x$ a fund would be created, the accumulated value of which n years hence would have to be

equal to $\$l_{x+n}$, so that each person alive at that time would receive \$1.00. Hence

$$(l_x)({}_nE_x)(1+i)^n = l_{x+n}$$

Solving for ${}_nE_x$, we have

$$(7) \qquad {}_nE_x = \frac{l_{x+n}}{(1+i)^n l_x} = v^n {}_np_x$$

If we desire the present value of an endowment of \$$R$, obviously all we have to do is multiply both sides of this equation of value by R. Hence

$$(7') \qquad R\,{}_nE_x = Rv^n{}_np_x$$

In order to compute ${}_nE_x$ it will be noticed that some interest rate must be specified. Table X specifies $i = .025$.

Example 1. Find the present value of a pure endowment of \$1000 paid to (40) provided he is alive at 65.

Solution.
$$\begin{aligned}(\$1000)({}_{25}E_{40}) &= \$1000v^{25}{}_{25}p_{40} \\ &= \$1000v^{25}\frac{l_{65}}{l_{40}} \\ &= (\$1000)\cdot .5393906\left(\tfrac{577882}{883342}\right) \\ &= \$539.3906(.6541996) \\ &= \$352.87\end{aligned}$$

This result means that if each person now aged 40 were to contribute \$352.87, and if this money were invested for 25 years at $2\frac{1}{2}\%$ compound interest, then at the end of that time a fund of \$577,882,000 would be created. This fund would be exactly sufficient to furnish \$1000 to each of the 577,882 survivors at age 65.

The amount of computation involved in determining the present value of a pure endowment by formula (7), for various ages and different values of n, can become extremely tedious. For this reason, the computation is simplified by the following invention. If both numerator and denominator of the right-hand side of (7) are multiplied by v^x, we have

$$(8) \qquad {}_nE_x = \frac{v^n l_{x+n}}{l_x} = \frac{v^{x+n} l_{x+n}}{v^x l_x} = \frac{D_{x+n}}{D_x}$$

where the symbol D_x is defined as being equal to $v^x l_x$. The advantage is that the value of this symbol may be computed for all ages in the table and listed once and for all as a separate column. Its value appears in the table in column 5. It is called a commutation function. It enables us to find the present value of any pure endowment of \$1.00 by a single division.

The solution of the example by this method would be as follows:

$$\$1000 \cdot {}_{25}E_{40} = \$1000\frac{D_{65}}{D_{40}} = \$1000\left(\frac{116088.15}{328983.61}\right)$$
$$= \$1000(.352869)$$
$$= \$352.87$$

The symbol ${}_nE_x$ may be thought of, not only as the present value of a pure endowment of \$1.00, but also as a contingent discount factor; that is, as the discounted value of \$1.00 for n years with benefit of both interest and survivorship. Similarly the symbol $\frac{1}{{}_nE_x}$ may be thought of as a contingent accumulation factor; that is, the accumulated value of \$1.00 for n years with benefit of interest and survivorship. We illustrate this point of view with the following example.

Example 2. A man aged 40 has \$1000 with which he wishes to purchase a pure endowment payable to him at age 65. How much endowment can he buy if $i = .025$?

Solution. Let \$$R$ be the amount of endowment that can be purchased for \$1000. Then \$1000 must be equal to the present value of an endowment of \$$R$ due in n years. Hence

$$R\,{}_{25}E_{40} = \$1000.00$$
$$R = \frac{\$1000}{{}_{25}E_{40}} = \frac{\$1000}{\frac{D_{65}}{D_{40}}}$$

$$\begin{aligned}
&= \frac{\$1000D_{40}}{D_{65}} \\
&= \$1000\left(\frac{328983.61}{116088.15}\right) \\
&= \$1000(2.833503) \\
&= \$2{,}833.50
\end{aligned}$$

That is, \$1000 invested now, by a man aged 40, in a pure endowment payable in 25 years will accumulate to \$2,833.50 with benefit of interest at $2\frac{1}{2}\%$ and survivorship.

17-6 CONTINGENT ANNUITIES

A contingent annuity is an annuity the payment of which depends on the happening or failing to happen of some event. (Notice the difference between this type of annuity and those discussed earlier in the text. The earlier annuities are sometimes called *annuities certain* because the payments are made regardless of circumstances.) We are interested in the type of contingent annuity whose payments depend on the life of a given individual. Such annuities may be of two kinds:

(I) *Ordinary*—payments are made at the end of the year.

(II) *Due*—payments are made at the beginning of the year.

Each of these life annuities is divisible into four different types:

(1) *Whole life annuities*—payments continue as long as an individual is alive.

(2) *Temporary life annuities*—payments are made for a certain number of years provided the individual is alive to receive them.

(3) *Deferred life annuities*—payments are postponed for a certain period of years and are made thereafter so long as the individual is alive to receive them.

(4) *Deferred-temporary life annuities*—payments are postponed for a certain period of years, then are made for a specified period of years provided the individual is alive to receive them.

(1) *Whole life annuities.* Let a_x stand for the present value of an ordinary whole life annuity of \$1.00 per year to (x). To determine this present value we may consider that a_x is made up of the sum of pure endowments of \$1.00 each, the first paid at age $x + 1$ if (x) is alive, the second at age $x + 2$ if (x) is alive, and so on out to the end of the table. Hence

$$
\begin{aligned}
a_x &= {}_1E_x + {}_2E_x + {}_3E_x + \cdots + {}_{\omega-x}E_x \\
&= \frac{D_{x+1}}{D_x} + \frac{D_{x+2}}{D_x} + \frac{D_{x+3}}{D_x} + \cdots + \frac{D_\omega}{D_x} \\
&= \frac{D_{x+1} + D_{x+2} + \cdots + D_\omega}{D_x}
\end{aligned}
\tag{9}
$$

This may be written as

$$a_x = \frac{N_{x+1}}{D_x} \tag{10}$$

where the symbol N_x represents the sum of all the D's from and including D_x out to the end of the table. This new commutation function may be formed by cumulative addition from the column of D's by starting with $D_{99} = N_{99}$, $N_{98} = D_{98} + D_{99}$, $N_{97} = D_{97} + D_{98} + D_{99} = D_{97} + N_{98}$, and so on back to the beginning of the table. It is possible, by the use of equation (10), then, to find the present value of any ordinary whole life annuity of \$1.00 by a single division.

It is a simple matter to find the present value of an ordinary whole life annuity of \$$R$ payable to (x) by multiplying both sides of equation (10) by R. Thus we have

$$Ra_x = R\frac{N_{x+1}}{D_x} \tag{10'}$$

The present value of a whole life annuity due of \$1.00 on the life of (x) may be found similarly. The only difference between it and a_x is that \$1.00 is due immediately. If we symbolize the present value of this annuity due by $\ddot{a}_x$, we have

$$
\begin{aligned}
\ddot{a}_x = 1 + a_x &= 1 + \frac{N_{x+1}}{D_x} \\
&= \frac{D_x + N_{x+1}}{D_x} \\
&= \frac{N_x}{D_x}
\end{aligned}
\tag{11}
$$

From this we immediately obtain

$$R\ddot{a}_x = R\frac{N_x}{D_x} \tag{11'}$$

when the annual payment is $\$R$ instead of \$1.00.

Example 1. (a) Find the present value of an ordinary whole life annuity of \$1000 per year issued to (40); (b) of a whole life annuity due.

Solution. (a) This is an example of formula (10′) where $R = \$1000$ and $x = 40$. Substituting, we have

$$\begin{aligned}\$1000a_{40} &= \$1000\,\frac{N_{41}}{D_{40}}\\ &= \$1000\,\frac{6379589.05}{328983.61}\\ &= \$1000(19.391814)\\ &= \$19{,}391.81\end{aligned}$$

(b) This is an example of formula (11′) where $R = \$1000$ and $x = 40$. Substituting, we have

$$\begin{aligned}\$1000\ddot{a}_{40} &= \$1000\,\frac{N_{40}}{D_{40}}\\ &= \$1000\,\frac{6708572.66}{328983.61}\\ &= \$1000(20.391814)\\ &= \$20{,}391.81\end{aligned}$$

(2) *Temporary life annuities.* The present value of an ordinary temporary life annuity of \$1.00 per year for a term of n years issued to (x) is symbolized by $a_{x:\overline{n}|}$. We may think of $a_{x:\overline{n}|}$ as the sum of n pure endowments of \$1.00 each, the first payable at age $x + 1$, the second at age $x + 2, \cdots$, the last at age $x + n$. Hence

$$\begin{aligned}a_{x:\overline{n}|} &= {}_1E_x + {}_2E_x + {}_3E_x + \cdots + {}_nE_x\\ &= \frac{D_{x+1}}{D_x} + \frac{D_{x+2}}{D_x} + \cdots + \frac{D_{x+n}}{D_x}\end{aligned}$$

By adding and subtracting $D_{x+n+1} + D_{x+n+2} + \cdots + D_\omega$ to and from the numerator, $a_{x:\overline{n}|}$ becomes

$$\frac{D_{x+1}+D_{x+2}+\cdots+D_{x+n}+(D_{x+n+1}+\cdots+D_\omega)-(D_{x+n+1}+\cdots+D_\omega)}{D_x}$$

$$=\frac{N_{x+1}-N_{x+n+1}}{D_x}$$

Hence we have the equation

(12) $$a_{x:\overline{n|}} = \frac{N_{x+1} - N_{x+n+1}}{D_x}$$

from which we obtain immediately

(12′) $$Ra_{x:\overline{n|}} = R\frac{N_{x+1} - N_{x+n+1}}{D_x}$$

where the annual payment is \$$R$ instead of \$1.00.

For the present value of a temporary life annuity due of \$1.00 payable at the beginning of each year for n years to (x) we use the symbol $\ddot{a}_{x:\overline{n|}}$. Using the same method as before

$$\ddot{a}_{x:\overline{n|}} = 1 + {}_1E_x + {}_2E_x + \cdots + {}_{n-1}E_x$$

$$= 1 + \frac{D_{x+1}}{D_x} + \frac{D_{x+2}}{D_x} + \cdots + \frac{D_{x+n-1}}{D_x}$$

$$= \frac{D_x + D_{x+1} + D_{x+2} + \cdots + D_{x+n-1}}{D_x}$$

By adding and subtracting $D_{x+n} + D_{x+n+1} + \cdots + D_\omega$ to and from the numerator, $\ddot{a}_{x:\overline{n|}}$ becomes

$$\frac{D_x+D_{x+1}+\cdots+D_{x+n-1}+(D_{x+n}+\cdots+D_\omega)-(D_{x+n}+\cdots+D_\omega)}{D_x}$$

$$=\frac{N_x-N_{x+n}}{D_x}$$

Hence we have the equation

(13) $$\ddot{a}_{x:\overline{n|}} = \frac{N_x - N_{x+n}}{D_x}$$

and from it we obtain immediately

(13′) $$R\ddot{a}_{x:\overline{n|}} = R\frac{N_x - N_{x+n}}{D_x}$$

where the annual payment is \$$R$ instead of \$1.00.

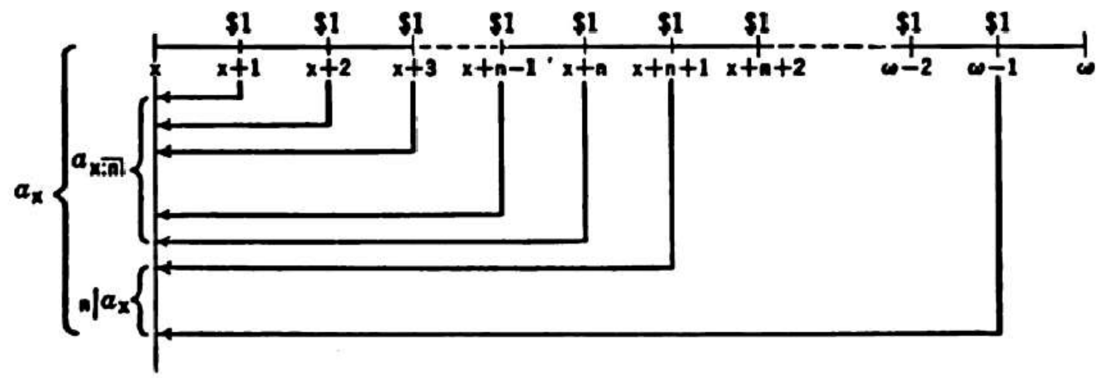
Example 2. Find the present value of an ordinary temporary life annuity of \$1000 issued to (40) to run for a term of 25 years if $i = .025$.

Solution. This is an example of formula (12′) where $R = \$1000$, $x = 40$, $n = 25$. Substituting in the formula

$$\begin{aligned}
\$1000a_{40:\overline{25|}} &= \$1000\frac{N_{41} - N_{66}}{D_{40}} \\
&= \$1000\frac{6379589.05 - 1056041.64}{328983.61} \\
&= \$1000\left(\frac{5323547.41}{328983.61}\right) \\
&= \$1000(16.181801) \\
&= \$16{,}181.80
\end{aligned}$$

(3) *Deferred life annuities.* The present value of an ordinary life annuity of \$1.00 per year, deferred n years, issued to (x), is symbolized by ${}_{n|}a_x$. Since this is an ordinary life annuity deferred n years, the first payment will not fall due until the end of the $(n + 1)$st year, that is, at age $x + n + 1$. We may think of ${}_{n|}a_x$ as the difference between the present value of a life annuity and the present value of a temporary life annuity whose term is n years. Thus we obtain

$$\begin{aligned}
{}_{n|}a_x &= a_x - a_{x:\overline{n|}} \\
&= \frac{N_{x+1}}{D_x} - \frac{N_{x+1} - N_{x+n+1}}{D_x} \\
&= \frac{N_{x+1} - N_{x+1} + N_{x+n+1}}{D_x} \\
&= \frac{N_{x+n+1}}{D_x}
\end{aligned}$$



Hence we have equation

$$(14)\qquad {}_{n|}a_x = \frac{N_{x+n+1}}{D_x}$$

from which we obtain immediately

$$(14')\qquad R_{n|}a_x = R\,\frac{N_{x+n+1}}{D_x}$$

where the annual payment is $\$R$.

In a similar manner, we obtain the formula for the present value ${}_{n|}\ddot{a}_x$ of a deferred life annuity due of \$1.00 per year:

$$\begin{aligned} {}_{n|}\ddot{a}_x = \ddot{a}_x - \ddot{a}_{x:\overline{n|}} &= \frac{N_x}{D_x} - \frac{N_x - N_{x+n}}{D_x} \\ &= \frac{N_{x+n}}{D_x} \end{aligned}$$

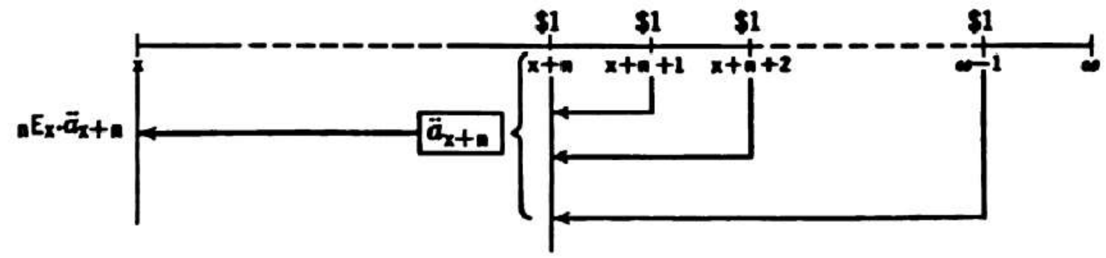
Hence we have the formulas

$$(15)\qquad {}_{n|}\ddot{a}_x = \frac{N_{x+n}}{D_x}$$

and

$$(15')\qquad R_{n|}\ddot{a}_x = R\,\frac{N_{x+n}}{D_x}$$

Formulas (14) and (15) might have been developed by using ${}_nE_x$ as the contingent discount factor on a whole life annuity which begins at age $x + n$. We will apply this method to the deferred life annuity due.



Actually, if (x) is alive at age $x + n$, the first payment of the annuity is then due. On this date, all payments have the value $\ddot{a}_{x+n}$. Since we want the value at age x, we must discount $\ddot{a}_{x+n}$ contingently for n years. Hence we have, in a second manner

(15)
$$_{n|}\ddot{a}_x = {}_nE_x \cdot \ddot{a}_{x+n}$$
$$= \frac{D_{x+n}}{D_x} \cdot \frac{N_{x+n}}{D_{x+n}} = \frac{N_{x+n}}{D_x}$$

(4) *Deferred temporary life annuities.* If the payments of $1.00 per year on the life of (x) are deferred n years, then run for a term of t years, and then cease, the symbols used for the present value are $_{n|t}a_x$ for the ordinary annuity and $_{n|t}\ddot{a}_x$ for the annuity due. The formulas for the present values are

(16)
$$_{n|t}a_x = \frac{N_{x+n+1} - N_{x+n+t+1}}{D_x}$$

and

(17)
$$_{n|t}\ddot{a}_x = \frac{N_{x+n} - N_{x+n+t}}{D_x}$$

The proof of these two formulas, which may be effected by methods analogous to those used for other formulas in this section, is left to the student.

Example 3. A man aged 40 has $15,000 which he invests in a deferred whole life annuity, first payment at age 65. If $i = .025$, what annual income can he look forward to receiving?

Solution. If R is his annual income, we have an example of the use of the formula (15′) where $x = 40$, $n = 25$, and $R_{25|}\ddot{a}_{40} = \$15{,}000$. Substituting, we have

$$\$15{,}000 = R\frac{N_{65}}{D_{40}}$$

and solving for R gives
$$R = \frac{\$15{,}000\, D_{40}}{N_{65}}$$
$$= \frac{\$15{,}000(328983.61)}{1172129.79}$$
$$= \$15{,}000(.28067165) = \$4{,}210.07$$

It must be understood by the student that if (40) does not survive to age 65 neither he nor his heirs receive any return on this investment.

Example 4. On reaching age 70, a man has accumulated $25,000 with which he purchases an annual retirement income that is certainly* payable

*This means that the income is paid for 15 years even though the man does not live that long.

for 15 years, and so long thereafter as he is alive. If the first payment comes at the end of the first year and if $i = .025$, what annual income is purchased?

Solution. First it should be noticed that this example combines the features of an annuity certain, payable for 15 years whether (70) is alive or not, with a deferred whole life annuity that will begin immediately after the certain period has expired, if (70) is then alive, and will continue for the rest of his life. Hence we may write

$$\begin{aligned}\$25{,}000 &= R(a_{\overline{15}|.025} + {}_{15|}a_{70}) \\ &= R\left(a_{\overline{15}|.025} + \frac{N_{86}}{D_{70}}\right) \\ &= R\left(12.3813777 + \frac{27896.5815}{80706.625}\right) \\ &= R(12.3813777 + .3456541)\end{aligned}$$

Solving for R, we have

$$R = \frac{\$25{,}000}{12.7270318} = \$1{,}964.32$$

Exercise 17-2

Find the present value at $2\frac{1}{2}\%$ of the pure endowments in Problems 1 through 3.

1. \$10,000 payable to a person aged 20, if he is alive at 65.
2. \$25,000 payable to a person aged 25, if he is alive at 50.
3. \$1000 payable to a person aged 30 if he is alive at 60.
4. (28) has \$5,000 with which he purchases a pure endowment payable at 68. How much can he buy?
5. What amount of endowment, payable at age 70, can be purchased for \$2500 by (50)?
6. Find the present value of a whole life annuity of \$2500 payable to a person aged 30 (a) if payments are made at the end of each year; (b) if payments are made at the beginning of each year.
7. Find the present value of a whole life annuity of \$1200 payable to a person aged 65 (a) if payments are made at the end of each year; (b) if payments are made at the beginning of each year.
8. A man aged 35 pays \$20,000 for an annuity payable to him at the end of each year for 25 years. What is his annual income from the annuity?

9. What is the present value of a 10-year ordinary temporary life annuity of $1500 issued to a person aged 33?

10. What is the present value of a 25-year temporary life annuity due of $2500 per year issued to a person aged 40?

11. A man aged 35 pays $5000 for an ordinary temporary life annuity payable to him for 15 years. What is his annual income from it?

12. A man aged 30 purchases a retirement life income of $3000 per year, first payment at age 65 if he is alive at that time. What does it cost him?

13. A man aged 40 purchases a deferred temporary life annuity of $2400 per year, first payment at age 70 and payments to continue for 15 years. What is the present value of this annuity?

14. On reaching age 65, a man has accumulated $30,000 with which he purchases an annual retirement income that is certainly payable for 20 years, and as long thereafter as he is alive. If the first payment comes at the end of the first year, what annual income is purchased?

15. On reaching age 60, a man has accumulated $20,000 with which he purchases an annual retirement income that is certainly payable for 18 years, and as long thereafter as he is alive. If the first payment comes at the beginning of the first year, what annual income is purchased?

17-7 WHOLE LIFE INSURANCE—NET PREMIUMS

A whole life insurance policy is a contract between a life insurance company, referred to as *the company*, and an individual, referred to as *the policyholder* or *the insured*, which agrees that on the death of the policyholder, the company will pay a certain sum, called *the face* or *benefits* of the policy, to a third party, named by the policyholder in advance, and called *the beneficiary*. If the insured does not name a beneficiary, the benefits are paid to his estate.

In order to purchase such a contract from the company, the policyholder must pay a sum of money whose net present value is equal to the present value of the benefits he will receive. If all this sum is paid when the policy is issued, it is called the *net single premium*. Actually, this net single premium is just sufficient to pay the cost of the benefits under the assumption that the individuals holding similar policies with the company have the same mortality as those listed in the table, and that the insurance company earns on its investments the rate of interest on which the commutation columns are based. It is usual, therefore, for the company to

charge gross or loaded premiums which are slightly greater than net premiums and are obtained from the net premiums by adding to the net premiums some amount determined by the company as being sufficient to take care of operating expenses and profit. In what follows, we will restrict our attention to net premiums, as the method of loading varies from company to company.

We will denote by A_x the net single premium charged (x) for a whole life policy of \$1.00, and will assume for purposes of this discussion that the benefit will be paid at the end of the policy year in which (x) dies. This assumption is accurate enough for our purposes. If each of l_x people aged x pay A_x at the beginning of the first policy year and the money is invested at $i\%$, a fund must be created that will be sufficiently large to take care of all death claims when they come due. Hence at the end of the

first year, $\$d_x$ must be available; its present value is vd_x;
second year, $\$d_{x+1}$ must be available; its present value is v^2d_{x+1};
third year, $\$d_{x+2}$ must be available; its present value is v^3d_{x+2};

. . . .

last year, $\$d_\omega$ must be available; its present value is $v^{\omega-x+1}d_\omega$

Hence, equating the present value of the premiums to the present value of the benefits, we have

$$l_xA_x = vd_x + v^2d_{x+1} + v^3d_{x+2} + \cdots + v^{\omega-x+1}d_\omega$$

Therefore $$A_x = \frac{vd_x + v^2d_{x+1} + v^3d_{x+2} + \cdots + v^{\omega-x+1}d_\omega}{l_x}$$

Multiplying numerator and denominator of the right-hand member by v^x, we have

$$A_x = \frac{v^{x+1}d_x + v^{x+2}d_{x+1} + v^{x+3}d_{x+2} + \cdots + v^{\omega+1}d_\omega}{v^xl_x}$$

Now if we introduce a new commutation symbol, $C_x = v^{x+1}d_x$, $C_{x+1} = v^{x+2}d_{x+1}$, etc., we have

$$A_x = \frac{C_x + C_{x+1} + C_{x+2} + \cdots + C_\omega}{D_x}$$

Introducing still another commutation symbol, $M_x = C_x + C_{x+1} + C_{x+2} + \cdots + C_\omega$, which is formed by cumulative addition of the C's, beginning at the last age in the table, just as the N's were formed from the D's, we have the equation

$$A_x = \frac{M_x}{D_x} \tag{18}$$

The commutation symbols C_x and M_x are also listed in Table X. Hence it is possible to find the net single premium on a whole life insurance policy issued to (x) by a single division.

If F is the face of the policy, it follows immediately that

$$FA_x = F \cdot \frac{M_x}{D_x} \tag{18'}$$

Example 1. Find the net single premium for a whole life policy of \$10,000 issued to (40) on the basis of $2\frac{1}{2}\%$ C.S.O., 1941, table of mortality.

Solution. Substituting the values $F = \$10{,}000$ and $x = 40$ in (18′), we have

$$\begin{aligned}
\$10{,}000A_{40} &= \$10{,}000\frac{M_{40}}{D_{40}} \\
&= \$10{,}000\frac{165359.8889}{328983.61} \\
&= \$10{,}000(.5026387) \\
&= \$5{,}026.39
\end{aligned}$$

Obviously if the company collected for insurance benefits by charging single premiums, the cost of insurance protection would be prohibitive for the majority of people. In order to remedy this situation, it is usual for the insured to pay premiums at the beginning of each year (half-year, quarter, or month) throughout life, or for a term of years contingent on his surviving the term. In these cases, we refer to the premiums as *net level premiums*.

We shall use the symbol P_x to represent the value of the net level annual premium on a whole life insurance policy of \$1.00, where the premiums are payable for the life of (x). These premiums will constitute a whole life annuity due of $\$P_x$ per year payable to the company by the insured. Their present value must be equal to the net single premium on the same insurance. Hence

$$P_x\ddot{a}_x = A_x$$

Solving for P_x, we have the equation

$$(19)\qquad P_x = \frac{A_x}{\ddot{a}_x}$$

$$= \frac{\dfrac{M_x}{D_x}}{\dfrac{N_x}{D_x}} = \frac{M_x}{N_x} \quad \text{by use of (18) and (11)}$$

Hence

$$(19')\qquad FP_x = F\frac{M_x}{N_x}$$

where F is the face of the policy. Again we see that it is possible to find the net level annual premium payable for life, on a whole life policy of \$1.00, by a single division.

Example 2. Find the net level annual premium on a whole life policy of \$10,000 issued to (40) on the basis of $2\frac{1}{2}\%$ C.S.O., 1941, table of mortality.

Solution. Using equation (19′) with $F = \$10{,}000$ and $x = 40$, we have

$$\begin{aligned} \$10{,}000P_{40} &= \$10{,}000\,\frac{M_{40}}{N_{40}} \\ &= \$10{,}000\,\frac{165{,}359.8889}{6{,}708{,}572.66} \\ &= \$10{,}000(.02464904) \\ &= \$246.49 \end{aligned}$$

If the net level annual premiums on a whole life policy of \$1.00 to (x) are payable for a term of n years, contingent on the life of (x), the symbol ${}_nP_x$ is used to represent each. If (x) survives the term of n years, his policy is said to be "paid up." After this term, he pays no more premiums, but continues to be insured for the rest of his life. The payment of ${}_nP_x$ at the beginning of each year by the insured constitutes a term life annuity due to the company. The present value of this term annuity must be the same as the net single premium on the insurance. Hence, ${}_nP_x \cdot \ddot{a}_{x:\overline{n|}} = A_x$, and solving for ${}_nP_x$, we have

$$(20)\qquad {}_nP_x = \frac{A_x}{\ddot{a}_{x:\overline{n|}}}$$

$$= \frac{\dfrac{M_x}{D_x}}{\dfrac{N_x - N_{x+n}}{D_x}} = \frac{M_x}{N_x - N_{x+n}} \quad \text{by use of (18) and (13)}$$

We also have the equation

(20′)
$$F\,{}_nP_x = F\frac{M_x}{N_x - N_{x+n}}$$

where F is, as usual, the face of the policy.

Example 3. Find the net level annual premium, payable for 20 years, on a whole life insurance of \$10,000 issued to (40) on the basis of $2\frac{1}{2}\%$ C.S.O., 1941, table of mortality.

Solution. Using equation (20′) with $F = \$10{,}000$, $n = 20$, and $x = 40$, we have

$$\begin{aligned}\$10{,}000\,{}_{20}P_{40} &= \$10{,}000\frac{M_{40}}{N_{40} - N_{60}}\\ &= \$10{,}000\frac{165359.8889}{6708572.66 - 1865613.58}\\ &= \$10{,}000\frac{165359.8889}{4842959.08}\\ &= \$10{,}000(.03414439)\\ &= \$341.44\end{aligned}$$

It should be noticed that in each of the three examples in this section, the same benefit—a \$10,000 whole life insurance on (40)—has been purchased. The three examples illustrate three different ways of paying for this benefit:

\$5026.39 payable at issue of the policy;
\$246.49 payable at the beginning of each year for life;
\$341.44 payable at the beginning of each year for 20 years provided (40) remains alive that long.

We wish to point out further that these premiums have been determined under the assumption that the mortality of the company is the same as that of the C.S.O. table, and that the company earns $2\frac{1}{2}\%$ on its investments. So long as this is true, no one of the three methods is more advantageous to the company than either of the other two.

Exercise 17-3

Prove the identities given in Problems 1 through 5.

1. $A_x = v\ddot{a}_x - a_x$.
2. $M_x = vN_x - N_{x+1}$.
3. $A_x = 1 - d\ddot{a}_x$.
4. $A_x = v(1 - ia_x) = v - da_x$.
5. $A_x = \dfrac{P_x}{P_x + d}$.

6. Find the net single premium and the net level annual premium on a $3000 whole life policy issued to a person aged 25.

7. Find the annual premium, payable for 10 years, on the policy of Problem 6.

8. Find the annual premium, payable for 20 years, on the policy of Problem 6.

9. Find the net single premium and the net level annual premium on a $5000 whole life policy issued to a person aged 48.

10. Find the annual premium payable for 15 years on the policy of Problem 9.

11. Find the annual premium payable for 25 years on the policy of Problem 9.

12. A benefactor, aged 55, makes a will that leaves $25,000 to a certain charity on his death. What is the present value of the bequest?

17-8 TERM INSURANCE

It often occurs that an individual needs a certain amount of insurance protection for a fixed period of years, but does not desire this protection throughout his whole life. A man may be buying a home on time, and may desire to protect his investment with an insurance policy large enough to pay off his debt in case of his death; or he may have a family of small children who, he wants to be sure, will have the opportunity of attending college. In these two instances and many others, it is to his advantage to be protected over a certain period of years, after which the protection may no longer be needed. This type of life insurance is called *term insurance*. We shall use the symbol $A^{1}_{x:\overline{n}|}$ to represent the net single premium for an insurance of $1.00 on (x), payable at the end of the year of death of (x) provided he dies within a term of n years.

If each of l_x people aged x pays $\$A^{1}_{x:\overline{n}|}$ at the beginning of the first policy year, and the money is invested at $i\%$, a fund must be created that will be sufficiently large to take care of all death claims when they come due. Hence at the end of the

first year, $\$d_x$ must be available; its present value is vd_x;
second year, $\$d_{x+1}$ must be available; its present value is v^2d_{x+1};
third year, $\$d_{x+2}$ must be available; its present value is v^3d_{x+2};
. . . .
nth year, $\$d_{x+n-1}$ must be available; its present value is v^nd_{x+n-1}.

Equating the present value of the premiums to the present value of the benefits, we have $l_xA^{1}_{x:\overline{n}|} = vd_x + v^2d_{x+1} + \cdots + v^nd_{x+n-1}$. Therefore

$$A^{1}_{x:\overline{n}|} = \frac{vd_x + v^2d_{x+1} + \cdots + v^nd_{x+n-1}}{l_x}$$

and multiplying numerator and denominator of the right-hand member by v^x, we have

$$A^1_{x:\overline{n|}} = \frac{v^{x+1}d_x + v^{x+2}d_{x+1} + \cdots + v^{x+n}d_{x+n-1}}{v^x l_x}$$
$$= \frac{C_x + C_{x+1} + \cdots + C_{x+n-1}}{D_x}$$

since $C_x = v^{x+1}d_x$, $C_{x+1} = v^{x+2}d_{x+1}$, etc. Adding and subtracting the sum of the C's from C_{x+n} out to the end of the table to and from the numerator of the right-hand member gives

$$A^1_{x:\overline{n|}} = \frac{C_x+C_{x+1}+\cdots+C_{x+n-1}+C_{x+n}+\cdots+C_\omega-(C_{x+n}+\cdots+C_\omega)}{D_x}$$
$$= \frac{M_x - M_{x+n}}{D_x}$$

since $M_x = C_x + C_{x+1} + \cdots + C_\omega$, and $M_{x+n} = C_{x+n} + C_{x+n+1} + \cdots + C_\omega$.

Hence we have

$$A^1_{x:\overline{n|}} = \frac{M_x - M_{x+n}}{D_x} \tag{21}$$

and

$$FA^1_{x:\overline{n|}} = F\frac{M_x - M_{x+n}}{D_x} \tag{21'}$$

F being the face of the policy.

The net single premium for term insurance is usually replaced by net annual premiums, which may be paid for any period of years equal to or less than the term of the insurance. If we wish to pay for an n-year term insurance of $1.00, annually for n years, we represent each annual premium by the symbol $P^1_{x:\overline{n|}}$. These premiums constitute a temporary life annuity due payable by the insured to the company. Hence their present value must equal the net single premium for the term insurance. Thus, we have the equation

$$P^1_{x:\overline{n|}}\ddot{a}_{x:\overline{n|}} = A^1_{x:\overline{n|}}$$

Solving for $P^1_{x:\overline{n|}}$, we have $P^1_{x:\overline{n|}} = \dfrac{A^1_{x:\overline{n|}}}{\ddot{a}_{x:\overline{n|}}}$

$$= \frac{\dfrac{M_x - M_{x+n}}{D_x}}{\dfrac{N_x - N_{x+n}}{D_x}} = \frac{M_x - M_{x+n}}{N_x - N_{x+n}}$$

Hence

$$P^{1}_{x:\overline{n}|} = \frac{M_x - M_{x+n}}{N_x - N_{x+n}} \tag{22}$$

and

$$FP^{1}_{x:\overline{n}|} = F\,\frac{M_x - M_{x+n}}{N_x - N_{x+n}} \tag{22'}$$

F being the face of the policy.

Example 1. Find (a) the net single premium and (b) the net level annual premium, payable for 20 years, on a 20-year term policy of \$10,000 issued to (40).

Solution. (a) Using formula (21′) with $F = \$10{,}000$, $n = 20$, and $x = 40$, we have

$$\begin{aligned}
\$10{,}000A^{1}_{40:\overline{20}|} &= \$10{,}000\,\frac{M_{40} - M_{60}}{D_{40}}\\
&= \$10{,}000\,\frac{165359.8889 - 108543.4550}{328983.61}\\
&= \$10{,}000\,\frac{56816.4339}{328983.61} = \$10{,}000(.17270293)\\
&= \$1{,}727.03
\end{aligned}$$

(b) Using formula (22′) with $F = \$10{,}000$, $n = 20$, and $x = 40$, we have

$$\begin{aligned}
\$10{,}000P^{1}_{40:\overline{20}|} &= \$10{,}000\,\frac{M_{40} - M_{60}}{N_{40} - N_{60}}\\
&= \$10{,}000\,\frac{165359.8889 - 108543.4550}{6708572.66 - 1865613.58}\\
&= \$10{,}000\,\frac{56816.4339}{4842959.08} = \$10{,}000(.01173176)\\
&= \$117.32
\end{aligned}$$

Example 2. (a) Find the net single premium on a 5-year term insurance of \$1000 issued to (40). (b) Under the assumption that l_{40} individuals pay this premium, make a table showing the sufficiency of the premium to take care of all death claims when they fall due, using $2\frac{1}{2}\%$ C.S.O., 1941, mortality table.

Solution. (a) Using formula (21′) with $F = \$1000$, $n = 5$, and $x = 40$ we have

$$
\begin{aligned}
\$1000\,A^{\;1}_{40:\overline{5}|} &= \$1000\,\frac{M_{40}-M_{45}}{D_{40}}\\
&= \$1000\,\frac{165359.8889-154736.6133}{328983.61}\\
&= \$1000\left(\frac{10623.2756}{328983.61}\right)\\
&= \$1000(.0322912)\\
&= \$32.2912
\end{aligned}
$$

(b) This premium of \$32.2912 is paid at the beginning of the first year by l_{40} = 883,342 people, yielding a fund of (883,342) · (\$32.2912) = \$28,524,173 and this is invested for one year at $2\frac{1}{2}\%$. Hence, at the end of the first year, the insurance company has available \$28,524,173(1.025) = \$29,237,277. Out of this must be paid the death claims of $\$1000d_{40}$ = \$1000(5459) = \$5,459,000. This leaves a balance of \$29,237,277 − \$5,459,000 = \$23,778,277, which is invested for the second year at $2\frac{1}{2}\%$. Continuing this process, we have the following table:

	COLUMN 1	COLUMN 2	COLUMN 3
Years	*Available for investment at beginning of year (Col. 2–Col. 3)*	*Accumulated value of Col. 1 at end of year at $2\frac{1}{2}\%$*	*Death claims paid at end of year* $= 1000\,d_x$
1	\$28,524,173	\$29,237,277	\$5,459,000
2	23,778,277	24,372,734	5,785,000
3	18,587,734	19,052,427	6,131,000
4	12,921,427	13,244,463	6,503,000
5	6,741,463	6,910,000	6,910,000

The computations above have been carried out to the nearest dollar.

17-9 ENDOWMENT INSURANCE

A very popular type of contract issued by insurance companies is one that calls for the payment of the benefits either in case of the death of the insured during a certain term of years or at the end of this term in case of his survival of it. This is really a combination of a term insurance on (x)

for n years and a pure endowment payable to (x) at the end of the nth year in case of his survival. Such a contract is called an *endowment insurance policy* or more commonly an *endowment policy*. If we designate the net single premium on such a contract for \$1.00 by $A_{x:\overline{n}|}$, we must have the present value of the premium equal to the present value of the benefits. Hence

$$\begin{aligned} A_{x:\overline{n}|} &= A^{1}_{x:\overline{n}|} + {}_nE_x \\ &= \frac{M_x - M_{x+n}}{D_x} + \frac{D_{x+n}}{D_x} \\ &= \frac{M_x - M_{x+n} + D_{x+n}}{D_x} \end{aligned}$$

We now have

$$A_{x:\overline{n}|} = \frac{M_x - M_{x+n} + D_{x+n}}{D_x} \tag{23}$$

and

$$FA_{x:\overline{n}|} = F\,\frac{M_x - M_{x+n} + D_{x+n}}{D_x} \tag{23'}$$

where F is the face of the policy.

If we wish to pay premiums in n annual installments, and let $P_{x:\overline{n}|}$ represent this net level annual premium payable for n years on an n-year endowment of \$1.00 issued to (x), we have

$$P_{x:\overline{n}|}\ddot{a}_{x:\overline{n}|} = A_{x:\overline{n}|}$$

Solving for $P_{x:\overline{n}|}$ gives

$$P_{x:\overline{n}|} = \frac{A_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}} = \frac{\dfrac{M_x - M_{x+n} + D_{x+n}}{D_x}}{\dfrac{N_x - N_{x+n}}{D_x}} = \frac{M_x - M_{x+n} + D_{x+n}}{N_x - N_{x+n}}$$

We now have

$$P_{x:\overline{n}|} = \frac{M_x - M_{x+n} + D_{x+n}}{N_x - N_{x+n}} \tag{24}$$

and

$$FP_{x:\overline{n}|} = F\,\frac{M_x - M_{x+n} + D_{x+n}}{N_x - N_{x+n}} \tag{24'}$$

F being the face of the policy.

Example. Find the net level annual premium, payable for 20 years on a 20-year endowment insurance of \$1000 issued to (40).

Solution. Using equation (24′) with $F = \$1000$, $x = 40$, and $n = 20$, we have

$$\begin{aligned}\$1000A_{40:\overline{25}|} &= \$1000\frac{M_{40} - M_{60} + D_{60}}{N_{40} - N_{60}}\\ &= \$1000\frac{165359.8889 - 108543.4550 + 154046.23}{6708572.66 - 1865613.58}\\ &= \$1000\frac{210862.66}{4842959.08}\\ &= \$43.54\end{aligned}$$

By reference to Example 1 of Section 17-8, we notice that the premium on the term insurance of \$1000 is \$11.73. Hence, our net annual premium of \$43.54 is composed of two parts, \$11.73 which pays for the term insurance and \$31.81 which pays for the endowment. This endowment part of the policy is really not insurance at all. It is simply an investment made by the insured which matures only if he survives the 20-year term. The combined features of the policy, however, are very attractive to the insured. Even if he survives the 20-year period, he has paid out in actual cash only 20(\$43.54) = \$870.80, for which he receives \$1000 in return. Meanwhile he has had and paid for insurance protection for the 20-year period. Of course, if he does not survive the 20-year period, his beneficiary collects the \$1000, no matter how few premiums have been paid.

Exercise 17-4

1. Find the net single premium and the net annual premium payable for 20 years on a \$1000, 20-year term policy issued at age 50.

2. Find the net annual premium payable for 10 years on the 20-year term policy of Problem 1.

3. Find the net single premium and the net annual premium payable for 25 years on a \$1000, 28-year term policy issued at age 37.

4. Find the net annual premium payable for 28 years on the policy of Problem 3.

5. Find the net single premium of a 20-year pure endowment of \$1000 issued at age 50.

6. Find the net single premium of a 28-year pure endowment of \$1000 issued at age 37.

7. What is the net single premium on a 20-year endowment insurance of \$1000 issued at age 50? Compare this with the sum of the results of Problem 5 and the first part of Problem 1.

8. Find the net annual premium payable for 20 years on the policy described in Problem 7.

9. Find the net single premium on $1000 endowment insurance on (37) payable at age 65. Compare this with the sum of the results of Problem 6 and the first part of Problem 3.

10. Find the net annual premium payable for 28 years on the policy described in Problem 9.

17-10 NET LEVEL RESERVES

When a person purchases an insurance contract and agrees to pay net level annual premiums for life or for a term of years, he agrees to pay a constant sum per year for his benefits. During the earlier years of the life of his policy, he is paying more than it actually costs the insurance company to insure him. For instance, a person aged 40 may pay $246.49 net level annual premium for the rest of his life for a $10,000 whole life policy (Example 2, Section 17-7) or he may pay $341.44 (Example 3, Section 17-7) for a contingent term of 20 years for the same benefits. We will compare these premiums with the actual net premium per annum from year to year. This net single premium on a one-year term policy at any age is called the *natural premium* and is designated by c_x. There is a 1000 c_x column in Table X. Referring to this column we notice that the natural premiums for $10,000 of insurance for the several ages indicated is as follows:

$$\begin{aligned} \$10{,}000\, c_{40} &= \$60.29 & \qquad \$10{,}000\, c_{63} &= \$329.37 \\ \$10{,}000\, c_{50} &= \$120.19 & \qquad \$10{,}000\, c_{64} &= \$356.89 \\ \$10{,}000\, c_{59} &= \$239.70 & \qquad \$10{,}000\, c_{70} &= \$578.54 \\ \$10{,}000\, c_{60} &= \$259.42 & \qquad \$10{,}000\, c_{80} &= \$1{,}286.36 \end{aligned}$$

It can be seen from these figures that it actually costs the insurance company only $60.29 to insure (40) for $10,000 for one year, and that it is not until (40) reaches age 60 that the natural premiums actually become as great as the net level premium charged for life, and age 64 before the natural premium reaches that charged for 20 years. If can be seen, also, that after these mentioned ages the natural premium rises quite rapidly, and hence the company is collecting, at these higher ages, much less than the natural premium. It must then create a financial cushion, or reserve, to take care of this contingency. This reserve is created each year by

investing, at the interest rate of the mortality table being used, the sum of the net level annual premium charged and the reserve from the preceding year minus the natural premium for the year. When created in this manner, it is called *net level reserve*. There are several methods of performing these computations, three of which will be discussed briefly.

(1) *Retrospective method*. The point of view taken in the retrospective method is that if a policy is issued to (x), its value t years after its issue date is the difference between the value of the net premiums paid in, accumulated to age $x + t$, and the value of the past benefits, accumulated to the same date. In other words, the retrospective method looks back over the past history of the policy and, choosing age $x + t$ as a comparison date, sets up an equation of value which states that the net level reserve at age $x + t$ is equal to the difference between what has been paid and what should have been paid for the benefits received. Representing the tth reserve on a whole life policy of \$1.00 to (x) with premiums payable for life by ${}_tV_x$, we may set up our equation of value as follows:

The present value of the premiums paid in is $P_x\ddot{a}_{x:\overline{t|}}$, since the premiums constitute an annuity due, payable for t years.

The present value of the benefits is $A^1_{x:\overline{t|}}$, since the benefits purchased are the same as those for a t-year term policy.

The difference is $P_x\ddot{a}_{x:\overline{t|}} - A^1_{x:\overline{t|}}$.

This difference, when accumulated contingently to age $x + t$, is $(1/{}_tE_x)\,(P_x\ddot{a}_{x:\overline{t|}} - A^1_{x:\overline{t|}})$. Hence, we have the following equation:

$$ {}_tV_x = \frac{P_x\ddot{a}_{x:\overline{t|}} - A^1_{x:\overline{t|}}}{{}_tE_x} \tag{25} $$

Expressing this in terms of commutation symbols gives

$$ {}_tV_x = \frac{\dfrac{M_x}{N_x}\left(\dfrac{N_x - N_{x+t}}{D_x}\right) - \left(\dfrac{M_x - M_{x+t}}{D_x}\right)}{\dfrac{D_{x+t}}{D_x}} = \frac{M_x}{N_x}\left(\frac{N_x - N_{x+t}}{D_{x+t}}\right) - \left(\frac{M_x - M_{x+t}}{D_{x+t}}\right) \tag{25'} $$

Using the same general procedure and indicating the tth net reserve on an n-year payment endowment insurance of \$1.00 on (x) by ${}_tV_{x:\overline{n|}}$, $(t \le n)$, we have

$$ {}_tV_{x:\overline{n|}} = \frac{P_{x:\overline{n|}}\ddot{a}_{x:\overline{t|}} - A^1_{x:\overline{t|}}}{{}_tE_x} \tag{26} $$

Example 1. Find the reserve at the end of the 10th year on a 20-year term policy of \$1000 issued to (40), if the premiums are payable for 20 years.

Solution. Representing the reserve at the end of the 10th year by $\$1000\,{}_{10}V^{1}_{40:\overline{20|}}$, where $t = 10$, $x = 40$, $n = 20$, $F = \$1000$, we have

$$\$1000\,{}_{10}V^{1}_{40:\overline{20|}} = \$1000\frac{P^{1}_{40:\overline{20|}}\ddot{a}_{40:\overline{10|}} - A^{1}_{40:\overline{10|}}}{{}_{10}E_{40}}$$

$$P^{1}_{40:\overline{20|}} = .01173176 \qquad \text{Example 1 (b), Section 17-8}$$

$$\ddot{a}_{40:\overline{10|}} = \frac{N_{40} - N_{50}}{D_{40}} = \frac{6708572.66 - 3849487.59}{328983.61} = 8.690661$$

$$A^{1}_{40:\overline{10|}} = \frac{M_{40} - M_{50}}{D_{40}} = \frac{165359.8889 - 142035.0986}{328983.61}$$

$$= \frac{23324.7903}{328983.61} = .07089955$$

$${}_{10}E_{40} = \frac{D_{50}}{D_{40}} = \frac{235925.04}{328983.61} = .7171331$$

$$\$1000\,{}_{10}V^{1}_{40:\overline{20|}} = \$1000\frac{(.01173176)(8.690661) - .07089955}{.7171331}$$

$$= \$1000\left(\frac{.10195675 - .07089955}{.7171331}\right)$$

$$= \$1000\left(\frac{.0310572}{.7171331}\right) = \$43.31$$

Example 2. Find the reserve at the end of the 5th year on a ten-payment 10-year endowment for \$1000 issued to (40).

Solution. Representing the reserve at the end of the 5th year by $\$1{,}000\ {}_{5}V_{40:\overline{10|}}$, where $F = \$1000$, $t = 5$, $n = 10$, $x = 40$, we have

$$\$1000\,{}_{5}V_{40:\overline{10|}} = \$1000\left(\frac{P_{40:\overline{10|}}\ddot{a}_{40:\overline{5|}} - A^{1}_{40:\overline{5|}}}{{}_{5}E_{40}}\right)$$

$$P_{40:\overline{10|}} = \frac{M_{40} - M_{50} + D_{50}}{N_{40} - N_{50}} = \frac{23324.7903 + 235925.04}{2859085.07}$$

$$= \frac{259249.83}{2859085.07} = .09067580$$

$$\ddot{a}_{40:\overline{5|}} = \frac{N_{40} - N_{45}}{D_{40}} = \frac{6708572.66 - 5161996}{328983.61} = \frac{1546576.66}{328983.61}$$

$$= 4.701075$$

$$A^{1}_{40:\overline{5}|} = \frac{M_{40} - M_{45}}{D_{40}} = \frac{165359.8889 - 154736.6133}{328983.61}$$

$$= \frac{10623.2756}{328983.61} = .03229120$$

$${}_5E_{40} = \frac{D_{45}}{D_{40}} = \frac{280638.95}{328983.61} = .8530484$$

$$\$1000\,{}_5V_{40:\overline{10}|} = \$1000\,\frac{(.09067580)(4.701075) - .03229120}{.8530484}$$

$$= \$1000\,\frac{(.4262737 - .03229120)}{.8530484} = \$\,\frac{393.9825}{.8530484} = \$461.85$$

(2) *Prospective method.* The prospective method of computing reserves looks ahead to the future of the policy. From this point of view the net level reserve at the end of the tth year is equal to the value at that date of the difference between future benefits and future premiums. Or, stating it another way, the reserve at the end of the tth year is the value at that time of the difference between what should be paid for future benefits and what will be paid for these benefits. Illustrating again with a whole life policy of \$1.00 to (x) with premiums payable for life, we have the following:

Value of future benefits at age $x + t$ is A_{x+t}, since this is the net single premium for a whole life insurance of \$1.00 at age x + t.

Value of future premiums at age $x + t$ is $P_x\ddot{a}_{x+t}$, since the premiums constitute an annuity on the life of a person aged $x + t$.

Hence we have the following equation:

(27)
$$\begin{aligned} {}_tV_x &= A_{x+t} - P_x\ddot{a}_{x+t} \\ &= P_{x+t}\ddot{a}_{x+t} - P_x\ddot{a}_{x+t} \\ &= (P_{x+t} - P_x)\ddot{a}_{x+t} \end{aligned}$$

Expressing this in terms of commutation symbols, we have

(27′)
$${}_tV_x = \left(\frac{M_{x+t}}{N_{x+t}} - \frac{M_x}{N_x}\right)\frac{N_{x+t}}{D_{x+t}}$$

Similarly, for an n-year n-payment endowment insurance of \$1.00 on (x), the tth reserve $(t \leq n)$ is the difference between the values at age $x + t$ of the future benefits, $A_{x+t:\overline{n-t}|}$, and the future premiums $P_{x:\overline{n}|}\,\ddot{a}_{x+t:\overline{n-t}|}$. Hence we have the following equation:

(28)
$${}_tV_{x:\overline{n}|} = A_{x+t:\overline{n-t}|} - P_{x:\overline{n}|}\ddot{a}_{x+t:\overline{n-t}|}$$

Example 3. Solve Example 1 of this section by the prospective method.

Solution. Using $F = \$1000$, $x = 40$, $n = 20$, and $t = 10$, the value at age 50 of future benefits is $1000\ A^1_{50:\overline{10}|}$ since this is the net single premium on a 10-year term policy issued to (50).

The value at age 50 of future premiums is $\$1000\ P^1_{40:\overline{20}|} \cdot \ddot{a}_{50:\overline{10}|}$. Hence

$$\$1000\ {}_{10}V^1_{40:\overline{20}|} = \$1000(A^1_{50:\overline{10}|} - P^1_{40:\overline{20}|}\ddot{a}_{50:\overline{10}|})$$

$$A^1_{50:\overline{10}|} = \frac{M_{50} - M_{60}}{D_{50}} = \frac{142035.096 - 108543.455}{235925.04} = \frac{33491.641}{235925.04}$$

$$= .14195882$$

$$P^1_{40:\overline{20}|} = .01173176 \qquad \text{Example 1(b), Section 17-8}$$

$$\ddot{a}_{50:\overline{10}|} = \frac{N_{50} - N_{60}}{D_{50}} = \frac{3849487.59 - 1865613.58}{235925.04} = \frac{1983874.01}{235925.04}$$

$$= 8.408917$$

$$\begin{aligned}\$1000\ {}_{10}V^1_{40:\overline{20}|} &= \$1000[.14195882 - (.01173176)(8.408917)]\\ &= \$1000(.14195882 - .09865140)\\ &= \$1000(.04330742)\\ &= \$43.31\end{aligned}$$

It will be noticed that this is precisely the result of Example 1.

Example 4. Solve Example 2 of this section by the prospective method.

Solution. Using $F = \$1000$, $t = 5$, $n = 10$, $x = 40$, we have

Value at age 45 of future benefits is $1000\ A_{45:\overline{5}|}$

Value at age 45 of future premiums is $1000\ P_{40:\overline{10}|}\ddot{a}_{45:\overline{5}|}$

Hence

$$\$1000\ {}_5V_{40:\overline{10}|} = \$1000(A_{45:\overline{5}|} - P_{40:\overline{10}|}\ddot{a}_{45:\overline{5}|})$$

$$A_{45:\overline{5}|} = \frac{M_{45} - M_{50} + D_{50}}{D_{45}} = \frac{154736.6133 + 93889.94}{280638.95}$$

$$= \frac{248626.55}{280638.95} = .8859303$$

$$P_{40:\overline{10}|} = \frac{M_{40} - M_{50} + D_{50}}{N_{40} - N_{50}} = \frac{259249.83}{2859085.07} = .09067580$$

$$\ddot{a}_{45:\overline{5}|} = \frac{N_{45} - N_{50}}{D_{45}} = \frac{5161996.00 - 3849487.59}{280638.95} = \frac{1312508.41}{280638.95}$$

$$= 4.676857$$

Therefore

$$\$1000\ {}_5V_{40:\overline{10|}} = \$1000[.8859303 - (.0906758)(4.676857)]$$
$$= \$1000(.8859303 - .4240777) = \$461.85$$

and this is the same as the result given in Example 2.

(3) *Fackler's Accumulation Formula.* The prospective and retrospective methods are valuable for finding the reserve at the end of any given year. If, however, we need a table listing the reserve on a policy at the end of each year for the life of the policy, the amount of computation involved in the use of either of these methods becomes quite tedious, because of the fact that a separate set of computations, none of which depends on any of the others, is necessary for each year. This obvious disadvantage is eliminated by the following method, due to Fackler, which is applicable to any type of policy.

Suppose policies of the same kind of \$1.00 face are issued to l_x people, each of whom pays a net level annual premium of \$$P$. This creates a fund of \$$l_xP$ which, if invested at $i\%$ for one year, will accumulate to \$$l_xP(1+i)$ at the end of the year. From this fund must be deducted the death claims of \$1.00 for each of the d_x people who do not survive the year. This leaves a fund of $l_xP(1+i) - d_x$ dollars which is the reserve on the l_{x+1} policies that still remain in force.

Hence $$l_{x+1} \cdot {}_1V = l_x(P)(1+i) - d_x$$

where ${}_1V$ represents the reserve on each policy at the end of the first year. Solving for ${}_1V$, we have

$${}_1V = \frac{l_x}{l_{x+1}} P(1+i) - \frac{d_x}{l_{x+1}}$$

At the beginning of the second year each of the l_{x+1} people remaining again contributes \$$P$ as an annual premium. Hence the total fund then available is

$$l_{x+1} \cdot {}_1V + l_{x+1}P = l_{x+1}({}_1V + P)$$

This is then put at interest for the second year, creating a fund of

$$l_{x+1}({}_1V + P)(1+i)$$

at the end of the year, out of which \$$d_{x+1}$ in death claims is paid. The total reserve at the end of the second year on the l_{x+2} policies remaining is

$$l_{x+2} \cdot {}_2V = l_{x+1}(P + {}_1V)(1+i) - d_{x+1}$$

Solving for ${}_2V$, we have

$$_2V = \frac{l_{x+1}}{l_{x+2}}(P + {}_1V)(1+i) - \frac{d_{x+1}}{l_{x+2}}$$

Continuing in this manner, we have

$$_3V = \frac{l_{x+2}}{l_{x+3}}(P + {}_2V)(1+i) - \frac{d_{x+2}}{l_{x+3}}$$

$$_tV = \frac{l_{x+t-1}}{l_{x+t}}(P + {}_{t-1}V)(1+i) - \frac{d_{x+t-1}}{l_{x+t}}$$

$$_{t+1}V = \frac{l_{x+t}}{l_{x+t+1}}(P + {}_tV)(1+i) - \frac{d_{x+t}}{l_{x+t+1}}$$

Multiplying numerator and denominator of each term of the right member by v^{x+t+1}, we have

$$_{t+1}V = \frac{v^{x+t}l_{x+t}}{v^{x+t+1}l_{x+t+1}}(P + {}_tV) - \frac{v^{x+t+1}d_{x+t}}{v^{x+t+1}l_{x+t+1}}$$

$$= \frac{D_{x+t}}{D_{x+t+1}}(P + {}_tV) - \frac{C_{x+t}}{D_{x+t+1}}$$

Now if we let $u_{x+t} = \dfrac{D_{x+t}}{D_{x+t+1}}$, and $k_{x+t} = \dfrac{C_{x+t}}{D_{x+t+1}}$, we have

(29) $$_{t+1}V = u_{x+t}(P + {}_tV) - k_{x+t}$$

Columns of values for u_x and 1000 k_x appear in Table X. It is possible with their use to compute, rapidly, the reserve on a policy from year to year by using equation (29), each new value obtained depending on that of the preceding year. It should be noticed that, when $t = 0$, (29) reduces to

$$_1V = u_xP - k_x$$

since $_0V = 0$, there being no reserve at the beginning of the first year. Also, if the policy being considered is paid up, that is, t is greater than, or equal to, the number of payments, then $P = 0$ and (29) reduces to

$$_{t+1}V = u_{x+t} \cdot {}_tV - k_{x+t}$$

Example 5. Using Fackler's Accumulation Formula, find the net level reserves on a 10-payment 10-year endowment insurance of \$1000 issued to (40).

Solution. The net level annual premium is

$$\$1000\, P_{40:\overline{10|}} = \$1000(.09067580)$$
$$= \$90.6758$$

as calculated in Example 4. Using tabular form, we have

	(1)	(2)	(3)	(4)	(5)
t	$1000\,(P + {}_{t-1}V)$	u_{x+t-1}	$(1) \times (2)$	$1000\, k_{x+t-1}$	$(3) - (4)\ {}_tV$
1	90.6758	1.0313738	93.52064	6.21837	87.3023
2	177.9781	1.0317993	183.63768	6.63343	177.0042
3	267.6800	1.0322569	276.31453	7.07995	269.2346
4	359.9104	1.0327555	371.69945	7.56634	364.1331
5	454.8089	1.0333077	469.95754	8.10506	461.8519
6	552.5277	1.0339013	571.25911	8.68419	562.5749
7	653.2507	1.0345485	675.8195	9.31559	666.5039
8	757.1797	1.0352597	783.8776	10.00948	773.8681
9	864.5439	1.0360236	895.6879	10.75471	884.9332
10	975.6090	1.0368717	1011.5814	11.58219	1000.0000

Exercise 17-5

1. Prove: ${}_tV_x = (P_{x+t} - P_x)a_{x+t}$.

2. Prove: ${}_tV_{x:\overline{n}|} = (P^{\,1}_{x+t:\overline{n-t}|} - P^{1}_{x:\overline{n}|})a_{x+t:\overline{n-t}|}$.

3. Prove that the reserve at the end of the tth year, on a whole life policy issued to (x) and paid for throughout life, is the same for the prospective and the retrospective methods.

4. Use the prospective method to compute the reserve at the end of the 5th year on a 20-payment whole life policy for \$1000 issued to (37).

5. Use the retrospective method to compute the reserve at the end of the 5th year on the policy of Problem 4.

6. Use Fackler's Accumulation Formula to compute the reserve for the first 5 years on the policy of Problem 4.

7. Find the reserve at the end of the 20th year on the policy of Problem 4.

8. Use the prospective method to compute the reserve at the end of the 10th year on a 20-payment 20-year endowment policy for \$1000 issued to (45).

9. Use the retrospective method to compute the reserve at the end of the 10th year on the policy of Problem 8.

10. Use Fackler's Accumulation Formula to compute the reserve for the first 10 years in the policy of Problem 8.

11. Find the reserve at the end of the 20th year on the policy of Problem 8.

SUMMARY

In this chapter we have given the student a brief look at the subject of contingent annuities and life insurance. We have discussed the mortality table and some of the commutation functions that are useful in evaluating the symbols defined.

We have proved formulas for the present value of a pure endowment, and for the present value of contingent annuities, both ordinary and due, of the following kinds: (a) whole life, (b) temporary, (c) deferred, (d) deferred temporary.

We have also proved formulas for the net single premium and net level annual premium on the following kinds of insurance benefits: (a) whole life, (b) term, (c) endowment.

Three different methods for computing net level reserves have been given. They are the prospective method, the retrospective method, and Fackler's Accumulation Formula.

Exercise 17-6 (Review)

1. Interpret verbally the following symbols:

(a) ${}_{6}q_{21}$ (b) ${}_{18}q_{21}$ (c) ${}_{3|7}q_{21}$

2. Find the value of each symbol in Problem 1.

3. A man aged 30 has \$5000 that he wishes to invest for 15 years. He can purchase a pure endowment at $2\frac{1}{2}\%$ or can lend the money at 3% compounded semi-annually. Assuming he is alive at age 45, which investment is preferable?

4. If an n-year term annuity due to (x) is forborne for the term, that is, if each payment is left in the fund to draw interest contingently to the end of the term, and is then paid to (x) provided he is alive, show that the amount at the end of the nth year is $\dfrac{N_x - N_{x+n}}{D_{x+n}}$.

5. Prove that $A_x = \dfrac{1 - ia_x}{1 + i}$.

6. Compute the numerical values of

(a) $A^{1}_{40:\overline{15}|}$ (b) $A_{40:\overline{15}|}$ (c) A_{40}

7. (30) wishes to purchase from an insurance company a policy of face \$10,000 that will insure him for life and also furnish him an endowment at age 65. Find the net single premium on this contract.

8. (a) Find the net level annual premium payable for 35 years on the policy of Problem 7. (b) Find the net level annual premium payable for 20 years on the policy of Problem 7.

9. A man aged 60 has \$25,000 with which he purchases an ordinary annuity that is certainly payable for 15 years and as long thereafter as he may live. Find the annual payment if $i = .025$.

10. A man aged 37 purchases a \$10,000, 10-year endowment insurance policy, with 10 annual premiums. Find the net level reserve at the end of each year by using Fackler's Accumulation Formula.

11. In Problem 10, check the reserves at the end of the 5th and the 10th year by using (a) the prospective method; (b) the retrospective method.

TABLES

Table I is taken by permission from *Trigonometric and Logarithmic Tables*, by George Wentworth and David Eugene Smith. Courtesy of Ginn and Company, Boston.

Tables II–V and VII–IX are reproduced from *Tables of Applied Mathematics in Finance, Insurance, and Statistics*, by James W. Glover, by arrangement with the publisher, George Wahr.

Table VI is reprinted from *Compound Interest and Annuity Tables*, by F. C. and M. E. Kent. Copyright 1926. Courtesy of McGraw-Hill Book Co., New York.

Table X is included by courtesy of the Society of Actuaries.

N	0	1	2	3	4	5	6	7	8	9
100	00 000	00 043	00 087	00 130	00 173	00 217	00 260	00 303	00 346	00 389
101	432	475	518	561	604	647	689	732	775	817
102	860	903	945	988	01 030	01 072	01 115	01 157	01 199	01 242
103	01 284	01 326	01 368	01 410	452	494	536	578	620	662
104	703	745	787	828	870	912	953	995	02 036	02 078
105	02 119	02 160	02 202	02 243	02 284	02 325	02 366	02 407	02 449	02 490
106	531	572	612	653	694	735	776	816	857	898
107	938	979	03 019	03 060	03 100	03 141	03 181	03 222	03 262	03 302
108	03 342	03 383	423	463	503	543	583	623	663	703
109	743	782	822	862	902	941	981	04 021	04 060	04 100
110	04 139	04 179	04 218	04 258	04 297	04 336	04 376	04 415	04 454	04 493
111	532	571	610	650	689	727	766	805	844	883
112	922	961	999	05 038	05 077	05 115	05 154	05 192	05 231	05 269
113	05 308	05 346	05 385	423	461	500	538	576	614	652
114	690	729	767	805	843	881	918	956	994	06 032
115	06 070	06 108	06 145	06 183	06 221	06 258	06 296	06 333	06 371	06 408
116	446	483	521	558	595	633	670	707	744	781
117	819	856	893	930	967	07 004	07 041	07 078	07 115	07 151
118	07 188	07 225	07 262	07 298	07 335	372	408	445	482	518
119	555	591	628	664	700	737	773	809	846	882
120	07 918	07 954	07 990	08 027	08 063	08 099	08 135	08 171	08 207	08 243
121	08 279	08 314	08 350	386	422	458	493	529	565	600
122	636	672	707	743	778	814	849	884	920	955
123	991	09 026	09 061	09 096	09 132	09 167	09 202	09 237	09 272	09 307
124	09 342	377	412	447	482	517	552	587	621	656
125	09 691	09 726	09 760	09 795	09 830	09 864	09 899	09 934	09 968	10 003
126	10 037	10 072	10 106	10 140	10 175	10 209	10 243	10 278	10 312	346
127	380	415	449	483	517	551	585	619	653	687
128	721	755	789	823	857	890	924	958	992	11 025
129	11 059	11 093	11 126	11 160	11 193	11 227	11 261	11 294	11 327	361
130	11 394	11 428	11 461	11 494	11 528	11 561	11 594	11 628	11 661	11 694
131	727	760	793	826	860	893	926	959	992	12 024
132	12 057	12 090	12 123	12 156	12 189	12 222	12 254	12 287	12 320	352
133	385	418	450	483	516	548	581	613	646	678
134	710	743	775	808	840	872	905	937	969	13 001
135	13 033	13 066	13 098	13 130	13 162	13 194	13 226	13 258	13 290	13 322
136	354	386	418	450	481	513	545	577	609	640
137	672	704	735	767	799	830	862	893	925	956
138	988	14 019	14 051	14 082	14 114	14 145	14 176	14 208	14 239	14 270
139	14 301	333	364	395	426	457	489	520	551	582
140	14 613	14 644	14 675	14 706	14 737	14 768	14 799	14 829	14 860	14 891
141	922	953	983	15 014	15 045	15 076	15 106	15 137	15 168	15 198
142	15 229	15 259	15 290	320	351	381	412	442	473	503
143	534	564	594	625	655	685	715	746	776	806
144	836	866	897	927	957	987	16 017	16 047	16 077	16 107
145	16 137	16 167	16 197	16 227	16 256	16 286	16 316	16 346	16 376	16 406
146	435	465	495	524	554	584	613	643	673	702
147	732	761	791	820	850	879	909	938	967	997
148	17 026	17 056	17 085	17 114	17 143	17 173	17 202	17 231	17 260	17 289
149	319	348	377	406	435	464	493	522	551	580
150	17 609	17 638	17 667	17 696	17 725	17 754	17 782	17 811	17 840	17 869
N	0	1	2	3	4	5	6	7	8	9

N	0	1	2	3	4	5	6	7	8	9
150	17 609	17 638	17 667	17 696	17 725	17 754	17 782	17 811	17 840	17 869
151	898	926	955	984	18 013	18 041	18 070	18 099	18 127	18 156
152	18 184	18 213	18 241	18 270	298	327	355	384	412	441
153	469	498	526	554	583	611	639	667	696	724
154	752	780	808	837	865	893	921	949	977	19 005
155	19 033	19 061	19 089	19 117	19 145	19 173	19 201	19 229	19 257	19 285
156	312	340	368	396	424	451	479	507	535	562
157	590	618	645	673	700	728	756	783	811	838
158	866	893	921	948	976	20 003	20 030	20 058	20 085	20 112
159	20 140	20 167	20 194	20 222	20 249	276	303	330	358	385
160	20 412	20 439	20 466	20 493	20 520	20 548	20 575	20 602	20 629	20 656
161	683	710	737	763	790	817	844	871	898	925
162	952	978	21 005	21 032	21 059	21 085	21 112	21 139	21 165	21 192
163	21 219	21 245	272	299	325	352	378	405	431	458
164	484	511	537	564	590	617	643	669	696	722
165	21 748	21 775	21 801	21 827	21 854	21 880	21 906	21 932	21 958	21 985
166	22 011	22 037	22 063	22 089	22 115	22 141	22 167	22 194	22 220	22 246
167	272	298	324	350	376	401	427	453	479	505
168	531	557	583	608	634	660	686	712	737	763
169	789	814	840	866	891	917	943	968	994	23 019
170	23 045	23 070	23 096	23 121	23 147	23 172	23 198	23 223	23 249	23 274
171	300	325	350	376	401	426	452	477	502	528
172	553	578	603	629	654	679	704	729	754	779
173	805	830	855	880	905	930	955	980	24 005	24 030
174	24 055	24 080	24 105	24 130	24 155	24 180	24 204	24 229	254	279
175	24 304	24 329	24 353	24 378	24 403	24 428	24 452	24 477	24 502	24 527
176	551	576	601	625	650	674	699	724	748	773
177	797	822	846	871	895	920	944	969	993	25 018
178	25 042	25 066	25 091	25 115	25 139	25 164	25 188	25 212	25 237	261
179	285	310	334	358	382	406	431	455	479	503
180	25 527	25 551	25 575	25 600	25 624	25 648	25 672	25 696	25 720	25 744
181	768	792	816	840	864	888	912	935	959	983
182	26 007	26 031	26 055	26 079	26 102	26 126	26 150	26 174	26 198	26 221
183	245	269	293	316	340	364	387	411	435	458
184	482	505	529	553	576	600	623	647	670	694
185	26 717	26 741	26 764	26 788	26 811	26 834	26 858	26 881	26 905	26 928
186	951	975	998	27 021	27 045	27 068	27 091	27 114	27 138	27 161
187	27 184	27 207	27 231	254	277	300	323	346	370	393
188	416	439	462	485	508	531	554	577	600	623
189	646	669	692	715	738	761	784	807	830	852
190	27 875	27 898	27 921	27 944	27 967	27 989	28 012	28 035	28 058	28 081
191	28 103	28 126	28 149	28 171	28 194	28 217	240	262	285	307
192	330	353	375	398	421	443	466	488	511	533
193	556	578	601	623	646	668	691	713	735	758
194	780	803	825	847	870	892	914	937	959	981
195	29 003	29 026	29 048	29 070	29 092	29 115	29 137	29 159	29 181	29 203
196	226	248	270	292	314	336	358	380	403	425
197	447	469	491	513	535	557	579	601	623	645
198	667	688	710	732	754	776	798	820	842	863
199	885	907	929	951	973	994	30 016	30 038	30 060	30 081
200	30 103	30 125	30 146	30 168	30 190	30 211	30 233	30 255	30 276	30 298
N	0	1	2	3	4	5	6	7	8	9

N	0	1	2	3	4	5	6	7	8	9
200	30 103	30 125	30 146	30 168	30 190	30 211	30 233	30 255	30 276	30 298
201	320	341	363	384	406	428	449	471	492	514
202	535	557	578	600	621	643	664	685	707	728
203	750	771	792	814	835	856	878	899	920	942
204	963	984	31 006	31 027	31 048	31 069	31 091	31 112	31 133	31 154
205	31 175	31 197	31 218	31 239	31 260	31 281	31 302	31 323	31 345	31 366
206	387	408	429	450	471	492	513	534	555	576
207	597	618	639	660	681	702	723	744	765	785
208	806	827	848	869	890	911	931	952	973	994
209	32 015	32 035	32 056	32 077	32 098	32 118	32 139	32 160	32 181	32 201
210	32 222	32 243	32 263	32 284	32 305	32 325	32 346	32 366	32 387	32 408
211	428	449	469	490	510	531	552	572	593	613
212	634	654	675	695	715	736	756	777	797	818
213	838	858	879	899	919	940	960	980	33 001	33 021
214	33 041	33 062	33 082	33 102	33 122	33 143	33 163	33 183	203	224
215	33 244	33 264	33 284	33 304	33 325	33 345	33 365	33 385	33 405	33 425
216	445	465	486	506	526	546	566	586	606	626
217	646	666	686	706	726	746	766	786	806	826
218	846	866	885	905	925	945	965	985	34 005	34 025
219	34 044	34 064	34 084	34 104	34 124	34 143	34 163	34 183	203	223
220	34 242	34 262	34 282	34 301	34 321	34 341	34 361	34 380	34 400	34 420
221	439	459	479	498	518	537	557	577	596	616
222	635	655	674	694	713	733	753	772	792	811
223	830	850	869	889	908	928	947	967	986	35 005
224	35 025	35 044	35 064	35 083	35 102	35 122	35 141	35 160	35 180	199
225	35 218	35 238	35 257	35 276	35 295	35 315	35 334	35 353	35 372	35 392
226	411	430	449	468	488	507	526	545	564	583
227	603	622	641	660	679	698	717	736	755	774
228	793	813	832	851	870	889	908	927	946	965
229	984	36 003	36 021	36 040	36 059	36 078	36 097	36 116	36 135	36 154
230	36 173	36 192	36 211	36 229	36 248	36 267	36 286	36 305	36 324	36 342
231	361	380	399	418	436	455	474	493	511	530
232	549	568	586	605	624	642	661	680	698	717
233	736	754	773	791	810	829	847	866	884	903
234	922	940	959	977	996	37 014	37 033	37 051	37 070	37 088
235	37 107	37 125	37 144	37 162	37 181	37 199	37 218	37 236	37 254	37 273
236	291	310	328	346	365	383	401	420	438	457
237	475	493	511	530	548	566	585	603	621	639
238	658	676	694	712	731	749	767	785	803	822
239	840	858	876	894	912	931	949	967	985	38 003
240	38 021	38 039	38 057	38 075	38 093	38 112	38 130	38 148	38 166	38 184
241	202	220	238	256	274	292	310	328	346	364
242	382	399	417	435	453	471	489	507	525	543
243	561	578	596	614	632	650	668	686	703	721
244	739	757	775	792	810	828	846	863	881	899
245	38 917	38 934	38 952	38 970	38 987	39 005	39 023	39 041	39 058	39 076
246	39 094	39 111	39 129	39 146	39 164	182	199	217	235	252
247	270	287	305	322	340	358	375	393	410	428
248	445	463	480	498	515	533	550	568	585	602
249	620	637	655	672	690	707	724	742	759	777
250	39 794	39 811	39 829	39 846	39 863	39 881	39 898	39 915	39 933	39 950
N	0	1	2	3	4	5	6	7	8	9

N	0	1	2	3	4	5	6	7	8	9
250	39 794	39 811	39 829	39 846	39 863	39 881	39 898	39 915	39 933	39 950
251	967	985	40 002	40 019	40 037	40 054	40 071	40 088	40 106	40 123
252	40 140	40 157	175	192	209	226	243	261	278	295
253	312	329	346	364	381	398	415	432	449	466
254	483	500	518	535	552	569	586	603	620	637
255	40 654	40 671	40 688	40 705	40 722	40 739	40 756	40 773	40 790	40 807
256	824	841	858	875	892	909	926	943	960	976
257	993	41 010	41 027	41 044	41 061	41 078	41 095	41 111	41 128	41 145
258	41 162	179	196	212	229	246	263	280	296	313
259	330	347	363	380	397	414	430	447	464	481
260	41 497	41 514	41 531	41 547	41 564	41 581	41 597	41 614	41 631	41 647
261	664	681	697	714	731	747	764	780	797	814
262	830	847	863	880	896	913	929	946	963	979
263	996	42 012	42 029	42 045	42 062	42 078	42 095	42 111	42 127	42 144
264	42 160	177	193	210	226	243	259	275	292	308
265	42 325	42 341	42 357	42 374	42 390	42 406	42 423	42 439	42 455	42 472
266	488	504	521	537	553	570	586	602	619	635
267	651	667	684	700	716	732	749	765	781	797
268	813	830	846	862	878	894	911	927	943	959
269	975	991	43 008	43 024	43 040	43 056	43 072	43 088	43 104	43 120
270	43 136	43 152	43 169	43 185	43 201	43 217	43 233	43 249	43 265	43 281
271	297	313	329	345	361	377	393	409	425	441
272	457	473	489	505	521	537	553	569	584	600
273	616	632	648	664	680	696	712	727	743	759
274	775	791	807	823	838	854	870	886	902	917
275	43 933	43 949	43 965	43 981	43 996	44 012	44 028	44 044	44 059	44 075
276	44 091	44 107	44 122	44 138	44 154	170	185	201	217	232
277	248	264	279	295	311	326	342	358	373	389
278	404	420	436	451	467	483	498	514	529	545
279	560	576	592	607	623	638	654	669	685	700
280	44 716	44 731	44 747	44 762	44 778	44 793	44 809	44 824	44 840	44 855
281	871	886	902	917	932	948	963	979	994	45 010
282	45 025	45 040	45 056	45 071	45 086	45 102	45 117	45 133	45 148	163
283	179	194	209	225	240	255	271	286	301	317
284	332	347	362	378	393	408	423	439	454	469
285	45 484	45 500	45 515	45 530	45 545	45 561	45 576	45 591	45 606	45 621
286	637	652	667	682	697	712	728	743	758	773
287	788	803	818	834	849	864	879	894	909	924
288	939	954	969	984	46 000	46 015	46 030	46 045	46 060	46 075
289	46 090	46 105	46 120	46 135	150	165	180	195	210	225
290	46 240	46 255	46 270	46 285	46 300	46 315	46 330	46 345	46 359	46 374
291	389	404	419	434	449	464	479	494	509	523
292	538	553	568	583	598	613	627	642	657	672
293	687	702	716	731	746	761	776	790	805	820
294	835	850	864	879	894	909	923	938	953	967
295	46 982	46 997	47 012	47 026	47 041	47 056	47 070	47 085	47 100	47 114
296	47 129	47 144	159	173	188	202	217	232	246	261
297	276	290	305	319	334	349	363	378	392	407
298	422	436	451	465	480	494	509	524	538	553
299	567	582	596	611	625	640	654	669	683	698
300	47 712	47 727	47 741	47 756	47 770	47 784	47 799	47 813	47 828	47 842
N	0	1	2	3	4	5	6	7	8	9

N	0	1	2	3	4	5	6	7	8	9
300	47 712	47 727	47 741	47 756	47 770	47 784	47 799	47 813	47 828	47 842
301	857	871	885	900	914	929	943	958	972	986
302	48 001	48 015	48 029	48 044	48 058	48 073	48 087	48 101	48 116	48 130
303	144	159	173	187	202	216	230	244	259	273
304	287	302	316	330	344	359	373	387	401	416
305	48 430	48 444	48 458	48 473	48 487	48 501	48 515	48 530	48 544	48 558
306	572	586	601	615	629	643	657	671	686	700
307	714	728	742	756	770	785	799	813	827	841
308	855	869	883	897	911	926	940	954	968	982
309	996	49 010	49 024	49 038	49 052	49 066	49 080	49 094	49 108	49 122
310	49 136	49 150	49 164	49 178	49 192	49 206	49 220	49 234	49 248	49 262
311	276	290	304	318	332	346	360	374	388	402
312	415	429	443	457	471	485	499	513	527	541
313	554	568	582	596	610	624	638	651	665	679
314	693	707	721	734	748	762	776	790	803	817
315	49 831	49 845	49 859	49 872	49 886	49 900	49 914	49 927	49 941	49 955
316	969	982	996	50 010	50 024	50 037	50 051	50 065	50 079	50 092
317	50 106	50 120	50 133	147	161	174	188	202	215	229
318	243	256	270	284	297	311	325	338	352	365
319	379	393	406	420	433	447	461	474	488	501
320	50 515	50 529	50 542	50 556	50 569	50 583	50 596	50 610	50 623	50 637
321	651	664	678	691	705	718	732	745	759	772
322	786	799	813	826	840	853	866	880	893	907
323	920	934	947	961	974	987	51 001	51 014	51 028	51 041
324	51 055	51 068	51 081	51 095	51 108	51 121	135	148	162	175
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596	525	532	539	546	554	561	568	576	583	590
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641	686	693	699	706	713	720	726	733	740	747
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697	323	330	336	342	348	354	361	367	373	379
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862	551	556	561	566	571	576	581	586	591	596
863	601	606	611	616	621	626	631	636	641	646
864	651	656	661	666	671	676	682	687	692	697
865	93 702	93 707	93 712	93 717	93 722	93 727	93 732	93 737	93 742	93 747
866	752	757	762	767	772	777	782	787	792	797
867	802	807	812	817	822	827	832	837	842	847
868	852	857	862	867	872	877	882	887	892	897
869	902	907	912	917	922	927	932	937	942	947
870	93 952	93 957	93 962	93 967	93 972	93 977	93 982	93 987	93 992	93 997
871	94 002	94 007	94 012	94 017	94 022	94 027	94 032	94 037	94 042	94 047
872	052	057	062	067	072	077	082	086	091	096
873	101	106	111	116	121	126	131	136	141	146
874	151	156	161	166	171	176	181	186	191	196
875	94 201	94 206	94 211	94 216	94 221	94 226	94 231	94 236	94 240	94 245
876	250	255	260	265	270	275	280	285	290	295
877	300	305	310	315	320	325	330	335	340	345
878	349	354	359	364	369	374	379	384	389	394
879	399	404	409	414	419	424	429	433	438	443
880	94 448	94 453	94 458	94 463	94 468	94 473	94 478	94 483	94 488	94 493
881	498	503	507	512	517	522	527	532	537	542
882	547	552	557	562	567	571	576	581	586	591
883	596	601	606	611	616	621	626	630	635	640
884	645	650	655	660	665	670	675	680	685	689
885	94 694	94 699	94 704	94 709	94 714	94 719	94 724	94 729	94 734	94 738
886	743	748	753	758	763	768	773	778	783	787
887	792	797	802	807	812	817	822	827	832	836
888	841	846	851	856	861	866	871	876	880	885
889	890	895	900	905	910	915	919	924	929	934
890	94 939	94 944	94 949	94 954	94 959	94 963	94 968	94 973	94 978	94 983
891	988	993	998	95 002	95 007	95 012	95 017	95 022	95 027	95 032
892	95 036	95 041	95 046	051	056	061	066	071	075	080
893	085	090	095	100	105	109	114	119	124	129
894	134	139	143	148	153	158	163	168	173	177
895	95 182	95 187	95 192	95 197	95 202	95 207	95 211	95 216	95 221	95 226
896	231	236	240	245	250	255	260	265	270	274
897	279	284	289	294	299	303	308	313	318	323
898	328	332	337	342	347	352	357	361	366	371
899	376	381	386	390	395	400	405	410	415	419
900	95 424	95 429	95 434	95 439	95 444	95 448	95 453	95 458	95 463	95 468
N	0	1	2	3	4	5	6	7	8	9

N	0	1	2	3	4	5	6	7	8	9
900	95 424	95 429	95 434	95 439	95 444	95 448	95 453	95 458	95 463	95 468
901	472	477	482	487	492	497	501	506	511	516
902	521	525	530	535	540	545	550	554	559	564
903	569	574	578	583	588	593	598	602	607	612
904	617	622	626	631	636	641	646	650	655	660
905	95 665	95 670	95 674	95 679	95 684	95 689	95 694	95 698	95 703	95 708
906	713	718	722	727	732	737	742	746	751	756
907	761	766	770	775	780	785	789	794	799	804
908	809	813	818	823	828	832	837	842	847	852
909	856	861	866	871	875	880	885	890	895	899
910	95 904	95 909	95 914	95 918	95 923	95 928	95 933	95 938	95 942	95 947
911	952	957	961	966	971	976	980	985	990	995
912	999	96 004	96 009	96 014	96 019	96 023	96 028	96 033	96 038	96 042
913	96 047	052	057	061	066	071	076	080	085	090
914	095	099	104	109	114	118	123	128	133	137
915	96 142	96 147	96 152	96 156	96 161	96 166	96 171	96 175	96 180	96 185
916	190	194	199	204	209	213	218	223	227	232
917	237	242	246	251	256	261	265	270	275	280
918	284	289	294	298	303	308	313	317	322	327
919	332	336	341	346	350	355	360	365	369	374
920	96 379	96 384	96 388	96 393	96 398	96 402	96 407	96 412	96 417	96 421
921	426	431	435	440	445	450	454	459	464	468
922	473	478	483	487	492	497	501	506	511	515
923	520	525	530	534	539	544	548	553	558	562
924	567	572	577	581	586	591	595	600	605	609
925	96 614	96 619	96 624	96 628	96 633	96 638	96 642	96 647	96 652	96 656
926	661	666	670	675	680	685	689	694	699	703
927	708	713	717	722	727	731	736	741	745	750
928	755	759	764	769	774	778	783	788	792	797
929	802	806	811	816	820	825	830	834	839	844
930	96 848	96 853	96 858	96 862	96 867	96 872	96 876	96 881	96 886	96 890
931	895	900	904	909	914	918	923	928	932	937
932	942	946	951	956	960	965	970	974	979	984
933	988	993	997	97 002	97 007	97 011	97 016	97 021	97 025	97 030
934	97 035	97 039	97 044	049	053	058	063	067	072	077
935	97 081	97 086	97 090	97 095	97 100	97 104	97 109	97 114	97 118	97 123
936	128	132	137	142	146	151	155	160	165	169
937	174	179	183	188	192	197	202	206	211	216
938	220	225	230	234	239	243	248	253	257	262
939	267	271	276	280	285	290	294	299	304	308
940	97 313	97 317	97 322	97 327	97 331	97 336	97 340	97 345	97 350	97 354
941	359	364	368	373	377	382	387	391	396	400
942	405	410	414	419	424	428	433	437	442	447
943	451	456	460	465	470	474	479	483	488	493
944	497	502	506	511	516	520	525	529	534	539
945	97 543	97 548	97 552	97 557	97 562	97 566	97 571	97 575	97 580	97 585
946	589	594	598	603	607	612	617	621	626	630
947	635	640	644	649	653	658	663	667	672	676
948	681	685	690	695	699	704	708	713	717	722
949	727	731	736	740	745	749	754	759	763	768
950	97 772	97 777	97 782	97 786	97 791	97 795	97 800	97 804	97 809	97 813
N	0	1	2	3	4	5	6	7	8	9

N	0	1	2	3	4	5	6	7	8	9
950	97 772	97 777	97 782	97 786	97 791	97 795	97 800	97 804	97 809	97 813
951	818	823	827	832	836	841	845	850	855	859
952	864	868	873	877	882	886	891	896	900	905
953	909	914	918	923	928	932	937	941	946	950
954	955	959	964	968	973	978	982	987	991	996
955	98 000	98 005	98 009	98 014	98 019	98 023	98 028	98 032	98 037	98 041
956	046	050	055	059	064	068	073	078	082	087
957	091	096	100	105	109	114	118	123	127	132
958	137	141	146	150	155	159	164	168	173	177
959	182	186	191	195	200	204	209	214	218	223
960	98 227	98 232	98 236	98 241	98 245	98 250	98 254	98 259	98 263	98 268
961	272	277	281	286	290	295	299	304	308	313
962	318	322	327	331	336	340	345	349	354	358
963	363	367	372	376	381	385	390	394	399	403
964	408	412	417	421	426	430	435	439	444	448
965	98 453	98 457	98 462	98 466	98 471	98 475	98 480	98 484	98 489	98 493
966	498	502	507	511	516	520	525	529	534	538
967	543	547	552	556	561	565	570	574	579	583
968	588	592	597	601	605	610	614	619	623	628
969	632	637	641	646	650	655	659	664	668	673
970	98 677	98 682	98 686	98 691	98 695	98 700	98 704	98 709	98 713	98 717
971	722	726	731	735	740	744	749	753	758	762
972	767	771	776	780	784	789	793	798	802	807
973	811	816	820	825	829	834	838	843	847	851
974	856	860	865	869	874	878	883	887	892	896
975	98 900	98 905	98 909	98 914	98 918	98 923	98 927	98 932	98 936	98 941
976	945	949	954	958	963	967	972	976	981	985
977	989	994	998	99 003	99 007	99 012	99 016	99 021	99 025	99 029
978	99 034	99 038	99 043	047	052	056	061	065	069	074
979	078	083	087	092	096	100	105	109	114	118
980	99 123	99 127	99 131	99 136	99 140	99 145	99 149	99 154	99 158	99 162
981	167	171	176	180	185	189	193	198	202	207
982	211	216	220	224	229	233	238	242	247	251
983	255	260	264	269	273	277	282	286	291	295
984	300	304	308	313	317	322	326	330	335	339
985	99 344	99 348	99 352	99 357	99 361	99 366	99 370	99 374	99 379	99 383
986	388	392	396	401	405	410	414	419	423	427
987	432	436	441	445	449	454	458	463	467	471
988	476	480	484	489	493	498	502	506	511	515
989	520	524	528	533	537	542	546	550	555	559
990	99 564	99 568	99 572	99 577	99 581	99 585	99 590	99 594	99 599	99 603
991	607	612	616	621	625	629	634	638	642	647
992	651	656	660	664	669	673	677	682	686	691
993	695	699	704	708	712	717	721	726	730	734
994	739	743	747	752	756	760	765	769	774	778
995	99 782	99 787	99 791	99 795	99 800	99 804	99 808	99 813	99 817	99 822
996	826	830	835	839	843	848	852	856	861	865
997	870	874	878	883	887	891	896	900	904	909
998	913	917	922	926	930	935	939	944	948	952
999	957	961	965	970	974	978	983	987	991	996
1000	00 000	00 004	00 009	00 013	00 017	00 022	00 026	00 030	00 035	00 039
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

Present Value of 1 at Compound Interest

$$v^n = (1+i)^{-n}$$

n	$\frac{1}{4}\%$	$\frac{7}{24}\%$	$\frac{1}{3}\%$	$\frac{5}{12}\%$	n
1	0.9975 0623	0.9970 9182	0.9966 7774	0.9958 5062	1
2	0.9950 1869	0.9941 9209	0.9933 6652	0.9917 1846	2
3	0.9925 3734	0.9913 0079	0.9900 6630	0.9876 0345	3
4	0.9900 6219	0.9884 1791	0.9867 7704	0.9835 0551	4
5	0.9875 9321	0.9855 4341	0.9834 9871	0.9794 2457	5
6	0.9851 3038	0.9826 7726	0.9802 3127	0.9753 6057	6
7	0.9826 7370	0.9798 1946	0.9769 7469	0.9713 1343	7
8	0.9802 2314	0.9769 6996	0.9737 2893	0.9672 8308	8
9	0.9777 7869	0.9741 2875	0.9704 9395	0.9632 6946	9
10	0.9753 4034	0.9712 9580	0.9672 6972	0.9592 7249	10
11	0.9729 0807	0.9684 7110	0.9640 5620	0.9552 9211	11
12	0.9704 8187	0.9656 5460	0.9608 5335	0.9513 2824	12
13	0.9680 6171	0.9628 4630	0.9576 6115	0.9473 8082	13
14	0.9656 4759	0.9600 4617	0.9544 7955	0.9434 4978	14
15	0.9632 3949	0.9572 5418	0.9513 0852	0.9395 3505	15
16	0.9608 3740	0.9544 7030	0.9481 4803	0.9356 3656	16
17	0.9584 4130	0.9516 9453	0.9449 9803	0.9317 5425	17
18	0.9560 5117	0.9489 2682	0.9418 5851	0.9278 8805	18
19	0.9536 6700	0.9461 6717	0.9387 2941	0.9240 3789	19
20	0.9512 8878	0.9434 1554	0.9356 1071	0.9202 0371	20
21	0.9489 1649	0.9406 7191	0.9325 0236	0.9163 8544	21
22	0.9465 5011	0.9379 3627	0.9294 0435	0.9125 8301	22
23	0.9441 8964	0.9352 0857	0.9263 1663	0.9087 9636	23
24	0.9418 3505	0.9324 8881	0.9232 3916	0.9050 2542	24
25	0.9394 8634	0.9297 7696	0.9201 7192	0.9012 7012	25
26	0.9371 4348	0.9270 7300	0.9171 1487	0.8975 3041	26
27	0.9348 0646	0.9243 7690	0.9140 6798	0.8938 0622	27
28	0.9324 7527	0.9216 8864	0.9110 3121	0.8900 9748	28
29	0.9301 4990	0.9190 0820	0.9080 0453	0.8864 0413	29
30	0.9278 3032	0.9163 3556	0.9049 8790	0.8827 2610	30
31	0.9255 1653	0.9136 7068	0.9019 8130	0.8790 6334	31
32	0.9232 0851	0.9110 1356	0.8989 8468	0.8754 1577	32
33	0.9209 0624	0.9083 6416	0.8959 9802	0.8717 8334	33
34	0.9186 0972	0.9057 2247	0.8930 2128	0.8681 6599	34
35	0.9163 1892	0.9030 8847	0.8900 5444	0.8645 6364	35
36	0.9140 3384	0.9004 6212	0.8870 9745	0.8609 7624	36
37	0.9117 5445	0.8978 4341	0.8841 5028	0.8574 0372	37
38	0.9094 8075	0.8952 3231	0.8812 1290	0.8538 4603	38
39	0.9072 1272	0.8926 2881	0.8782 8528	0.8503 0310	39
40	0.9049 5034	0.8900 3288	0.8753 6739	0.8467 7487	40
41	0.9026 9361	0.8874 4450	0.8724 5920	0.8432 6128	41
42	0.9004 4250	0.8848 6365	0.8695 6066	0.8397 6227	42
43	0.8981 9701	0.8822 9030	0.8666 7175	0.8362 7778	43
44	0.8959 5712	0.8797 2444	0.8637 9245	0.8328 0775	44
45	0.8937 2281	0.8771 6604	0.8609 2270	0.8293 5211	45
46	0.8914 9407	0.8746 1508	0.8580 6249	0.8259 1082	46
47	0.8892 7090	0.8720 7153	0.8552 1179	0.8224 8380	47
48	0.8870 5326	0.8695 3539	0.8523 7055	0.8190 7100	48
49	0.8848 4116	0.8670 0662	0.8495 3876	0.8156 7237	49
50	0.8826 3457	0.8644 8520	0.8467 1637	0.8122 8784	50

$$v^n = (1+i)^{-n}$$

n	$\frac{1}{4}\%$	$\frac{7}{24}\%$	$\frac{1}{3}\%$	$\frac{5}{12}\%$	n
51	0.8804 3349	0.8619 7112	0.8439 0336	0.8089 1735	51
52	0.8782 3790	0.8594 6435	0.8410 9969	0.8055 6084	52
53	0.8760 4778	0.8569 6487	0.8383 0534	0.8022 1827	53
54	0.8738 6312	0.8544 7266	0.8355 2027	0.7988 8956	54
55	0.8716 8391	0.8519 8769	0.8327 4446	0.7955 7467	55
56	0.8695 1013	0.8495 0995	0.8299 7787	0.7922 7353	56
57	0.8673 4178	0.8470 3942	0.8272 2047	0.7889 8608	57
58	0.8651 7883	0.8445 7608	0.8244 7222	0.7857 1228	58
59	0.8630 2128	0.8421 1989	0.8217 3311	0.7824 5207	59
60	0.8608 6911	0.8396 7085	0.8190 0310	0.7792 0538	60
61	0.8587 2230	0.8372 2893	0.8162 8216	0.7759 7216	61
62	0.8565 8085	0.8347 9412	0.8135 7026	0.7727 5236	62
63	0.8544 4474	0.8323 6638	0.8108 6737	0.7695 4591	63
64	0.8523 1395	0.8299 4571	0.8081 7346	0.7663 5278	64
65	0.8501 8848	0.8275 3207	0.8054 8850	0.7631 7289	65
66	0.8480 6831	0.8251 2545	0.8028 1246	0.7600 0620	66
67	0.8459 5343	0.8227 2584	0.8001 4531	0.7568 5265	67
68	0.8438 4382	0.8203 3320	0.7974 8702	0.7537 1218	68
69	0.8417 3947	0.8179 4752	0.7948 3756	0.7505 8474	69
70	0.8396 4037	0.8155 6878	0.7921 9690	0.7474 7028	70
71	0.8375 4650	0.8131 9695	0.7895 6502	0.7443 6874	71
72	0.8354 5786	0.8108 3202	0.7869 4188	0.7412 8008	72
73	0.8333 7442	0.8084 7397	0.7843 2745	0.7382 0423	73
74	0.8312 9618	0.8061 2278	0.7817 2171	0.7351 4114	74
75	0.8292 2312	0.8037 7843	0.7791 2463	0.7320 9076	75
76	0.8271 5523	0.8014 4089	0.7765 3618	0.7290 5304	76
77	0.8250 9250	0.7991 1015	0.7739 5632	0.7260 2792	77
78	0.8230 3491	0.7967 8619	0.7713 8504	0.7230 1536	78
79	0.8209 8246	0.7944 6899	0.7688 2230	0.7200 1529	79
80	0.8189 3512	0.7921 5853	0.7662 6807	0.7170 2768	80
81	0.8168 9289	0.7898 5479	0.7637 2233	0.7140 5246	81
82	0.8148 5575	0.7875 5774	0.7611 8505	0.7110 8959	82
83	0.8128 2369	0.7852 6738	0.7586 5619	0.7081 3901	83
84	0.8107 9670	0.7829 8368	0.7561 3574	0.7052 0067	84
85	0.8087 7476	0.7807 0662	0.7536 2366	0.7022 7453	85
86	0.8067 5787	0.7784 3618	0.7511 1993	0.6993 6052	86
87	0.8047 4600	0.7761 7234	0.7486 2451	0.6964 5861	87
88	0.8027 3915	0.7739 1509	0.7461 3739	0.6935 6874	88
89	0.8007 3731	0.7716 6440	0.7436 5853	0.6906 9086	89
90	0.7987 4046	0.7694 2026	0.7411 8790	0.6878 2493	90
91	0.7967 4859	0.7671 8264	0.7387 2548	0.6849 7088	91
92	0.7947 6168	0.7649 5153	0.7362 7125	0.6821 2868	92
93	0.7927 7973	0.7627 2691	0.7338 2516	0.6792 9827	93
94	0.7908 0273	0.7605 0876	0.7313 8720	0.6764 7960	94
95	0.7888 3065	0.7582 9706	0.7289 5735	0.6736 7263	95
96	0.7868 6349	0.7560 9179	0.7265 3556	0.6708 7731	96
97	0.7849 0124	0.7538 9294	0.7241 2182	0.6680 9359	97
98	0.7829 4388	0.7517 0048	0.7217 1610	0.6653 2141	98
99	0.7809 9140	0.7495 1439	0.7193 1837	0.6625 6074	99
100	0.7790 4379	0.7473 3467	0.7169 2861	0.6598 1153	100

Present Value of 1 at Compound Interest

$$v^n = (1+i)^{-n}$$

n	¼%	7/24%	⅓%	5/12%	n
101	0.7771 0104	0.7451 6128	0.7145 4681	0.6570 7372	101
102	0.7751 6313	0.7429 9421	0.7121 7290	0.6543 4727	102
104	0.7732 3006	0.7408 3345	0.7098 0688	0.6516 3214	103
104	0.7713 0180	0.7386 7897	0.7074 4872	0.6489 2827	104
105	0.7693 7836	0.7365 3075	0.7050 9839	0.6462 3562	105
106	0.7674 5971	0.7343 8879	0.7027 5587	0.6435 5415	106
107	0.7655 4584	0.7322 5305	0.7004 2114	0.6408 8380	107
108	0.7636 3675	0.7301 2352	0.6980 9416	0.6382 2453	108
109	0.7617 3242	0.7280 0019	0.6957 7491	0.6355 7630	109
110	0.7598 3284	0.7258 8303	0.6934 6336	0.6329 3905	110
111	0.7579 3799	0.7237 7203	0.6911 5950	0.6303 1275	111
112	0.7560 4787	0.7216 6716	0.6888 6329	0.6276 9734	112
113	0.7541 6247	0.7195 6842	0.6865 7470	0.6250 9279	113
114	0.7522 8176	0.7174 7578	0.6842 9372	0.6224 9904	114
115	0.7504 0575	0.7153 8923	0.6820 2032	0.6199 1606	115
116	0.7485 3441	0.7133 0875	0.6797 5448	0.6173 4379	116
117	0.7466 6774	0.7112 3431	0.6774 9616	0.6147 8220	117
118	0.7448 0573	0.7091 6591	0.6752 4534	0.6122 3123	118
119	0.7429 4836	0.7071 0353	0.6730 0200	0.6096 9086	119
120	0.7410 9562	0.7050 4714	0.6707 6611	0.6071 6102	120
121	0.7392 4750	0.7029 9673	0.6685 3765	0.6046 4168	121
122	0.7374 0399	0.7009 5229	0.6663 1660	0.6021 3279	122
123	0.7355 6508	0.6989 1379	0.6641 0292	0.5996 3431	123
124	0.7337 3075	0.6968 8122	0.6618 9660	0.5971 4620	124
125	0.7319 0100	0.6948 5456	0.6596 9761	0.5946 6842	125
126	0.7300 7581	0.6928 3379	0.6575 0592	0.5922 0091	126
127	0.7282 5517	0.6908 1890	0.6553 2152	0.5897 4365	127
128	0.7264 3907	0.6888 0988	0.6531 4437	0.5872 9658	128
129	0.7246 2750	0.6868 0669	0.6509 7445	0.5848 5966	129
130	0.7228 2045	0.6848 0933	0.6488 1175	0.5824 3286	130
131	0.7210 1791	0.6828 1778	0.6466 5623	0.5800 1613	131
132	0.7192 1986	0.6808 3202	0.6445 0787	0.5776 0942	132
133	0.7174 2629	0.6788 5203	0.6423 6665	0.5752 1270	133
134	0.7156 3720	0.6768 7780	0.6402 3254	0.5728 2593	134
135	0.7138 5257	0.6749 0932	0.6381 0552	0.5704 4906	135
136	0.7120 7239	0.6729 4656	0.6359 8557	0.5680 8205	136
137	0.7102 9664	0.6709 8950	0.6338 7266	0.5657 2486	137
138	0.7085 2533	0.6690 3814	0.6317 6677	0.5633 7745	138
139	0.7067 5843	0.6670 9246	0.6296 6788	0.5610 3979	139
140	0.7049 9595	0.6651 5243	0.6275 7596	0.5587 1182	140
141	0.7032 3785	0.6632 1804	0.6254 9099	0.5563 9351	141
142	0.7014 8414	0.6612 8928	0.6234 1295	0.5540 8483	142
143	0.6997 3480	0.6593 6613	0.6213 4181	0.5517 8572	143
144	0.6979 8983	0.6574 4857	0.6192 7755	0.5494 9615	144
145	0.6962 4921	0.6555 3659	0.6172 2015	0.5472 1609	145
146	0.6945 1292	0.6536 3017	0.6151 6958	0.5449 4548	146
147	0.6927 8097	0.6517 2929	0.6131 2583	0.5426 8429	147
148	0.6910 5334	0.6498 3394	0.6110 8887	0.5404 3249	148
149	0.6893 3001	0.6479 4410	0.6090 5867	0 5381 9003	149
150	0.6876 1098	0.6460 5976	0.6070 3522	0 5359 5688	150

$$v^n = (1+i)^{-n}$$

n	¼%	7/24%	⅓%	5/12%	n
151	0.6858 9624	0.6441 8090	0.6050 1849	0.5337 3299	151
152	0.6841 8578	0.6423 0750	0.6030 0847	0.5315 1833	152
153	0.6824 7958	0.6404 3956	0.6010 0512	0.5293 1286	153
154	0.6807 7764	0.6385 7704	0.5990 0842	0.5271 1654	154
155	0.6790 7994	0.6367 1994	0.5970 1836	0.5249 2934	155
156	0.6773 8647	0.6348 6824	0.5950 3491	0.5227 5121	156
157	0.6756 9723	0.6330 2193	0.5930 5805	0.5205 8211	157
158	0.6740 1220	0.6311 8098	0.5910 8776	0.5184 2202	158
159	0.6723 3137	0.6293 4539	0.5891 2401	0.5162 7089	159
160	0.6706 5473	0.6275 1514	0.5871 6679	0.5141 2869	160
161	0.6689 8228	0.6256 9021	0.5852 1607	0.5119 9538	161
162	0.6673 1399	0.6238 7058	0.5832 7183	0.5098 7091	162
163	0.6656 4987	0.6220 5625	0.5813 3405	0.5077 5527	163
164	0.6639 8989	0.6202 4720	0.5794 0271	0.5056 4840	164
165	0.6623 3406	0.6184 4341	0.5774 7778	0.5035 5027	165
166	0.6606 8235	0.6166 4486	0.5755 5925	0.5014 6085	166
167	0.6590 3476	0.6148 5154	0.5736 4710	0.4993 8010	167
168	0.6573 9129	0.6130 6344	0.5717 4129	0.4973 0798	168
169	0.6557 5191	0.6112 8054	0.5698 4182	0.4952 4447	169
170	0.6541 1661	0.6095 0282	0.5679 4866	0.4931 8951	170
171	0.6524 8540	0.6077 3027	0.5660 6178	0.4911 4308	171
172	0.6508 5826	0.6059 6288	0.5641 8118	0.4891 0514	172
173	0.6492 3517	0.6042 0063	0.5623 0682	0.4870 7566	173
174	0.6476 1613	0.6024 4350	0.5604 3870	0.4850 5460	174
175	0.6460 0112	0.6006 9149	0.5585 7677	0.4830 4192	175
176	0.6443 9015	0.5989 4456	0.5567 2104	0.4810 3760	176
177	0.6427 8319	0.5972 0272	0.5548 7147	0.4790 4159	177
178	0.6411 8024	0.5954 6595	0.5530 2804	0.4770 5387	178
179	0.6395 8129	0.5937 3422	0.5511 9074	0.4750 7439	179
180	0.6379 8632	0.5920 0753	0.5493 5954	0.4731 0313	180
181	0.6363 9533	0.5902 8586	0.5475 3442	0.4711 4005	181
182	0.6348 0831	0.5885 6920	0.5457 1537	0.4691 8511	182
183	0.6332 2525	0.5868 5754	0.5439 0237	0.4672 3828	183
184	0.6316 4613	0.5851 5085	0.5420 9538	0.4652 9953	184
185	0.6300 7096	0.5834 4912	0.5402 9440	0.4633 6883	185
186	0.6284 9971	0.5817 5234	0.5384 9940	0.4614 4614	186
187	0.6269 3238	0.5800 6050	0.5367 1037	0.4595 3142	187
188	0.6253 6895	0.5783 7357	0.5349 2728	0.4576 2465	188
189	0.6238 0943	0.5766 9156	0.5331 5011	0.4557 2580	189
190	0.6222 5380	0.5750 1443	0.5313 7885	0.4538 3482	190
191	0.6207 0204	0.5733 4218	0.5296 1347	0.4519 5168	191
192	0.6191 5416	0.5716 7480	0.5278 5396	0.4500 7637	192
193	0.6176 1013	0.5700 1226	0.5261 0029	0.4482 0883	193
194	0.6160 6996	0.5683 5456	0.5243 5245	0.4463 4904	194
195	0.6145 3362	0.5667 0168	0.5226 1041	0.4444 9697	195
196	0.6130 0112	0.5650 5361	0.5208 7417	0.4426 5258	196
197	0.6114 7244	0.5634 1033	0.5191 4369	0.4408 1585	197
198	0.6099 4757	0.5617 7183	0.5174 1896	0.4389 8674	198
199	0.6084 2650	0.5601 3809	0.5156 9996	0.4371 6522	199
200	0.6069 0923	0.5585 0911	0.5139 8667	0.4353 5125	200

Present Value of 1 at Compound Interest

$$v^n = (1+i)^{-n}$$

n	½%	7/12%	⅝%	⅔%	n
1	0.9950 2488	0.9942 0050	0.9937 8882	0.9933 7748	1
2	0.9900 7450	0.9884 3463	0.9876 1622	0.9867 9882	2
3	0.9851 4876	0.9827 0220	0.9814 8196	0.9802 6373	3
4	0.9802 4752	0.9770 0302	0.9753 8580	0.9737 7192	4
5	0.9753 7067	0.9713 3688	0.9693 2750	0.9673 2310	5
6	0.9705 1808	0.9657 0361	0.9633 0683	0.9609 1699	6
7	0.9656 8963	0.9601 0301	0.9573 2356	0.9545 5330	7
8	0.9608 8520	0.9545 3489	0.9513 7745	0.9482 3175	8
9	0.9561 0468	0.9489 9907	0.9454 6827	0.9419 5207	9
10	0.9513 4794	0.9434 9534	0.9395 9580	0.9357 1398	10
11	0.9466 1489	0.9380 2354	0.9337 5980	0.9295 1720	11
12	0.9419 0534	0.9325 8347	0.9279 6005	0.9233 6145	12
13	0.9372 1924	0.9271 7495	0.9221 9632	0.9172 4648	13
14	0.9325 5646	0.9217 9780	0.9164 6840	0.9111 7200	14
15	0.9279 1688	0.9164 5183	0.9107 7604	0.9051 3775	15
16	0.9233 0037	0.9111 3686	0.9051 1905	0.8991 4346	16
17	0.9187 0684	0.9058 5272	0.8994 9719	0.8931 8886	17
18	0.9141 3616	0.9005 9923	0.8939 1025	0.8872 7371	18
19	0.9095 8822	0.8953 7620	0.8883 5802	0.8813 9772	19
20	0.9050 6290	0.8901 8346	0.8828 4027	0.8755 6065	20
21	0.9005 6010	0.8850 2084	0.8773 5679	0.8697 6224	21
22	0.8960 7971	0.8798 8816	0.8719 0736	0.8640 0222	22
23	0.8916 2160	0.8747 8525	0.8664 9179	0.8582 8035	23
24	0.8871 8567	0.8697 1193	0.8611 0985	0.8525 9638	24
25	0.8827 7181	0.8646 6803	0.8557 6135	0.8469 5004	25
26	0.8783 7991	0.8596 5339	0.8504 4606	0.8413 4110	26
27	0.8740 0986	0.8546 6782	0.8451 6378	0.8357 6931	27
28	0.8696 6155	0.8497 1118	0.8399 1432	0.8302 3441	28
29	0.8653 3488	0.8447 8327	0.8346 9746	0.8247 3617	29
30	0.8610 2973	0.8398 8395	0.8295 1300	0.8192 7434	30
31	0.8567 4600	0.8350 1304	0.8243 6075	0.8138 4868	31
32	0.8524 8358	0.8301 7038	0.8192 4050	0.8084 5896	32
33	0.8482 4237	0.8253 5581	0.8141 5205	0.8031 0492	33
34	0.8440 2226	0.8205 6915	0.8090 9520	0.7977 8635	34
35	0.8398 2314	0.8158 1026	0.8040 6976	0.7925 0299	35
36	0.8356 4492	0.8110 7897	0.7990 7554	0.7872 5463	36
37	0.8314 8748	0.8063 7511	0.7941 1234	0.7820 4102	37
38	0.8273 5073	0.8016 9854	0.7891 7997	0.7768 6194	38
39	0.8232 3455	0.7970 4908	0.7842 7823	0.7717 1716	39
40	0.8191 3886	0.7924 2660	0.7794 0693	0.7666 0645	40
41	0.8150 6354	0.7878 3092	0.7745 6590	0.7615 2959	41
42	0.8110 0850	0.7832 6189	0.7697 5493	0.7564 8635	42
43	0.8069 7363	0.7787 1936	0.7649 7384	0.7514 7650	43
44	0.8029 5884	0.7742 0317	0.7602 2245	0.7464 9984	44
45	0.7989 6402	0.7697 1318	0.7555 0057	0.7415 5613	45
46	0.7949 8907	0.7652 4923	0.7508 0802	0.7366 4516	46
47	0.7910 3390	0.7608 1116	0.7461 4462	0.7317 6672	47
48	0.7870 9841	0.7563 9884	0.7415 1018	0.7269 2058	48
49	0.7831 8250	0.7520 1210	0.7369 0453	0.7221 0654	49
50	0.7792 8607	0.7476 5080	0.7323 2748	0.7173 2437	50

$$v^n = (1+i)^{-n}$$

n	$\frac{1}{2}\%$	$\frac{7}{12}\%$	$\frac{5}{8}\%$	$\frac{2}{3}\%$	n
51	0.7754 0902	0.7433 1480	0.7277 7886	0.7125 7388	51
52	0.7715 5127	0.7390 0394	0.7232 5849	0.7078 5485	52
53	0.7677 1270	0.7347 1809	0.7187 6620	0.7031 6707	53
54	0.7638 9324	0.7304 5709	0.7143 0182	0.6985 1033	54
55	0.7600 9277	0.7262 2080	0.7098 6516	0.6938 8444	55
56	0.7563 1122	0.7220 0908	0.7054 5606	0.6892 8918	56
57	0.7525 4847	0.7178 2179	0.7010 7434	0.6847 2435	57
58	0.7488 0445	0.7136 5878	0.6967 1985	0.6801 8975	58
59	0.7450 7906	0.7095 1991	0.6923 9239	0.6756 8518	59
60	0.7413 7220	0.7054 0505	0.6880 9182	0.6712 1044	60
61	0.7376 8378	0.7013 1405	0.6838 1796	0.6667 6534	61
62	0.7340 1371	0.6972 4678	0.6795 7064	0.6623 4968	62
63	0.7303 6190	0.6932 0310	0.6753 4970	0.6579 6326	63
64	0.7267 2826	0.6891 8286	0.6711 5499	0.6536 0588	64
65	0.7231 1269	0.6851 8594	0.6669 8632	0.6492 7737	65
66	0.7195 1512	0.6812 1221	0.6628 4355	0.6449 7752	66
67	0.7159 3544	0.6772 6151	0.6587 2651	0.6407 0614	67
68	0.7123 7357	0.6733 3373	0.6546 3504	0.6364 6306	68
69	0.7088 2943	0.6694 2873	0.6505 6898	0.6322 4807	69
70	0.7053 0291	0.6655 4638	0.6465 2818	0.6280 6100	70
71	0.7017 9394	0.6616 8654	0.6425 1248	0.6239 0165	71
72	0.6983 0243	0.6578 4909	0.6385 2172	0.6197 6985	72
73	0.6948 2829	0.6540 3389	0.6345 5574	0.6156 6541	73
74	0.6913 7143	0.6502 4082	0.6306 1440	0.6115 8816	74
75	0.6879 3177	0.6464 6975	0.6266 9754	0.6075 3791	75
76	0.6845 0923	0.6427 2054	0.6228 0501	0.6035 1448	76
77	0.6811 0371	0.6389 9308	0.6189 3666	0.5995 1769	77
78	0.6777 1513	0.6352 8724	0.6150 9233	0.5955 4738	78
79	0.6743 4342	0.6316 0289	0.6112 7188	0.5916 0336	79
80	0.6709 8847	0.6279 3991	0.6074 7516	0.5876 8545	80
81	0.6676 5022	0.6242 9817	0.6037 0203	0.5837 9350	81
82	0.6643 2858	0.6206 7755	0.5999 5232	0.5799 2732	82
83	0.6610 2346	0.6170 7793	0.5962 2591	0.5760 8674	83
84	0.6577 3479	0.6134 9919	0.5925 2264	0.5722 7159	84
85	0.6544 6248	0.6099 4120	0.5888 4238	0.5684 8171	85
86	0.6512 0644	0.6064 0384	0.5851 8497	0.5647 1693	86
87	0.6479 6661	0.6028 8700	0.5815 5028	0.5609 7709	87
88	0.6447 4290	0.5993 9056	0.5779 3817	0.5572 6201	88
89	0.6415 3522	0.5959 1439	0.5743 4849	0.5535 7153	89
90	0.6383 4350	0.5924 5838	0.5707 8111	0.5499 0549	90
91	0.6351 6766	0.5890 2242	0.5672 3589	0.5462 6374	91
92	0.6320 0763	0.5856 0638	0.5637 1268	0.5426 4610	92
93	0.6288 6331	0.5822 1015	0.5602 1136	0.5390 5241	93
94	0.6257 3464	0.5788 3363	0.5567 3179	0.5354 8253	94
95	0.6226 2153	0.5754 7668	0.5532 7383	0.5319 3629	95
96	0.6195 2391	0.5721 3920	0.5498 3734	0.5284 1353	96
97	0.6164 4170	0.5688 2108	0.5464 2220	0.5249 1410	97
98	0.6133 7483	0.5655 2220	0.5430 2828	0.5214 3785	98
99	0.6103 2321	0.5622 4245	0.5396 5543	0.5179 8462	99
100	0.6072 8678	0.5589 8172	0.5363 0353	0.5145 8426	100

Present Value of 1 at Compound Interest

$$v^n = (1+i)^{-n}$$

n	$\frac{1}{2}\%$	$\frac{7}{12}\%$	$\frac{5}{8}\%$	$\frac{2}{3}\%$	n
101	0.6042 6545	0.5557 3991	0.5329 7246	0.5111 4660	101
102	0.6012 5915	0.5525 1689	0.5296 6207	0.5077 6152	102
103	0.5982 6781	0.5493 1257	0.5263 7225	0.5043 9886	103
104	0.5952 9136	0.5461 2683	0.5231 0285	0.5010 5847	104
105	0.5923 2971	0.5429 5957	0.5198 5377	0.4977 4020	105
106	0.5893 8279	0.5398 1067	0.5166 2486	0.4944 4391	106
107	0.5864 5054	0.5366 8004	0.5134 1601	0.4911 6945	107
108	0.5835 3288	0.5335 6756	0.5102 2709	0.4879 1667	108
109	0.5806 2973	0.5304 7313	0.5070 5798	0.4846 8543	109
110	0.5777 4102	0.5273 9665	0.5039 0855	0.4814 7559	110
111	0.5748 6669	0.5243 3801	0.5007 7868	0.4782 8701	111
112	0.5720 0666	0.5212 9711	0.4976 6826	0.4751 1955	112
113	0.5691 6085	0.5182 7385	0.4945 7715	0.4719 7306	113
114	0.5663 2921	0.5152 6812	0.4915 0524	0.4688 4741	114
115	0.5635 1165	0.5122 7982	0.4884 5242	0.4657 4246	115
116	0.5607 0811	0.5093 0885	0.4854 1855	0.4626 5808	116
117	0.5579 1852	0.5063 5512	0.4824 0353	0.4595 9411	117
118	0.5551 4280	0.5034 1851	0.4794 0723	0.4565 5044	118
119	0.5523 8090	0.5004 9893	0.4764 2955	0.4535 2693	119
120	0.5496 3273	0.4975 9629	0.4734 7036	0.4505 2344	120
121	0.5468 9824	0.4947 1047	0.4705 2955	0.4475 3984	121
122	0.5441 7736	0.4918 4140	0.4676 0700	0.4445 7600	122
123	0.5414 7001	0.4889 8896	0.4647 0261	0.4416 3179	123
124	0.5387 7612	0.4861 5307	0.4618 1626	0.4387 0708	124
125	0.5360 9565	0.4833 3363	0.4589 4784	0.4358 0173	125
126	0.5334 2850	0.4805 3053	0.4560 9723	0.4329 1563	126
127	0.5307 7463	0.4777 4369	0.4532 6433	0.4300 4864	127
128	0.5281 3396	0.4749 7302	0.4504 4902	0.4272 0063	128
129	0.5255 0643	0.4722 1841	0.4476 5120	0.4243 7149	129
130	0.5228 9197	0.4694 7978	0.4448 7076	0.4215 6108	130
131	0.5202 9052	0.4667 5703	0.4421 0759	0.4187 6929	131
132	0.5177 0201	0.4640 5007	0.4393 6158	0.4159 9598	132
133	0.5151 2637	0.4613 5881	0.4366 3262	0.4132 4104	133
134	0.5125 6356	0.4586 8316	0.4339 2062	0.4105 0434	134
135	0.5100 1349	0.4560 2303	0.4312 2546	0.4077 8577	135
136	0.5074 7611	0.4533 7832	0.4285 4704	0.4050 8520	136
137	0.5049 5135	0.4507 4895	0.4258 8526	0.4024 0252	137
138	0.5024 3916	0.4481 3483	0.4232 4001	0.3997 3760	138
139	0.4999 3946	0.4455 3587	0.4206 1119	0.3970 9033	139
140	0.4974 5220	0.4429 5198	0.4179 9870	0.3944 6059	140
141	0.4949 7731	0.4403 8308	0.4154 0243	0.3918 4827	141
142	0.4925 1474	0.4378 2908	0.4128 2229	0.3892 5325	142
143	0.4900 6442	0.4352 8989	0.4102 5818	0.3866 7541	143
144	0.4876 2628	0.4327 6542	0.4077 0999	0.3841 1465	144
145	0.4852 0028	0.4302 5560	0.4051 7763	0.3815 7084	145
146	0.4827 8635	0.4277 6033	0.4026 6100	0.3790 4389	146
147	0.4803 8443	0.4252 7953	0.4001 6000	0.3765 3366	147
148	0.4779 9446	0.4228 1312	0.3976 7453	0.3740 4006	148
149	0.4756 1637	0.4203 6102	0.3952 0451	0.3715 6297	159
150	0.4732 5012	0.4179 2313	0.3927 4982	0.3691 0229	150

$$v^n = (1+i)^{-n}$$

n	½%	7/12%	⅝%	⅔%	n
151	0.4708 9565	0.4154 9939	0.3903 1038	0.3666 5791	151
152	0.4685 5288	0.4130 8970	0.3878 8609	0.3642 2971	152
153	0.4662 2177	0.4106 9398	0.3854 7686	0.3618 1759	153
154	0.4639 0226	0.4083 1216	0.3830 8259	0.3594 2145	154
155	0.4615 9429	0.4059 4416	0.3807 0320	0.3570 4117	155
156	0.4592 9780	0.4035 8988	0.3783 3858	0.3546 7666	156
157	0.4570 1274	0.4012 4926	0.3759 8865	0.3523 2781	157
158	0.4547 3904	0.3989 2221	0.3736 5332	0.3499 9451	158
159	0.4524 7666	0.3966 0866	0.3713 3249	0.3476 7667	159
160	0.4502 2553	0.3943 0853	0.3690 2608	0.3453 7417	160
161	0.4479 8560	0.3920 2174	0.3667 3399	0.3430 8693	161
162	0.4457 5682	0.3897 4821	0.3644 5614	0.3408 1483	162
163	0.4435 3912	0.3874 8786	0.3621 9244	0.3385 5778	163
164	0.4413 3246	0.3852 4062	0.3599 4280	0.3363 1567	164
165	0.4391 3678	0.3830 0642	0.3577 0713	0.3340 8841	165
166	0.4369 5202	0.3807 8517	0.3554 8534	0.3318 7591	166
167	0.4347 7813	0.3785 7681	0.3532 7736	0.3296 7805	167
168	0.4326 1505	0.3763 8125	0.3510 8309	0.3274 9476	168
169	0.4304 6274	0.3741 9843	0.3489 0245	0.3253 2592	169
170	0.4283 2113	0.3720 2826	0.3467 3535	0.3231 7144	170
171	0.4261 9018	0.3698 7068	0.3445 8172	0.3210 3123	171
172	0.4240 6983	0.3677 2562	0.3424 4146	0.3189 0520	172
173	0.4219 6003	0.3655 9299	0.3403 1449	0.3167 9324	173
174	0.4198 6073	0.3634 7273	0.3382 0074	0.3146 9527	174
175	0.4177 7187	0.3613 6477	0.3361 0011	0.3126 1120	175
176	0.4156 9340	0.3592 6904	0.3340 1254	0.3105 4093	176
177	0.4136 2528	0.3571 8546	0.3319 3792	0.3084 8436	177
178	0.4115 6744	0.3551 1396	0.3298 7620	0.3064 4142	178
179	0.4095 1984	0.3530 5447	0.3278 2728	0.3044 1201	179
180	0.4074 8243	0.3510 0693	0.3257 9108	0.3023 9603	180
181	0.4054 5515	0.3489 7127	0.3237 6754	0.3003 9341	181
182	0.4034 3796	0.3469 4741	0.3217 5656	0.2984 0405	182
183	0.4014 3081	0.3449 3529	0.3197 5807	0.2964 2786	183
184	0.3994 3364	0.3429 3483	0.3177 7199	0.2944 6477	184
185	0.3974 4641	0.3409 4598	0.3157 9825	0.2925 1467	185
186	0.3954 6906	0.3389 6866	0.3138 3677	0.2905 7748	186
187	0.3935 0155	0.3370 0281	0.3118 8748	0.2886 5313	187
188	0.3915 4383	0.3350 4837	0.3099 5029	0.2867 4152	188
189	0.3895 9586	0.3331 0525	0.3080 2513	0.2848 4257	189
190	0.3876 5757	0.3311 7341	0.3061 1193	0.2829 5619	190
191	0.3857 2892	0.3292 5277	0.3042 1062	0.2810 8231	191
192	0.3838 0987	0.3273 4326	0.3023 2111	0.2792 2084	192
193	0.3819 0037	0.3254 4484	0.3004 4334	0.2773 7170	193
194	0.3800 0037	0.3235 5742	0.2985 7723	0.2755 3480	194
195	0.3781 0982	0.3216 8095	0.2967 2271	0.2737 1006	195
196	0.3762 2868	0.3198 1536	0.2948 7972	0.2718 9741	196
197	0.3743 5689	0.3179 6059	0.2930 4816	0.2700 9677	197
198	0.3724 9442	0.3161 1657	0.2912 2799	0.2683 0805	198
199	0.3706 4121	0.3142 8325	0.2894 1912	0.2665 3117	199
200	0.3687 9723	0.3124 6057	0.2876 2149	0.2647 6607	200

Present Value of 1 at Compound Interest

$$v^n = (1+i)^{-n}$$

n	$\frac{3}{4}\%$	$\frac{7}{8}\%$	1%	$1\frac{1}{8}\%$	n
1	0.9925 5583	0.9913 2590	0.9900 9901	0.9888 7515	1
2	0.9851 6708	0.9827 2704	0.9802 9605	0.9778 7407	2
3	0.9778 3333	0.9742 0276	0.9705 9015	0.9669 9537	3
4	0.9705 5417	0.9657 5243	0.9609 8034	0.9562 3770	4
5	0.9633 2920	0.9573 7539	0.9514 6569	0.9455 9970	5
6	0.9561 5802	0.9490 7102	0.9420 4524	0.9350 8005	6
7	0.9490 4022	0.9408 3868	0.9327 1805	0.9246 7743	7
8	0.941 7540	0.9326 7775	0.9234 8322	0.9143 9054	8
9	0.9349 6318	0.9245 8761	0.9143 3982	0.9042 1808	9
10	0.9280 0315	0.9165 6765	0.9052 8695	0.8941 5881	10
11	0.9210 9494	0.9086 1724	0.8963 2372	0.8842 1142	11
12	0.9142 3815	0.9007 3581	0.8874 4923	0.8743 7470	12
13	0.9074 3241	0.8929 2273	0.8786 6260	0.8646 4742	13
14	0.9006 7733	0.8851 7743	0.8699 6297	0.8550 2835	14
15	0.8939 7254	0.8774 9931	0.8613 4947	0.8455 1629	15
16	0.8873 1766	0.8698 8779	0.8528 2126	0.8361 1005	16
17	0.8807 1231	0.8623 4230	0.8443 7749	0.8268 0846	17
18	0.8741 5614	0.8548 6225	0.8360 1731	0.8176 1034	18
19	0.8676 4878	0.8474 4709	0.8277 3992	0.8085 1455	19
20	0.8611 8985	0.8400 9624	0.8195 4447	0.7995 1995	20
21	0.8547 7901	0.8328 0917	0.8114 3017	0.7906 2542	21
22	0.8484 1589	0.8255 8530	0.8033 9621	0.7818 2983	22
23	0.8421 0014	0.8184 2409	0.7954 4179	0.7731 3210	23
24	0.8358 3140	0.8113 2499	0.7875 6613	0.7645 3112	24
25	0.8296 0933	0.8042 8748	0.7797 6844	0.7560 2583	25
26	0.8234 3358	0.7973 1101	0.7720 4796	0.7476 1516	26
27	0.8173 0380	0.7903 9505	0.7644 0392	0.7392 9806	27
28	0.8112 1966	0.7835 3908	0.7568 3557	0.7310 7348	28
29	0.8051 8080	0.7767 4258	0.7493 4215	0.7229 4040	29
30	0.7991 8690	0.7700 0504	0.7419 2292	0.7148 9780	30
31	0.7932 3762	0.7633 2594	0.7345 7715	0.7069 4467	31
32	0.7873 3262	0.7567 0477	0.7273 0411	0.6990 8002	32
33	0.7814 7158	0.7501 4104	0.7201 0307	0.6913 0287	33
34	0.7756 5418	0.7436 3424	0.7129 7334	0.6836 1223	34
35	0.7698 8008	0.7371 8388	0.7059 1420	0.6760 0715	35
36	0.7641 4896	0.7307 8947	0.6989 2495	0.6684 8667	36
37	0.7584 6051	0.7244 5053	0.6920 0490	0.6610 4986	37
38	0.7528 1440	0.7181 6657	0.6851 5337	0.6536 9578	38
39	0.7472 1032	0.7119 3712	0.6783 6967	0.6464 2352	39
40	0.7416 4796	0.7057 6171	0.6716 5314	0.6392 3216	40
41	0.7361 2701	0.6996 3986	0.6650 0311	0.6321 2080	41
42	0.7306 4716	0.6935 7111	0.6584 1892	0.6250 8855	42
43	0.7252 0809	0.6875 5500	0.6518 9992	0.6181 3454	43
44	0.7198 0952	0.6815 9108	0.6454 4546	0.6112 5789	44
45	0.7144 5114	0.6756 7889	0.6390 5492	0.6044 5774	45
46	0.7091 3264	0.6698 1798	0.6327 2764	0.5977 3324	46
47	0.7038 5374	0.6640 0792	0.6264 6301	0.5910 8355	47
48	0.6986 1414	0.6582 4824	0.6202 6041	0.5845 0784	48
49	0.6934 1353	0.6525 3853	0.6141 1921	0.5780 0528	49
50	0.6882 5165	0.6468 7835	0.6080 3882	0.5715 7506	50

$$v^n = (1+i)^{-n}$$

n	¾%	⅞%	1%	1⅛%	n
51	0.6831 2819	0.6412 6726	0.6020 1864	0.5652 1637	51
52	0.6780 4286	0.6357 0484	0.5960 5806	0.5589 2843	52
53	0.6729 9540	0.6301 9067	0.5901 5649	0.5527 1044	53
54	0.6679 8551	0.6247 2433	0.5843 1336	0.5465 6162	54
55	0.6630 1291	0.6193 0541	0.5785 2808	0.5404 8120	55
56	0.6580 7733	0.6139 3349	0.5728 0008	0.5344 6843	56
57	0.6531 7849	0.6086 0817	0.5671 2879	0.5285 2256	57
58	0.6483 1612	0.6033 2904	0.5615 1365	0.5226 4282	58
59	0.6434 8995	0.5980 9571	0.5559 5411	0.5168 2850	59
60	0.6386 9970	0.5929 0776	0.5504 4962	0.5110 7887	60
61	0.6339 4511	0.5877 6482	0.5449 9962	0.5053 9319	61
62	0.6292 2592	0.5826 6649	0.5396 0358	0.4997 7077	62
63	0.6245 4185	0.5776 1238	0.5342 6097	0.4942 1090	63
64	0.6198 9266	0.5726 0211	0.5289 7126	0.4887 1288	64
65	0.6152 7807	0.5676 3530	0.5237 3392	0.4832 7602	65
66	0.6106 9784	0.5627 1158	0.5185 4844	0.4778 9965	66
67	0.6061 5170	0.5578 3056	0.5134 1429	0.4725 8309	67
68	0.6016 3940	0.5529 9188	0.5083 3099	0.4673 2568	68
69	0.5971 6070	0.5481 9517	0.5032 9801	0.4621 2675	69
70	0.5927 1533	0.5434 4007	0.4983 1486	0.4569 8566	70
71	0.5883 0306	0.5387 2622	0.4933 8105	0.4519 0177	71
72	0.5839 2363	0.5340 5325	0.4884 9609	0.4468 7443	72
73	0.5795 7681	0.5294 2082	0.4836 5949	0.4419 0302	73
74	0.5752 6234	0.5248 2857	0.4788 7078	0.4369 8692	74
75	0.5709 7999	0.5202 7615	0.4741 2949	0.4321 2551	75
76	0.5667 2952	0.5157 6322	0.4694 3514	0.4273 1818	76
77	0.5625 1069	0.5112 8944	0.4647 8726	0.4225 6433	77
78	0.5583 2326	0.5068 5447	0.4601 8541	0.4178 6337	78
79	0.5541 6701	0.5024 5796	0.4556 2912	0.4132 1470	79
80	0.5500 4170	0.4980 9959	0.4511 1794	0.4086 1775	80
81	0.5459 4710	0.4937 7902	0.4466 5142	0.4040 7194	81
82	0.5418 8297	0.4894 9593	0.4422 2913	0.3995 7670	82
83	0.5378 4911	0.4852 4999	0.4378 5063	0.3951 3148	83
84	0.5338 4527	0.4810 4089	0.4335 1547	0.3907 3570	84
85	0.5298 7123	0.4768 6829	0.4292 2324	0.3863 8882	85
86	0.5259 2678	0.4727 3188	0.4249 7350	0.3820 9031	86
87	0.5220 1169	0.4686 3136	0.4207 6585	0.3778 3961	87
88	0.5181 2575	0.4645 6640	0.4165 9985	0.3736 3621	88
89	0.5142 6873	0.4605 3671	0.4124 7510	0.3694 7956	89
90	0.5104 4043	0.4565 4197	0.4083 9119	0.3653 6916	90
91	0.5066 4063	0.4525 8187	0.4043 4771	0.3613 0448	91
92	0.5028 6911	0.4486 5613	0.4003 4427	0.3572 8503	92
93	0.4991 2567	0.4447 6444	0.3963 8046	0.3533 1029	93
94	0.4954 1009	0.4409 0651	0.3924 5590	0.3493 7976	94
95	0.4917 2217	0.4370 8204	0.3885 7020	0.3454 9297	95
96	0.4880 6171	0.4332 9075	0.3847 2297	0.3416 4941	96
97	0.4844 2850	0.4295 3234	0.3809 1383	0.3378 4861	97
98	0.4808 2233	0.4258 0654	0.3771 4241	0.3340 9010	98
99	0.4772 4301	0.4221 1305	0.3734 0832	0.3303 7340	99
100	0.4736 9033	0.4184 5159	0.3697 1121	0.3266 9805	100

Present Value of 1 at Compound Interest

$$v^n = (1+i)^{-n}$$

n	$\frac{3}{4}\%$	$\frac{7}{8}\%$	1%	$1\frac{1}{8}\%$	n
101	0.4701 6410	0.4148 2190	0.3660 5071	0.3230 6358	101
102	0.4666 6412	0.4112 2370	0.3624 2644	0.3194 6955	102
103	0.4631 9019	0.4076 5670	0.3588 3806	0.3159 1550	103
104	0.4597 4213	0.4041 2064	0.3552 8521	0.3124 0099	104
105	0.4563 1973	0.4006 1526	0.3517 6753	0.3089 2558	105
106	0.4529 2281	0.3971 4028	0.3482 8469	0.3054 8883	106
107	0.4495 5117	0.3936 9545	0.3448 3632	0.3020 9031	107
108	0.4462 0464	0.3902 8049	0.3414 2210	0.2987 2960	108
109	0.4428 8302	0.3868 9516	0.3380 4168	0.2954 0628	109
110	0.4395 8612	0.3835 3919	0.3346 9474	0.2921 1993	110
111	0.4363 1377	0.3802 1233	0.3213 8093	0.2888 7014	111
112	0.4330 6577	0.3769 1433	0.3280 9993	0.2856 5651	112
113	0.4298 4196	0.3736 4494	0.3248 5141	0.2824 7862	113
114	0.4266 4124	0.3704 0391	0.3216 3506	0.2793 3609	114
115	0.4234 6615	0.3671 9099	0.3184 5056	0.2762 2852	115
116	0.4203 1379	0.3640 0593	0.3152 9758	0.2731 5552	116
117	0.4171 8491	0.3608 4851	0.3121 7582	0.2701 1671	117
118	0.4140 7931	0.3577 1847	0.3090 8497	0.2671 1170	118
119	0.4109 9683	0.3546 1559	0.3060 2473	0.2641 4013	119
120	0.4079 3730	0.3515 3961	0.3029 9478	0.2612 0161	120
121	0.4049 0055	0.3484 9032	0.2999 9483	0.2582 9578	121
122	0.4018 8640	0.3454 6748	0.2970 2459	0.2554 2228	122
123	0.3988 9469	0.3424 7086	0.2940 8375	0.2525 8075	123
124	0.3959 2525	0.3395 0024	0.2911 7203	0.2497 7082	124
125	0.3929 7792	0.3365 5538	0.2882 8914	0.2469 9216	125
126	0.3900 5252	0.3336 3606	0.2854 3479	0.2442 4441	126
127	0.3871 4891	0.3307 4207	0.2826 0870	0.2415 2723	127
128	0.3842 6691	0.3278 7318	0.2798 1060	0.2388 4028	128
129	0.3814 0636	0.3250 2917	0.2770 4019	0.2361 8322	129
130	0.3785 6711	0.3222 0984	0.2742 9722	0.2335 5572	130
131	0.3757 4899	0.3194 1496	0.2715 8141	0.2309 5744	131
132	0.3729 5185	0.3166 4432	0.2688 9248	0.2283 8808	132
133	0.3701 7553	0.3138 9771	0.2662 3018	0.2258 4730	133
134	0.3674 1988	0.3111 7493	0.2635 9424	0.2233 3478	134
135	0.3646 8475	0.3084 7577	0.2609 8439	0.2208 5021	135
136	0.3619 6997	0.3058 0002	0.2584 0039	0.2183 9329	136
137	0.3592 7541	0.3031 4748	0.2558 4197	0.2159 6370	137
138	0.3566 0090	0.3005 1795	0.2533 0888	0.2135 6114	138
139	0.3539 4630	0.2979 1122	0.2508 0087	0.2111 8530	139
140	0.3513 1147	0.2953 2711	0.2483 1770	0.2088 3590	140
141	0.3486 9625	0.2927 6541	0.2458 5911	0.2065 1263	141
142	0.3461 0049	0.2902 2594	0.2434 2486	0.2042 1521	142
143	0.3435 2406	0.2877 0849	0.2410 1471	0.2019 4335	143
144	0.3409 6681	0.2852 1288	0.2386 2843	0.1996 9676	144
145	0.3384 2860	0.2827 3891	0.2362 6577	0.1974 7516	145
146	0.3359 0928	0.2802 8640	0.2339 2650	0.1952 7828	146
147	0.3334 0871	0.2778 5517	0.2316 1040	0.1931 6584	147
148	0.3309 2676	0.2754 4503	0.2293 1723	0.1909 5757	148
149	0.3284 6329	0.2730 5579	0.2270 4676	0.1888 3320	149
150	0.3260 1815	0.2706 8728	0.2247 9877	0.1867 3245	150

$$v^n = (1+i)^{-n}$$

n	¾%	⅞%	1%	1⅛%	n
151	0.3235 9122	0.2683 3931	0.2225 7304	0.1846 5509	151
152	0.3211 8235	0.2660 1170	0.2203 6935	0.1826 0083	152
153	0.3187 9141	0.2637 0429	0.2181 8747	0.1805 6942	153
154	0.3164 1828	0.2614 1689	0.2160 2720	0.1785 6061	154
155	0.3140 6280	0.2591 4934	0.2138 8832	0.1765 7415	155
156	0.3117 2487	0.2569 0145	0.2117 7061	0.1746 0979	156
157	0.3094 0434	0.2546 7306	0.2096 7387	0.1726 6729	157
158	0.3071 0108	0.2524 6400	0.2075 9789	0.1707 4639	158
159	0.3048 1496	0.2502 7410	0.2055 4247	0.1688 4686	159
160	0.3025 4587	0.2481 0320	0.2035 0739	0.1669 6847	160
161	0.3002 9367	0.2459 5113	0.2014 9247	0.1651 1097	161
162	0.2980 5823	0.2438 1772	0.1994 9750	0.1632 7413	162
163	0.2958 3944	0.2417 0282	0.1975 2227	0.1614 5774	163
164	0.2936 3716	0.2396 0627	0.1955 6661	0.1596 6154	164
165	0.2914 5127	0.2375 2790	0.1936 3030	0.1578 8533	165
166	0.2892 8166	0.2354 6756	0.1917 1317	0.1561 2888	166
167	0.2871 2820	0.2334 2509	0.1898 1502	0.1543 9197	167
168	0.2849 9077	0.2314 0033	0.1879 3566	0.1526 7439	168
169	0.2828 6925	0.2293 9314	0.1860 7492	0.1509 7591	169
170	0.2807 6352	0.2274 0336	0.1842 3259	0.1492 9632	170
171	0.2786 7347	0.2254 3084	0.1824 0850	0.1476 3543	171
172	0.2765 9898	0.2234 7543	0.1806 0248	0.1459 9300	172
173	0.2745 3993	0.2215 3699	0.1788 1434	0.1443 6885	173
174	0.2724 9621	0.2196 1535	0.1770 4390	0.1427 6277	174
175	0.2704 6770	0.2177 1039	0.1752 9099	0.1411 7456	175
176	0.2684 5429	0.2158 2194	0.1735 5543	0.1396 0401	176
177	0.2664 5587	0.2139 4988	0.1718 3706	0.1380 5094	177
178	0.2644 7233	0.2120 9406	0.1701 3571	0.1365 1515	178
179	0.2625 0355	0.2102 5433	0.1684 5119	0.1349 9644	179
180	0.2605 4943	0.2084 3057	0.1667 8336	0.1334 9462	180
181	0.2586 0986	0.2066 2262	0.1651 3204	0.1320 0951	181
182	0.2566 8472	0.2048 3035	0.1634 9707	0.1305 4093	182
183	0.2547 7392	0.2030 5363	0.1618 7829	0.1290 8868	183
184	0.2528 7734	0.2012 9233	0.1602 7553	0.1276 5259	184
185	0.2509 9488	0.1995 4630	0.1586 8864	0.1262 3247	185
186	0.2491 2643	0.1978 1541	0.1571 1747	0.1248 2816	186
187	0.2472 7189	0.1960 9954	0.1555 6185	0.1234 3946	187
188	0.2454 3116	0.1943 9855	0.1540 2163	0.1220 6622	188
189	0.2436 0413	0.1927 1232	0.1524 9667	0.1207 0825	189
190	0.2417 9070	0.1910 4071	0.1509 8680	0.1193 6539	190
191	0.2399 9077	0.1893 8361	0.1494 9188	0.1180 3747	191
192	0.2382 0423	0.1877 4087	0.1480 1176	0.1167 2432	192
193	0.2364 3100	0.1861 1239	0.1465 4630	0.1154 2578	193
194	0.2346 7097	0.1844 9803	0.1450 9535	0.1141 4169	194
195	0.2329 2404	0.1828 9768	0.1436 5876	0.1128 7188	195
196	0.2311 9011	0.1813 1121	0.1422 3640	0.1116 1620	196
197	0.2294 6909	0.1797 3849	0.1408 2811	0.1103 7448	197
198	0.2277 6089	0.1781 7942	0.1394 3378	0.1091 4658	198
199	0.2260 6540	0.1766 3388	0.1380 5324	0.1079 3234	199
200	0.2243 8253	0.1751 0174	0.1366 8638	0.1067 3161	200

Present Value of 1 at Compound Interest

$$v^n = (1+i)^{-n}$$

n	$1\frac{1}{4}\%$	$1\frac{3}{8}\%$	$1\frac{1}{2}\%$	$1\frac{3}{4}\%$	n
1	0.9876 5432	0.9864 3650	0.9852 2167	0.9828 0098	1
2	0.9754 6106	0.9730 5696	0.9706 6175	0.9658 9777	2
3	0.9634 1833	0.9598 5890	0.9563 1699	0.9492 8528	3
4	0.9515 2428	0.9468 3986	0.9421 8423	0.9329 5851	4
5	0.9397 7706	0.9339 9739	0.9282 6033	0.9169 1254	5
6	0.9281 7488	0.9213 2912	0.9145 4219	0.9011 4254	6
7	0.9167 1593	0.9088 3267	0.9010 2679	0.8856 4378	7
8	0.9053 9845	0.8965 0571	0.8877 1112	0.8704 1157	8
9	0.8942 2069	0.8843 4596	0.8745 9224	0.8554 4135	9
10	0.8831 8093	0.8723 5113	0.8616 6723	0.8407 2860	10
11	0.8722 7746	0.8605 1899	0.8489 3323	0.8262 6889	11
12	0.8615 0860	0.8488 4734	0.8363 8742	0.8120 5788	12
13	0.8508 7269	0.8373 3400	0.8240 2702	0.7980 9128	13
14	0.8403 6809	0.8259 7682	0.8118 4928	0.7843 6490	14
15	0.8299 9318	0.8147 7368	0.7998 5150	0.7708 7459	15
16	0.8197 4635	0.8037 2250	0.7880 3104	0.7576 1631	16
17	0.8096 2602	0.7928 2120	0.7763 8526	0.7445 8605	17
18	0.7996 3064	0.7820 6777	0.7649 1159	0.7317 7990	18
19	0.7897 5866	0.7714 6020	0.7536 0747	0.7191 9401	19
20	0.7800 0855	0.7609 9649	0.7424 7042	0.7068 2458	20
21	0.7703 7881	0.7506 7472	0.7314 9795	0.6946 6789	21
22	0.7608 6796	0.7404 9294	0.7206 8763	0.6827 2028	22
23	0.7514 7453	0.7304 4926	0.7100 3708	0.6709 7817	23
24	0.7421 9707	0.7205 4181	0.6995 4392	0.6594 3800	24
25	0.7330 3414	0.7107 6874	0.6892 0583	0.6480 9632	25
26	0.7239 8434	0.7011 2823	0.6790 2052	0.6369 4970	26
27	0.7150 4626	0.6916 1847	0.6689 8574	0.6259 9479	27
28	0.7062 1853	0.6822 3771	0.6590 9925	0.6152 2829	28
29	0.6974 9978	0.6729 8417	0.6493 5887	0.6046 4697	29
30	0.6888 8867	0.6638 5615	0.6397 6243	0.5942 4764	30
31	0.6803 8387	0.6548 5194	0.6303 0781	0.5840 2716	31
32	0.6719 8407	0.6459 6985	0.6209 9292	0.5739 8247	32
33	0.6636 8797	0.6372 0824	0.6118 1568	0.5641 1053	33
34	0.6554 9429	0.6285 6546	0.6027 7407	0.5544 0839	34
35	0.6474 0177	0.6200 3991	0.5938 6608	0.5448 7311	35
36	0.6394 0916	0.6116 3000	0.5850 8974	0.5355 0183	36
37	0.6315 1522	0.6033 3416	0.5764 4309	0.5262 9172	37
38	0.6237 1873	0.5951 5083	0.5679 2423	0.5172 4002	38
39	0.6160 1850	0.5870 7850	0.5595 3126	0.5083 4400	39
40	0.6084 1334	0.5791 1566	0.5512 6232	0.4996 0098	40
41	0.6009 0286	0.5712 6083	0.5431 1559	0.4910 0834	41
42	0.5934 8352	0.5635 1253	0.5350 8925	0.4825 6348	42
43	0.5861 5656	0.5558 6933	0.5271 8153	0.4742 6386	43
44	0.5789 2006	0.5483 2979	0.5193 9067	0.4661 0699	44
45	0.5717 7290	0.5408 9252	0.5117 1494	0.4580 9040	45
46	0.5647 1397	0.5335 5612	0.5041 5265	0.4502 1170	46
47	0.5577 4219	0.5263 1923	0.4967 0212	0.4424 6850	47
48	0.5508 5649	0.5191 8050	0.4893 6170	0.4348 5848	48
49	0.5440 5579	0.5121 3860	0.4821 2975	0.4273 7934	49
50	0.5373 3905	0.5051 9220	0.4750 0468	0.4200 2883	50

$$v^n = (1+i)^{-n}$$

n	$1\frac{1}{4}\%$	$1\frac{3}{8}\%$	$1\frac{1}{2}\%$	$1\frac{3}{4}\%$	n
51	0.5307 0524	0.4983 4003	0.4679 8491	0.4128 0475	51
52	0.5241 5332	0.4915 8079	0.4610 6887	0.4057 0492	52
53	0.5176 8229	0.4849 1323	0.4542 5505	0.3987 2719	53
54	0.5112 9115	0.4783 3611	0.4475 4192	0.3918 6947	54
55	0.5049 7892	0.4718 4820	0.4409 2800	0.3851 2970	55
56	0.4987 4461	0.4654 4829	0.4344 1182	0.3785 0585	56
57	0.4925 8727	0.4591 3518	0.4279 9194	0.3719 9592	57
58	0.4865 0594	0.4529 0770	0.4216 6694	0.3655 9796	58
59	0.4804 9970	0.4467 6468	0.4154 3541	0.3593 1003	59
60	0.4745 6760	0.4407 0499	0.4092 9597	0.3531 3025	60
61	0.4687 0874	0.4347 2749	0.4032 4726	0.3470 5676	61
62	0.4629 2222	0.4288 3106	0.3972 8794	0.3410 8772	62
63	0.4572 0713	0.4230 1461	0.3914 1669	0.3352 2135	63
64	0.4515 6259	0.4172 7705	0.3856 3221	0.3294 5587	64
65	0.4459 8775	0.4116 1731	0.3799 3321	0.3237 8956	65
66	0.4404 8173	0.4060 3434	0.3743 1843	0.3182 2069	66
67	0.4350 4368	0.4005 2709	0.3687 8663	0.3127 4761	67
68	0.4296 7277	0.3950 9454	0.3633 3658	0.3073 6866	68
69	0.4243 6817	0.3897 3568	0.3579 6708	0.3020 8222	69
70	0.4191 2905	0.3844 4949	0.3526 7692	0.2968 8670	70
71	0.4139 5462	0.3792 3501	0.3474 6495	0.2917 8054	71
72	0.4088 4407	0.3740 9126	0.3423 3000	0.2867 6221	72
73	0.4037 9661	0.3690 1727	0.3372 7093	0.2818 3018	73
74	0.3988 1147	0.3640 1210	0.3322 8663	0.2769 8298	74
75	0.3938 8787	0.3590 7483	0.3273 7599	0.2722 1914	75
76	0.3890 2506	0.3542 0451	0.3225 3793	0.2675 3724	76
77	0.3842 2228	0.3494 0026	0.3177 7136	0.2629 3586	77
78	0.3794 7879	0.3446 6117	0.3130 7523	0.2584 1362	78
79	0.3747 9387	0.3399 8636	0.3084 4850	0.2539 6916	79
80	0.3701 6679	0.3353 7495	0.3038 9015	0.2496 0114	80
81	0.3655 9683	0.3308 2609	0.2993 9916	0.2453 0825	81
82	0.3610 8329	0.3263 3893	0.2949 7454	0.2410 8919	82
83	0.3566 2547	0.3219 1263	0.2906 1531	0.2369 4269	83
84	0.3522 2268	0.3175 4637	0.2863 2050	0.2328 6751	84
85	0.3478 7426	0.3132 3933	0.2820 8917	0.2288 6242	85
86	0.3435 7951	0.3089 9071	0.2779 2036	0.2249 2621	86
87	0.3393 3779	0.3047 9971	0.2738 1316	0.2210 5770	87
88	0.3351 4843	0.3006 6556	0.2697 6666	0.2172 5572	88
89	0.3310 1080	0.2965 8748	0.2657 7997	0.2135 1914	89
90	0.3269 2425	0.2925 6472	0.2618 5218	0.2098 4682	90
91	0.3228 8814	0.2885 9652	0.2579 8245	0.2062 3766	91
92	0.3189 0187	0.2846 8214	0.2541 6990	0.2026 9057	92
93	0.3149 6481	0.2808 2085	0.2504 1369	0.1992 0450	93
94	0.3110 7636	0.2770 1194	0.2467 1300	0.1957 7837	94
95	0.3072 3591	0.2732 5468	0.2430 6699	0.1924 1118	95
96	0.3034 4287	0.2695 4839	0.2394 7487	0.1891 0190	96
97	0.2996 9666	0.2658 9237	0.2359 3583	0.1858 4953	97
98	0.2959 9670	0.2622 8594	0.2324 4909	0.1826 5310	98
99	0.2923 4242	0.2587 2843	0.2290 1389	0.1795 1165	99
100	0.2887 3326	0.2552 1916	0.2256 2944	0.1764 2422	100

Present Value of 1 at Compound Interest

$$v^n = (1+i)^{-n}$$

n	2%	2¼%	2½%	2¾%	n
1	0.9803 9216	0.9779 9511	0.9756 0976	0.9732 3601	1
2	0.9611 6878	0.9564 7444	0.9518 1440	0.9471 8833	2
3	0.9423 2233	0.9354 2732	0.9285 9941	0.9218 3779	3
4	0.9238 4543	0.9148 4335	0.9059 5064	0.8971 6573	4
5	0.9057 3081	0.8947 1232	0.8838 5429	0.8731 5400	5
6	0.8879 7138	0.8750 2427	0.8622 9687	0.8497 8491	6
7	0.8705 6018	0.8557 6946	0.8412 6524	0.8270 4128	7
8	0.8534 9037	0.8369 3835	0.8207 4657	0.8049 0635	8
9	0.8367 5527	0.8185 2161	0.8007 2836	0.7833 6385	9
10	0.8203 4830	0.8005 1013	0.7811 9840	0.7623 9791	10
11	0.8042 6304	0.7828 9499	0.7621 4478	0.7419 9310	11
12	0.7884 9318	0.7656 6748	0.7435 5589	0.7221 3440	12
13	0.7730 3253	0.7488 1905	0.7254 2038	0.7028 0720	13
14	0.7578 7502	0.7323 4137	0.7077 2720	0.6839 9728	14
15	0.7430 1473	0.7162 2628	0.6904 6556	0.6656 9078	15
16	0.7284 4581	0.7004 6580	0.6736 2493	0.6478 7424	16
17	0.7141 6256	0.6850 5212	0.6571 9506	0.6305 3454	17
18	0.7001 5937	0.6699 7763	0.6411 6591	0.6136 5892	18
19	0.6864 3076	0.6552 3484	0.6255 2772	0.5972 3496	19
20	0.6729 7133	0.6408 1647	0.6102 7094	0.5812 5057	20
21	0.6597 7582	0.6267 1538	0.5953 8629	0.5656 9398	21
22	0.6468 3904	0.6129 2457	0.5808 6467	0.5505 5375	22
23	0.6341 5592	0.5994 3724	0.5666 9724	0.5358 1874	23
24	0.6217 2149	0.5862 4668	0.5528 7535	0.5214 7809	24
25	0.6095 3087	0.5733 4639	0.5393 9059	0.5075 2126	25
26	0.5975 7928	0.5607 2997	0.5262 3472	0.4939 3796	26
27	0.5858 6204	0.5483 9117	0.5133 9973	0.4807 1821	27
28	0.5743 7455	0.5363 2388	0.5008 7778	0.4678 5227	28
29	0.5631 1231	0.5245 2213	0.4886 6125	0.4553 3068	29
30	0.5520 7089	0.5129 8008	0.4767 4269	0.4431 4421	30
31	0.5412 4597	0.5016 9201	0.4651 1481	0.4312 8391	31
32	0.5306 3330	0.4906 5233	0.4537 7055	0.4197 4103	32
33	0.5202 2873	0.4798 5558	0.4427 0298	0.4085 0708	33
34	0.5100 2817	0.4692 9641	0.4319 0534	0.3975 7380	34
35	0.5000 2761	0.4589 6960	0.4213 7107	0.3869 3314	35
36	0.4902 2315	0.4488 7002	0.4110 9372	0.3765 7727	36
37	0.4806 1093	0.4389 9268	0.4010 6705	0.3664 9856	37
38	0.4711 8719	0.4293 3270	0.3912 8492	0.3566 8959	38
39	0.4619 4822	0.4198 8528	0.3817 4139	0.3471 4316	39
40	0.4528 9042	0.4106 4575	0.3724 3062	0.3378 5222	40
41	0.4440 1021	0.4016 0954	0.3633 4695	0.3288 0995	41
42	0.4353 0413	0.3927 7216	0.3544 8483	0.3200 0968	42
43	0.4267 6875	0.3841 2925	0.3458 3886	0.3114 4495	43
44	0.4184 0074	0.3756 7653	0.3374 0376	0.3031 0944	44
45	0.4101 9680	0.3674 0981	0.3291 7440	0.2949 9702	45
46	0.4021 5373	0.3593 2500	0.3211 4576	0.2871 0172	46
47	0.3942 6836	0.3514 1809	0.3133 1294	0.2794 1773	47
48	0.3865 3761	0.3436 8518	0.3056 7116	0.2719 3940	48
49	0.3789 5844	0.3361 2242	0.2982 1576	0.2646 6122	49
50	0.3715 2788	0.3287 2608	0.2909 4221	0.2575 7783	50

$$v^n = (1+i)^{-n}$$

n	2%	2¼%	2½%	2¾%	n
51	0.3642 4302	0.3214 9250	0.2838 4606	0.2506 8402	51
52	0.3571 0100	0.3144 1810	0.2769 2298	0.2439 7471	52
53	0.3500 9902	0.3074 9936	0.2701 6876	0.2374 4497	53
54	0.3432 3433	0.3007 3287	0.2635 7928	0.2310 9000	54
55	0.3365 0425	0.2941 1528	0.2571 5052	0.2249 0511	55
56	0.3299 0613	0.2876 4330	0.2508 7855	0.2188 8575	56
57	0.3234 3738	0.2813 1374	0.2447 5956	0.2130 2749	57
58	0.3170 9547	0.2751 2347	0.2387 8982	0.2073 2603	58
59	0.3108 7791	0.2690 6940	0.2329 6568	0.2017 7716	59
60	0.3047 8227	0.2631 4856	0.2272 8359	0.1963 7679	60
61	0.2988 0614	0.2573 5801	0.2217 4009	0.1911 2097	61
62	0.2929 4720	0.2516 9487	0.2163 3179	0.1860 0581	62
63	0.2872 0314	0.2461 5635	0.2110 5541	0.1810 2755	63
64	0.2815 7170	0.2407 3971	0.2059 0771	0.1761 8253	64
65	0.2760 5069	0.2354 4226	0.2008 8557	0.1714 6718	65
66	0.2706 3793	0.2302 6138	0.1959 8593	0.1668 7804	66
67	0.2653 3130	0.2251 9450	0.1912 0578	0.1624 1172	67
68	0.2601 2873	0.2202 3912	0.1865 4223	0.1580 6493	68
69	0.2550 2817	0.2153 9278	0.1819 9241	0.1538 3448	69
70	0.2500 2761	0.2106 5309	0.1775 5358	0.1497 1726	70
71	0.2451 2511	0.2060 1769	0.1732 2300	0.1457 1023	71
72	0.2403 1874	0.2014 8429	0.1689 9805	0.1418 1044	72
73	0.2356 0661	0.1970 5065	0.1648 7615	0.1380 1503	73
74	0.2309 8687	0.1927 1458	0.1608 5478	0.1343 2119	74
75	0.2264 5771	0.1884 7391	0.1569 3149	0.1307 2622	75
76	0.2220 1737	0.1843 2657	0.1531 0389	0.1272 2747	76
77	0.2176 6408	0.1802 7048	0.1493 6965	0.1238 2235	77
78	0.2133 9616	0.1763 0365	0.1457 2649	0.1205 0837	78
79	0.2092 1192	0.1724 2411	0.1421 7218	0.1172 8309	79
80	0.2051 0973	0.1686 2993	0.1387 0457	0.1141 4412	80
81	0.2010 8797	0.1649 1925	0.1353 2153	0.1110 8917	81
82	0.1971 4507	0.1612 9022	0.1320 2101	0.1081 1598	82
83	0.1932 7948	0.1577 4105	0.1288 0098	0.1052 2237	83
84	0.1894 8968	0.1542 6997	0.1256 5949	0.1024 0620	84
85	0.1857 7420	0.1508 7528	0.1225 9463	0.0996 6540	85
86	0.1821 3157	0.1475 5528	0.1196 0452	0.0969 9795	86
87	0.1785 6036	0.1443 0835	0.1166 8733	0.0944 0190	87
88	0.1750 5918	0.1411 3286	0.1138 4130	0.0918 7533	88
89	0.1716 2665	0.1380 2724	0.1110 6468	0.0894 1638	89
90	0.1682 6142	0.1349 8997	0.1083 5579	0.0870 2324	90
91	0.1649 6217	0.1320 1953	0.1057 1296	0.0846 9415	91
92	0.1617 2762	0.1291 1445	0.1031 3460	0.0824 2740	92
93	0.1585 5649	0.1262 7331	0.1006 1912	0.0802 2131	93
94	0.1554 4754	0.1234 9468	0.0981 6500	0.0780 7427	94
95	0.1523 9955	0.1207 7719	0.0957 7073	0.0759 8469	95
96	0.1494 1132	0.1181 1950	0.0934 3486	0.0739 5104	96
97	0.1464 8169	0.1155 2029	0.0911 5596	0.0719 7181	97
98	0.1436 0950	0.1129 7828	0.0889 3264	0.0700 4556	98
99	0.1407 9363	0.1104 9221	0.0867 6355	0.0681 7086	99
100	0.1380 3297	0.1080 6084	0.0846 4737	0.0663 4634	100

$$v^n = (1+i)^{-n}$$

n	3%	3½%	4%	4½%	n
1	0.9708 7379	0.9661 8357	0.9615 3846	0.9569 3780	1
2	0.9425 9591	0.9335 1070	0.9245 5621	0.9157 2995	2
3	0.9151 4166	0.9019 4271	0.8889 9636	0.8762 9660	3
4	0.8884 8705	0.8714 4223	0.8548 0419	0.8385 6134	4
5	0.8626 0878	0.8419 7317	0.8219 2711	0.8024 5105	5
6	0.8374 8426	0.8135 0064	0.7903 1453	0.7678 9574	6
7	0.8130 9151	0.7859 9096	0.7599 1781	0.7348 2846	7
8	0.7894 0923	0.7594 1156	0.7306 9021	0.7031 8513	8
9	0.7664 1673	0.7337 3097	0.7025 8674	0.6729 0443	9
10	0.7440 9391	0.7089 1881	0.6755 6417	0.6439 2768	10
11	0.7224 2128	0.6849 4571	0.6495 8093	0.6161 9874	11
12	0.7013 7988	0.6617 8330	0.6245 9705	0.5896 6386	12
13	0.6809 5134	0.6394 0415	0.6005 7409	0.5642 7164	13
14	0.6611 1781	0.6177 8179	0.5774 7508	0.5399 7286	14
15	0.6418 6195	0.5968 9062	0.5552 6450	0.5167 2044	15
16	0.6231 6694	0.5767 0591	0.5339 0818	0.4944 6932	16
17	0.6050 1645	0.5572 0378	0.5133 7325	0.4731 7639	17
18	0.5873 9461	0.5383 6114	0.4936 2812	0.4528 0037	18
19	0.5702 8603	0.5201 5569	0.4746 4242	0.4333 0179	19
20	0.5536 7575	0.5025 6588	0.4563 8695	0.4146 4286	20
21	0.5375 4928	0.4855 7090	0.4388 3360	0.3967 8743	21
22	0.5218 9250	0.4691 5063	0.4219 5539	0.3797 0089	22
23	0.5066 9175	0.4532 8563	0.4057 2633	0.3633 5013	23
24	0.4919 3374	0.4379 5713	0.3901 2147	0.3477 0347	24
25	0.4776 0557	0.4231 4699	0.3751 1680	0.3327 3060	25
26	0.4636 9473	0.4088 3767	0.3606 8923	0.3184 0248	26
27	0.4501 8906	0.3950 1224	0.3468 1657	0.3046 9137	27
28	0.4370 7675	0.3816 5434	0.3334 7747	0.2915 7069	28
29	0.4243 4636	0.3687 4815	0.3206 5141	0.2790 1502	29
30	0.4119 8676	0.3562 7841	0.3083 1867	0.2670 0002	30
31	0.3999 8715	0.3442 3035	0.2964 6026	0.2555 0241	31
32	0.3883 3703	0.3325 8971	0.2850 5794	0.2444 9991	32
33	0.3770 2625	0.3213 4271	0.2740 9417	0.2339 7121	33
34	0.3660 4490	0.3104 7605	0.2635 5209	0.2238 9589	34
35	0.3553 8340	0.2999 7686	0.2534 1547	0.2142 5444	35
36	0.3450 3243	0.2898 3272	0.2436 6872	0.2050 2817	36
37	0.3349 8294	0.2800 3161	0.2342 9685	0.1961 9921	37
38	0.3252 2615	0.2705 6194	0.2252 8543	0.1877 5044	38
39	0.3157 5355	0.2614 1250	0.2166 2061	0.1796 6549	39
40	0.3065 5684	0.2525 7247	0.2082 8904	0.1719 2870	40
41	0.2976 2800	0.2440 3137	0.2002 7793	0.1645 2507	41
42	0.2889 5922	0.2357 7910	0.1925 7493	0.1574 4026	42
43	0.2805 4294	0.2278 0590	0.1851 6820	0.1506 6054	43
44	0.2723 7178	0.2201 0231	0.1780 4635	0.1441 7276	44
45	0.2644 3862	0.2126 5924	0.1711 9841	0.1379 6437	45
46	0.2567 3653	0.2054 6787	0.1646 1386	0.1320 2332	46
47	0.2492 5876	0.1985 1968	0.1582 8256	0.1263 3810	47
48	0.2419 9880	0.1918 0645	0.1521 9476	0.1208 9771	48
49	0.2349 5029	0.1853 2024	0.1463 4112	0.1156 9158	49
50	0.2281 0708	0.1790 5337	0.1407 1262	0.1107 0965	50

$$v^n = (1+i)^{-n}$$

n	3%	3½%	4%	4½%	n
51	0.2214 6318	0.1729 9843	0.1353 0059	0.1059 4225	51
52	0.2150 1280	0.1671 4824	0.1300 9672	0.1013 8014	52
53	0.2087 5029	0.1614 9589	0.1250 9300	0.0970 1449	53
54	0.2026 7019	0.1560 3467	0.1202 8173	0.0928 3683	54
55	0.1967 6717	0.1507 5814	0.1156 5551	0.0888 3907	55
56	0.1910 3609	0.1456 6004	0.1112 0722	0.0850 1347	56
57	0.1854 7193	0.1407 3433	0.1069 3002	0.0813 5260	57
58	0.1800 6984	0.1359 7520	0.1028 1733	0.0778 4938	58
59	0.1748 2508	0.1313 7701	0.0988 6282	0.0744 9701	59
60	0.1697 3309	0.1269 3431	0.0950 6040	0.0712 8901	60
61	0.1647 8941	0.1226 4184	0.0914 0423	0.0682 1915	61
62	0.1599 8972	0.1184 9453	0.0878 8868	0.0652 8148	62
63	0.1553 2982	0.1144 8747	0.0845 0835	0.0624 7032	63
64	0.1508 0565	0.1106 1591	0.0812 5803	0.0597 8021	64
65	0.1464 1325	0.1068 7528	0.0781 3272	0.0572 0594	65
66	0.1421 4879	0.1032 6114	0.0751 2762	0.0547 4253	66
67	0.1380 0853	0.0997 6922	0.0722 3809	0.0523 8519	67
68	0.1339 8887	0.0963 9538	0.0694 5970	0.0501 2937	68
69	0.1300 8628	0.0931 3563	0.0667 8818	0.0479 7069	69
70	0.1262 9736	0.0899 8612	0.0642 1940	0.0459 0497	70
71	0.1226 1880	0.0869 4311	0.0617 4942	0.0439 2820	71
72	0.1190 4737	0.0840 0300	0.0593 7445	0.0420 3655	72
73	0.1155 7998	0.0811 6232	0.0570 9081	0.0402 2637	73
74	0.1122 1357	0.0784 1770	0.0548 9501	0.0384 9413	74
75	0.1089 4521	0.0757 6590	0.0527 8367	0.0368 3649	75
76	0.1057 7205	0.0732 0376	0.0507 5353	0.0352 5023	76
77	0.1026 9131	0.0707 2827	0.0488 0147	0.0337 3228	77
78	0.0997 0030	0.0683 3650	0.0469 2449	0.0322 7969	78
79	0.0967 9641	0.0660 2560	0.0451 1970	0.0308 8965	79
80	0.0939 7710	0.0637 9285	0.0433 8433	0.0295 5948	80
81	0.0912 3990	0.0616 3561	0.0417 1570	0.0282 8658	81
82	0.0885 8243	0.0595 5131	0.0401 1125	0.0270 6850	82
83	0.0860 0236	0.0575 3750	0.0385 6851	0.0259 0287	83
84	0.0834 9743	0.0555 9178	0.0370 8510	0.0247 8744	84
85	0.0810 6547	0.0537 1187	0.0356 5875	0.0237 2003	85
86	0.0787 0434	0.0518 9553	0.0342 8726	0.0226 9860	86
87	0.0764 1198	0.0501 4060	0.0329 6852	0.0217 2115	87
88	0.0741 8639	0.0484 4503	0.0317 0050	0.0207 8579	88
89	0.0720 2562	0.0468 0679	0.0304 8125	0.0198 9070	89
90	0.0699 2779	0.0452 2395	0.0293 0890	0.0190 3417	90
91	0.0678 9105	0.0436 9464	0.0281 8163	0.0182 1451	91
92	0.0659 1364	0.0422 1704	0.0270 9772	0.0174 3016	92
93	0.0639 9383	0.0407 8941	0.0260 5550	0.0166 7958	93
94	0.0621 2993	0.0394 1006	0.0250 5337	0.0159 6132	94
95	0.0603 2032	0.0380 7735	0.0240 8978	0.0152 7399	95
96	0.0585 6342	0.0367 8971	0.0231 6325	0.0146 1626	96
97	0.0568 5769	0.0355 4562	0.0222 7235	0.0139 8685	97
98	0.0552 0164	0.0343 4359	0.0214 1572	0.0133 8454	98
99	0.0535 9383	0.0331 8221	0.0205 9204	0.0128 0817	99
100	0.0520 3284	0.0320 6011	0.0198 0004	0.0122 5663	100

Present Value of 1 at Compound Interest

$$v^n = (1+i)^{-n}$$

n	5%	5½%	6%	6½%	n
1	0.9523 8095	0.9478 6730	0.9433 9623	0.9389 6714	1
2	0.9070 2948	0.8984 5242	0.8899 9644	0.8816 5928	2
3	0.8638 3760	0.8516 1366	0.8396 1928	0.8278 4909	3
4	0.8227 0247	0.8072 1674	0.7920 9366	0.7773 2309	4
5	0.7835 2617	0.7651 3435	0.7472 5817	0.7298 8084	5
6	0.7462 1540	0.7252 4583	0.7049 6054	0.6853 3412	6
7	0.7106 8133	0.6874 3681	0.6650 5711	0.6435 0621	7
8	0.6768 3936	0.6515 9887	0.6274 1237	0.6042 3119	8
9	0.6446 0892	0.6176 2926	0.5918 9846	0.5673 5323	9
10	0.6139 1325	0.5854 3058	0.5583 9478	0.5327 2604	10
11	0.5846 7929	0.5549 1050	0.5267 8753	0.5002 1224	11
12	0.5568 3742	0.5259 8152	0.4969 6936	0.4696 8285	12
13	0.5303 2135	0.4985 6068	0.4688 3902	0.4410 1676	13
14	0.5050 6795	0.4725 6937	0.4423 0096	0.4141 0025	14
15	0.4810 1710	0.4479 3305	0.4172 6506	0.3888 2652	15
16	0.4581 1152	0.4245 8109	0.3936 4628	0.3650 9533	16
17	0.4362 9669	0.4024 4653	0.3713 6442	0.3428 1251	17
18	0.4155 2065	0.3814 6590	0.3503 4379	0.3218 8969	18
19	0.3957 3396	0.3615 7906	0.3305 1301	0.3022 4384	19
20	0.3768 8948	0.3427 2896	0.3118 0473	0.2837 9703	20
21	0.3589 4236	0.3248 6158	0.2941 5540	0.2664 7608	21
22	0.3418 4987	0.3079 2567	0.2775 0510	0.2502 1228	22
23	0.3255 7131	0.2918 7267	0.2617 9726	0.2349 4111	23
24	0.3100 6791	0.2766 5656	0.2469 7855	0.2206 0198	24
25	0.2953 0277	0.2622 3370	0.2329 9863	0.2071 3801	25
26	0.2812 4073	0.2485 6275	0.2198 1003	0.1944 9579	26
27	0.2678 4832	0.2356 0450	0.2073 6795	0.1826 2515	27
28	0.2550 9364	0.2233 2181	0.1956 3014	0.1714 7902	28
29	0.2429 4632	0.2116 7944	0.1845 5674	0.1610 1316	29
30	0.2313 7745	0.2006 4402	0.1741 1013	0.1511 8607	30
31	0.2203 5947	0.1901 8390	0.1642 5484	0.1419 5875	31
32	0.2098 6617	0.1802 6910	0.1549 5740	0.1332 9460	32
33	0.1998 7254	0.1708 7119	0.1461 8622	0.1251 5925	33
34	0.1903 5480	0.1619 6321	0.1379 1153	0.1175 2042	34
35	0.1812 9029	0.1535 1963	0.1301 0522	0.1103 4781	35
36	0.1726 5741	0.1455 1624	0.1227 4077	0.1036 1297	36
37	0.1644 3563	0.1379 3008	0.1157 9318	0.0972 8917	37
38	0.1566 0536	0.1307 3941	0.1092 3885	0.0913 5134	38
39	0.1491 4797	0.1239 2362	0.1030 5552	0.0857 7590	39
40	0.1420 4568	0.1174 6314	0.0972 2219	0.0805 4075	40
41	0.1352 8160	0.1113 3947	0.0917 1905	0.0756 2512	41
42	0.1288 3962	0.1055 3504	0.0865 2740	0.0710 0950	42
43	0.1227 0440	0.1000 3322	0.0816 2962	0.0666 7559	43
44	0.1168 6133	0.0948 1822	0.0770 0908	0.0626 0619	44
45	0.1112 9651	0.0898 7509	0.0726 5007	0.0587 8515	45
46	0.1059 9668	0.0851 8965	0.0685 3781	0.0551 9733	46
47	0.1009 4921	0.0807 4849	0.0646 5831	0.0518 2848	47
48	0.0961 4211	0.0765 3885	0.0609 9840	0.0486 6524	48
49	0.0915 6391	0.0725 4867	0.0575 4566	0.0456 9506	49
50	0.0872 0373	0.0687 6652	0.0542 8836	0.0429 0616	50

$$v^n = (1+i)^{-n}$$

n	5%	5½%	6%	6½%	n
51	0.0830 5117	0.0651 8153	0.0512 1544	0.0402 8747	51
52	0.0790 9635	0.0617 8344	0.0483 1645	0.0378 2861	52
53	0.0753 2986	0.0585 6250	0.0455 8156	0.0355 1982	53
54	0.0717 4272	0.0555 0948	0.0430 0147	0.0333 5195	54
55	0.0683 2640	0.0526 1562	0.0405 6742	0.0313 1638	55
56	0.0650 7276	0.0498 7263	0.0382 7115	0.0294 0505	56
57	0.0619 7406	0.0472 7263	0.0361 0486	0.0276 1038	57
58	0.0590 2291	0.0448 0818	0.0340 6119	0.0259 2524	58
59	0.0562 1230	0.0424 7221	0.0321 3320	0.0243 4295	59
60	0.0535 3552	0.0402 5802	0.0303 1434	0.0228 5723	60
61	0.0509 8621	0.0381 5926	0.0285 9843	0.0214 6218	61
62	0.0485 5830	0.0361 6992	0.0269 7965	0.0201 5229	62
63	0.0462 4600	0.0342 8428	0.0254 5250	0.0189 2233	63
64	0.0440 4381	0.0324 9695	0.0240 1179	0.0177 6745	64
65	0.0419 4648	0.0308 0279	0.0226 5264	0.0166 8305	65
66	0.0399 4903	0.0291 9696	0.0213 7041	0.0156 6484	66
67	0.0380 4670	0.0276 7485	0.0201 6077	0.0147 0877	67
68	0.0362 3495	0.0262 3208	0.0190 1959	0.0138 1105	68
69	0.0345 0948	0.0248 6453	0.0179 4301	0.0129 6812	69
70	0.0328 6617	0.0235 6828	0.0169 2737	0.0121 7664	70
71	0.0313 0111	0.0223 3960	0.0159 6921	0.0114 3346	71
72	0.0298 1058	0.0211 7498	0.0150 6530	0.0107 3565	72
73	0.0283 9103	0.0200 7107	0.0142 1254	0.0100 8042	73
74	0.0270 3908	0.0190 2471	0.0134 0806	0.0094 6518	74
75	0.0257 5150	0.0180 3290	0.0126 4911	0.0088 8750	75
76	0.0245 2524	0.0170 9279	0.0119 3313	0.0083 4507	76
77	0.0233 5737	0.0162 0170	0.0112 5767	0.0078 3574	77
78	0.0222 4512	0.0153 5706	0.0106 2044	0.0073 5751	78
79	0.0211 8582	0.0145 5646	0.0100 1928	0.0069 0846	79
80	0.0201 7698	0.0137 9759	0.0094 5215	0.0064 8681	80
81	0.0192 1617	0.0130 7828	0.0089 1713	0.0060 9090	81
82	0.0183 0111	0.0123 9648	0.0084 1238	0.0057 1916	82
83	0.0174 2963	0.0117 5022	0.0079 3621	0.0053 7010	83
84	0.0165 9965	0.0111 3765	0.0074 8699	0.0050 4235	84
85	0.0158 0919	0.0105 5701	0.0070 6320	0.0047 3460	85
86	0.0150 5637	0.0100 0664	0.0066 6340	0.0044 4563	86
87	0.0143 3940	0.0094 8497	0.0062 8622	0.0041 7430	87
88	0.0136 5657	0.0089 9049	0.0059 3040	0.0039 1953	88
89	0.0130 0626	0.0085 2180	0.0055 9472	0.0036 8031	89
90	0.0123 8691	0.0080 7753	0.0052 7803	0.0034 5569	90
91	0.0117 9706	0.0076 5643	0.0049 7928	0.0032 4478	91
92	0.0112 3530	0.0072 5728	0.0046 9743	0.0030 4674	92
93	0.0107 0028	0.0068 7894	0.0044 3154	0.0028 6079	93
94	0.0101 9074	0.0065 2032	0.0041 8070	0.0026 8619	94
95	0.0097 0547	0.0061 8040	0.0039 4405	0.0025 2224	95
96	0.0092 4331	0.0058 5820	0.0037 2081	0.0023 6831	96
97	0.0088 0315	0.0055 5279	0.0035 1019	0.0022 2376	97
98	0.0083 8395	0.0052 6331	0.0033 1150	0.0020 8804	98
99	0.0079 8471	0.0049 8892	0.0031 2406	0.0019 6060	99
100	0.0076 0449	0.0047 2883	0.0029 4723	0.0018 4094	100

$$v^n = (1+i)^{-n}$$

n	7%	7½%	8%	8½%	n
1	0.9345 7944	0.9302 3256	0.9259 2593	0.9216 5899	1
2	0.8734 3873	0.8653 3261	0.8573 3882	0.8494 5529	2
3	0.8162 9788	0.8049 6057	0.7938 3224	0.7829 0810	3
4	0.7628 9521	0.7488 0053	0.7350 2985	0.7215 7428	4
5	0.7129 8618	0.6965 5863	0.6805 8320	0.6650 4542	5
6	0.6663 4222	0.6479 6152	0.6301 6963	0.6129 4509	6
7	0.6227 4974	0.6027 5490	0.5834 9040	0.5649 2635	7
8	0.5820 0910	0.5607 0223	0.5402 6888	0.5206 6945	8
9	0.5439 3374	0.5215 8347	0.5002 4897	0.4798 7968	9
10	0.5083 4929	0.4851 9393	0.4631 9349	0.4422 8542	10
11	0.4750 9280	0.4513 4319	0.4288 8286	0.4076 3633	11
12	0.4440 1196	0.4198 5413	0.3971 1376	0.3757 0168	12
13	0.4149 6445	0.3905 6198	0.3676 9792	0.3462 6883	13
14	0.3878 1724	0.3633 1347	0.3404 6104	0.3191 4178	14
15	0.3624 4602	0.3379 6602	0.3152 4170	0.2941 3989	15
16	0.3387 3460	0.3143 8699	0.2918 9047	0.2710 9667	16
17	0.3165 7439	0.2924 5302	0.2702 6895	0.2498 5869	17
18	0.2958 6392	0.2720 4932	0.2502 4903	0.2302 8450	18
19	0.2765 0832	0.2530 6913	0.2317 1206	0.2122 4378	19
20	0.2584 1900	0.2354 1315	0.2145 4821	0.1956 1639	20
21	0.2415 1309	0.2189 8897	0.1986 5575	0.1802 9160	21
22	0.2257 1317	0.2037 1067	0.1839 4051	0.1661 6738	22
23	0.2109 4688	0.1894 9830	0.1703 1528	0.1531 4965	23
24	0.1971 4662	0.1762 7749	0.1576 9934	0.1411 5176	24
25	0.1842 4918	0.1639 7906	0.1460 1790	0.1300 9378	25
26	0.1721 9549	0.1525 3866	0.1352 0176	0.1199 0210	26
27	0.1609 3037	0.1418 9643	0.1251 8682	0.1105 0885	27
28	0.1504 0221	0.1319 9668	0.1159 1372	0.1018 5148	28
29	0.1405 6282	0.1227 8761	0.1073 2752	0.0938 7233	29
30	0.1313 6712	0.1142 2103	0.0993 7733	0.0865 1828	30
31	0.1227 7301	0.1062 5212	0.0920 1605	0.0797 4035	31
32	0.1147 4113	0.0988 3918	0.0852 0005	0.0734 9341	32
33	0.1072 3470	0.0919 4343	0.0788 8893	0.0677 3586	33
34	0.1002 1934	0.0855 2877	0.0730 4531	0.0624 2936	34
35	0.0936 6294	0.0795 6164	0.0676 3454	0.0575 3858	35
36	0.0875 3546	0.0740 1083	0.0626 2458	0.0530 3095	36
37	0.0818 0884	0.0688 4729	0.0579 8572	0.0488 7645	37
38	0.0764 5686	0.0640 4399	0.0536 9048	0.0450 4742	38
39	0.0714 5501	0.0595 7580	0.0497 1341	0.0415 1836	39
40	0.0667 8038	0.0554 1935	0.0460 3093	0.0382 6577	40
41	0.0624 1157	0.0515 5288	0.0426 2123	0.0352 6799	41
42	0.0583 2857	0.0479 5617	0.0394 6411	0.0325 0506	42
43	0.0545 1268	0.0446 1039	0.0365 4084	0.0299 5858	43
44	0.0509 4643	0.0414 9804	0.0338 3411	0.0276 1160	44
45	0.0476 1349	0.0386 0283	0.0313 2788	0.0254 4848	45
46	0.0444 9859	0.0359 0961	0.0290 0730	0.0234 5482	46
47	0.0415 8747	0.0334 0428	0.0268 5861	0.0216 1734	47
48	0.0388 6679	0.0310 7375	0.0248 6908	0.0199 2382	48
49	0.0363 2410	0.0289 0582	0.0230 2693	0.0183 6297	49
50	0.0339 4776	0.0268 8913	0.0213 2123	0.0169 2439	50

$$v^n = (1+i)^{-n}$$

n	7%	7½%	8%	8½%	n
51	0.0317 2688	0.0250 1315	0.0197 4188	0.0155 9852	51
52	0.0296 5129	0.0232 6804	0.0182 7952	0.0143 7651	52
53	0.0277 1148	0.0216 4469	0.0169 2548	0.0132 5024	53
54	0.0258 9858	0.0201 3460	0.0156 7174	0.0122 1221	54
55	0.0242 0428	0.0187 2986	0.0145 1087	0.0112 5549	55
56	0.0226 2083	0.0174 2312	0.0134 3599	0.0103 7372	56
57	0.0211 4096	0.0162 0756	0.0124 4073	0.0095 6104	57
58	0.0197 5791	0.0150 7680	0.0115 1920	0.0088 1201	58
59	0.0184 6533	0.0140 2493	0.0106 6592	0.0081 2167	59
60	0.0172 5732	0.0130 4644	0.0098 7585	0.0074 8541	60
61	0.0161 2834	0.0121 3623	0.0091 4431	0.0068 9900	61
62	0.0150 7321	0.0112 8951	0.0084 6695	0.0063 5852	62
63	0.0140 8711	0.0105 0187	0.0078 3977	0.0058 6039	63
64	0.0131 6553	0.0097 6918	0.0072 5905	0.0054 0128	64
65	0.0123 0423	0.0090 8761	0.0067 2134	0.0049 7814	65
66	0.0114 9928	0.0084 5359	0.0062 2346	0.0045 8815	66
67	0.0107 4699	0.0078 6381	0.0057 6247	0.0042 2871	67
68	0.0100 4392	0.0073 1517	0.0053 3562	0.0038 9743	68
69	0.0093 8684	0.0068 0481	0.0049 4039	0.0035 9210	69
70	0.0087 7275	0.0063 3006	0.0045 7443	0.0033 1069	70
71	0.0081 9883	0.0058 8842	0.0042 3558	0.0030 5133	71
72	0.0076 6246	0.0054 7760	0.0039 2184	0.0028 1228	72
73	0.0071 6117	0.0050 9544	0.0036 3133	0.0025 9196	73
74	0.0066 9269	0.0047 3995	0.0033 6234	0.0023 8891	74
75	0.0062 5485	0.0044 0925	0.0031 1328	0.0022 0176	75
76	0.0058 4565	0.0041 0163	0.0028 8267	0.0020 2927	76
77	0.0054 6323	0.0038 1547	0.0026 6914	0.0018 7030	77
78	0.0051 0582	0.0035 4928	0.0024 7142	0.0017 2377	78
79	0.0047 7179	0.0033 0165	0.0022 8835	0.0015 8873	79
80	0.0044 5962	0.0030 7130	0.0021 1885	0.0014 6427	80
81	0.0041 6787	0.0028 5703	0.0019 6190	0.0013 4956	81
82	0.0038 9520	0.0026 5770	0.0018 1657	0.0012 4383	82
83	0.0036 4038	0.0024 7228	0.0016 8201	0.0011 4639	83
84	0.0034 0222	0.0022 9979	0.0015 5742	0.0010 5658	84
85	0.0031 7965	0.0021 3934	0.0014 4205	0.0009 7381	85
86	0.0029 7163	0.0019 9009	0.0013 3523	0.0008 9752	86
87	0.0027 7723	0.0018 5124	0.0012 3633	0.0008 2720	87
88	0.0025 9554	0.0017 2209	0.0011 4475	0.0007 6240	88
89	0.0024 2574	0.0016 0194	0.0010 5995	0.0007 0267	89
90	0.0022 6704	0.0014 9018	0.0009 8144	0.0006 4762	90
91	0.0021 1873	0.0013 8621	0.0009 0874	0.0005 9689	91
92	0.0019 8012	0.0012 8950	0.0008 4142	0.0005 5013	92
93	0.0018 5058	0.0011 9953	0.0007 7910	0.0005 0703	93
94	0.0017 2952	0.0011 1585	0.0007 2138	0.0004 6731	94
95	0.0016 1637	0.0010 3800	0.0006 6795	0.0004 3070	95
96	0.0015 1063	0.0009 6558	0.0006 1847	0.0003 9696	96
97	0.0014 1180	0.0008 9821	0.0005 7266	0.0003 6586	97
98	0.0013 1944	0.0008 3555	0.0005 3024	0.0003 3720	98
99	0.0012 3312	0.0007 7725	0.0004 9096	0.0003 1078	99
100	0.0011 5245	0.0007 2303	0.0004 5459	0.0002 8644	100

Amount of 1 at Compound Interest

$(1+i)^n$

n	$\frac{1}{4}\%$	$\frac{7}{24}\%$	$\frac{1}{3}\%$	$\frac{5}{12}\%$	n
1	1.0025 0000	1.0029 1667	1.0033 3333	1.0041 6667	1
2	1.0050 0625	1.0058 4184	1.0066 7778	1.0083 5069	2
3	1.0075 1877	1.0087 7555	1.0100 3337	1.0125 5216	3
4	1.0100 3756	1.0117 1781	1.0134 0015	1.0167 7112	4
5	1.0125 6266	1.0146 6865	1.0167 7815	1.0210 0767	5
6	1.0150 9406	1.0176 2810	1.0201 6741	1.0252 6187	6
7	1.0176 3180	1.0205 9618	1.0235 6797	1.0295 3379	7
8	1.0201 7588	1.0235 7292	1.0269 7986	1.0338 2352	8
9	1.0227 2632	1.0265 5834	1.0304 0313	1.0381 3111	9
10	1.0252 8313	1.0295 5247	1.0338 3780	1.0424 5666	10
11	1.0278 4634	1.0325 5533	1.0372 8393	1.0468 0023	11
12	1.0304 1596	1.0355 6695	1.0407 4154	1.0511 6190	12
13	1.0329 9200	1.0385 8736	1.0442 1068	1.0555 4174	13
14	1.0355 7448	1.0416 1657	1.0476 9138	1.0599 3983	14
15	1.0381 6341	1.0446 5462	1.0511 8369	1.0643 5625	15
16	1.0407 5882	1.0477 0153	1.0546 8763	1.0687 9106	16
17	1.0433 6072	1.0507 5732	1.0582 0326	1.0732 4436	17
18	1.0459 6912	1.0538 2203	1.0617 3060	1.0777 1621	18
19	1.0485 8404	1.0568 9568	1.0652 6971	1.0822 0670	19
20	1.0512 0550	1.0599 7829	1.0688 2060	1.0867 1589	20
21	1.0538 3352	1.0630 6990	1.0723 8334	1.0912 4387	21
22	1.0564 6810	1.0661 7052	1.0759 5795	1.0957 9072	22
23	1.0591 0927	1.0692 8018	1.0795 4448	1.1003 5652	23
24	1 0617 5704	1.0723 9891	1.0831 4296	1.1049 4134	24
25	1.0644 1144	1.0755 2674	1.0867 5344	1.1095 4526	25
26	1.0670 7247	1.0786 6370	1.0903 7595	1.1141 6836	26
27	1.0697 4015	1.0818 0980	1.0940 1053	1.1188 1073	27
28	1.0724 1450	1.0849 6508	1.0976 5724	1.1234 7244	28
29	1.0750 9553	1.0881 2956	1.1013 1609	1.1281 5358	29
30	1.0777 8327	1.0913 0327	1.1049 8715	1.1328 5422	30
31	1.0804 7773	1.0944 8624	1.1086 7044	1.1375 7444	31
32	1.0831 7892	1.0976 7849	1.1123 6601	1.1423 1434	32
33	1.0858 8687	1.1008 8005	1.1160 7389	1.1470 7398	33
34	1.0886 0159	1.1040 9095	1.1197 9414	1.1518 5346	34
35	1.0913 2309	1.1073 1122	1.1235 2679	1.1566 5284	35
36	1.0940 5140	1.1105 4088	1.1272 7187	1.1614 7223	36
37	1.0967 8653	1.1137 7995	1.1310 2945	1.1663 1170	37
38	1.0995 2850	1.1170 2848	1.1347 9955	1.1711 7133	38
39	1.1022 7732	1.1202 8648	1.1385 8221	1.1760 5121	39
40	1.1050 3301	1.1235 5398	1.1423 7748	1.1809 5142	40
41	1.1077 9559	1.1268 3101	1.1461 8541	1.1858 7206	41
42	1.1105 6508	1.1301 1760	1.1500 0603	1.1908 1319	42
43	1.1133 4149	1.1334 1378	1.1538 3938	1.1957 7491	43
44	1.1161 2485	1.1367 1957	1.1576 8551	1.2007 5731	44
45	1.1189 1516	1.1400 3500	1.1615 4446	1.2057 6046	45
46	1.1217 1245	1.1433 6010	1.1654 1628	1.2107 8446	46
47	1.1245 1673	1.1466 9490	1.1693 0100	1.2158 2940	47
48	1.1273 2802	1.1500 3943	1.1731 9867	1.2208 9536	48
49	1.1301 4634	1.1533 9371	1.1771 0933	1.2259 8242	49
50	1.1329 7171	1.1567 5778	1.1810 3303	1.2310 9068	50

$(1+i)^n$

n	$\frac{1}{4}\%$	$\frac{7}{24}\%$	$\frac{1}{3}\%$	$\frac{5}{12}\%$	n
51	1.1358 0414	1.1601 3165	1.1849 6981	1.2362 2002	51
52	1.1386 4365	1.1635 1537	1.1889 1971	1.2413 7114	52
53	1.1414 9026	1.1669 0896	1.1928 8277	1.2465 4352	53
54	1.1443 4398	1.1703 1244	1.1968 5905	1.2517 3745	54
55	1.1472 0484	1.1737 2585	1.2008 4858	1.2569 5302	55
56	1.1500 7285	1.1771 4922	1.2048 5141	1.2621 9033	56
57	1.1529 4804	1.1805 8257	1.2088 6758	1.2674 4946	57
58	1.1558 3041	1.1840 2594	1.2128 9714	1.2727 3050	58
59	1.1587 1998	1.1874 7935	1.2169 4013	1.2780 3354	59
60	1.1616 1678	1.1909 4283	1.2209 9659	1.2833 5868	60
61	1.1645 2082	1.1944 1641	1.2250 6658	1.2887 0601	61
62	1.1674 3213	1.1979 0013	1.2291 5014	1.2940 7561	62
63	1.1703 5071	1.2013 9400	1.2332 4730	1.2994 6760	63
64	1.1732 7658	1.2048 9807	1.2373 5813	1.3048 8204	64
65	1.1762 0977	1.2084 1235	1.2414 8266	1.3103 1905	65
66	1.1791 5030	1.2119 3689	1.2456 2093	1.3157 7872	66
67	1.1820 9817	1.2154 7171	1.2497 7300	1.3212 6113	67
68	1.1850 5342	1.2190 1683	1.2539 3891	1.3267 6638	68
69	1.1880 1605	1.2225 7230	1.2581 1871	1.3322 9458	69
70	1.1909 8609	1.2261 3813	1.2623 1244	1.3378 4580	70
71	1.1939 6356	1.2297 1437	1.2665 2015	1.3434 2016	71
72	1.1969 4847	1.2333 0104	1.2707 4188	1.3490 1774	72
73	1.1999 4084	1.2368 9816	1.2749 7769	1.3546 3865	73
74	1.2029 4069	1.2405 0578	1.2792 2761	1.3602 8298	74
75	1.2059 4804	1.2441 2393	1.2834 9170	1.3659 5082	75
76	1.2089 6291	1.2477 5262	1.2877 7001	1.3716 4229	76
77	1.2119 8532	1.2513 9190	1.2920 6258	1.3773 5746	77
78	1.2150 1528	1.2550 4179	1.2963 6945	1.3830 9645	78
79	1.2180 5282	1.2587 0233	1.3006 9068	1.3888 5935	79
80	1.2210 9795	1.2623 7355	1.3050 2632	1.3946 4627	80
81	1.2241 5070	1.2660 5547	1.3093 7641	1.4004 5729	81
82	1.2272 1108	1.2697 4813	1.3137 4099	1.4062 9253	82
83	1.2302 7910	1.2734 5156	1.3181 2013	1.4121 5209	83
84	1.2333 5480	1.2771 6580	1.3225 1386	1.4180 3605	84
85	1.2364 3819	1.2808 9086	1.3269 2224	1.4239 4454	85
86	1.2395 2928	1.2846 2680	1.3313 4532	1.4298 7764	86
87	1.2426 2811	1.2883 7362	1.3357 8314	1.4358 3546	87
88	1.2457 3468	1.2921 3138	1.3402 3575	1.4418 1811	88
89	1.2488 4901	1.2959 0010	1.3447 0320	1.4478 2568	89
90	1.2519 7114	1.2996 7980	1.3491 8554	1.4538 5829	90
91	1.2551 0106	1.3034 7054	1.3536 8283	1.4599 1603	91
92	1.2582 3882	1.3072 7233	1.3581 9510	1.4659 9902	92
93	1.2613 8441	1.3110 8520	1.3627 2242	1.4721 0735	93
94	1.2645 3787	1.3149 0920	1.3672 6483	1.4782 4113	94
95	1.2676 9922	1.3187 4435	1.3718 2238	1.4844 0047	95
96	1.2708 6847	1.3225 9069	1.3763 9512	1.4905 8547	96
97	1.2740 4564	1.3264 4825	1.3809 8310	1.4967 9624	97
98	1.2772 3075	1.3303 1706	1.3855 8638	1.5030 3289	98
99	1.2804 2383	1.3341 9715	1.3902 0500	1.5092 9553	99
100	1.2836 2489	1.3380 8856	1.3948 3902	1.5155 8426	100

Amount of 1 at Compound Interest

$(1+i)^n$

n	$\frac{1}{4}\%$	$\frac{7}{24}\%$	$\frac{1}{3}\%$	$\frac{5}{12}\%$	n
101	1.2868 3395	1.3419 9131	1.3994 8848	1.5218 9919	101
102	1.2900 5104	1.3459 0546	1.4041 5344	1.5282 4044	102
103	1.2932 7616	1.3498 3101	1.4088 3395	1.5346 0811	103
104	1.2965 0935	1.3537 6802	1.4135 3007	1.5410 0231	104
105	1.2997 5063	1.3577 1651	1.4182 4183	1.5474 2315	105
106	1.3030 0000	1.3616 7652	1.4229 6931	1.5538 7075	106
107	1.3062 5750	1.3656 4807	1.4277 1254	1.5603 4521	107
108	1.3095 2315	1.3696 3121	1.4324 7158	1.5668 4665	108
109	1.3127 9696	1.3736 2597	1.4372 4649	1.5733 7518	109
110	1.3160 7895	1.3776 3238	1.4420 3731	1.5799 3091	110
111	1.3193 6915	1.3816 5047	1.4468 4410	1.5865 1395	111
112	1.3226 6757	1.3856 8029	1.4516 6691	1.5931 2443	112
113	1.3259 7424	1.3897 2186	1.4565 0580	1.5997 6245	113
114	1.3292 8917	1.3937 7521	1.4613 6082	1.6064 2812	114
115	1.3326 1240	1.3978 4039	1.4662 3202	1.6131 2157	115
116	1.3359 4393	1.4019 1742	1.4711 1946	1.6198 4291	116
117	1.3392 8379	1.4060 0635	1.4760 2320	1.6265 9226	117
118	1.3426 3200	1.4101 0720	1.4809 4327	1.6333 6973	118
119	1.3459 8858	1.4142 2001	1.4858 7979	1.6401 7543	119
120	1.3493 5355	1.4183 4482	1.4908 3268	1.6470 0950	120
121	1.3527 2693	1.4224 8166	1.4958 0212	1.6538 7204	121
122	1.3561 0875	1.4266 3057	1.5007 8813	1.6607 6317	122
123	1.3594 9902	1.4307 9157	1.5057 9076	1.6676 8302	123
124	1.3628 9777	1.4349 6471	1.5108 1006	1.6746 3170	124
125	1.3663 0501	1.4391 5003	1.5158 4609	1.6816 0933	125
126	1.3697 2077	1.4433 4755	1.5208 9892	1.6886 1603	126
127	1.3731 4508	1.4475 5731	1.5259 6858	1.6956 5193	127
128	1.3765 7794	1.4517 7935	1.5310 5514	1.7027 1715	128
129	1.3800 1938	1.4560 1371	1.5361 5866	1.7098 1181	129
130	1.3834 6943	1.4602 6042	1.5412 7919	1.7169 3602	130
131	1.3869 2811	1.4645 1951	1.5464 1678	1.7240 8992	131
132	1.3903 9543	1.4687 9103	1.5515 7151	1.7312 7363	132
133	1.3938 7142	1.4730 7500	1.5567 4341	1.7384 8727	133
134	1.3973 5609	1.4773 7147	1.5619 3256	1.7457 3097	134
135	1.4008 4948	1.4816 8047	1.5671 3900	1.7530 0485	135
136	1.4043 5161	1.4860 0204	1.5723 6279	1.7603 0903	136
137	1.4078 6249	1.4903 3621	1.5776 0400	1.7676 4365	137
138	1.4113 8214	1.4946 8302	1.5828 6268	1.7750 0884	138
139	1.4149 1060	1.4990 4252	1.5881 3889	1.7824 0471	139
140	1.4184 4787	1.5034 1472	1.5934 3269	1.7898 3139	140
141	1.4219 9399	1.5077 9968	1.5987 4413	1.7972 8902	141
142	1.4255 4898	1.5121 9743	1.6040 7328	1.8047 7773	142
143	1.4291 1285	1.5166 0801	1.6094 2019	1.8122 9763	143
144	1.4326 8563	1.5210 3145	1.6147 8492	1.8198 4887	144
145	1.4362 6735	1.5254 6779	1.6201 6754	1.8274 3158	145
146	1.4398 5802	1.5299 1707	1.6255 6810	1.8350 4588	146
147	1.4434 5766	1.5343 7933	1.6309 8666	1.8426 9190	147
148	1.4470 6631	1.5388 5460	1.6364 2328	1.8503 6978	148
149	1.4506 8397	1.5433 4293	1.6418 7802	1.8580 7966	149
150	1.4543 1068	1.5478 4434	1.6473 5095	1.8658 2166	150

$(1+i)^n$

n	$\frac{1}{4}\%$	$\frac{7}{24}\%$	$\frac{1}{3}\%$	$\frac{5}{12}\%$	n
151	1.4579 4646	1.5523 5889	1.6528 4212	1.8735 9591	151
152	1.4615 9132	1.5568 8660	1.6583 5160	1.8814 0256	152
153	1.4652 4530	1.5614 2752	1.6638 7943	1.8892 4174	153
154	1.4689 0842	1.5659 8169	1.6694 2570	1.8971 1358	154
155	1.4725 8069	1.5705 4913	1.6749 9045	1.9050 1822	155
156	1.4762 6214	1.5751 2990	1.6805 7375	1.9129 5580	156
157	1.4799 5279	1.5797 2403	1.6861 7566	1.9209 2645	157
158	1.4836 5268	1.5843 3156	1.6917 9625	1.9289 3031	158
159	1.4873 6181	1.5889 5253	1.6974 3557	1.9369 6752	159
160	1.4910 8021	1.5935 8697	1.7030 9369	1.9450 3821	160
161	1.4948 0791	1.5982 3493	1.7087 7067	1.9531 4254	161
162	1.4985 4493	1.6028 9645	1.7144 6657	1.9612 8063	162
163	1.5022 9129	1.6075 7157	1.7201 8146	1.9694 5264	163
164	1.5060 4702	1.6122 6032	1.7259 1540	1.9776 5869	164
165	1.5098 1214	1.6169 6274	1.7316 6845	1.9858 9893	165
166	1.5135 8667	1.6216 7888	1.7374 4068	1.9941 7351	166
167	1.5173 7064	1.6264 0878	1.7432 3215	2.0024 8257	167
168	1.5211 6406	1.6311 5247	1.7490 4292	2.0108 2625	168
169	1.5249 6697	1.6359 1000	1.7548 7306	2.0192 0469	169
170	1.5287 7939	1.6406 8140	1.7607 2264	2.0276 1804	170
171	1.5326 0134	1.6454 6672	1.7665 9172	2.0360 6645	171
172	1.5364 3284	1.6502 6600	1.7724 8035	2.0445 5006	172
173	1.5402 7393	1.6550 7928	1.7783 8862	2.0530 6902	173
174	1.5441 2461	1.6599 0659	1.7843 1658	2.0616 2347	174
175	1.5479 8492	1.6647 4799	1.7902 6431	2.0702 1357	175
176	1.5518 5488	1.6696 0350	1.7962 3185	2.0788 3946	176
177	1.5557 3452	1.6744 7318	1.8022 1929	2.0875 0129	177
178	1.5596 2386	1.6793 5706	1.8082 2669	2.0961 9921	178
179	1.5635 2292	1.6842 5518	1.8142 5411	2.1049 3338	179
180	1.5674 3172	1.6891 6760	1.8203 0163	2.1137 0393	180
181	1.5713 5030	1.6940 9433	1.8263 6930	2.1225 1103	181
182	1.5752 7868	1.6990 3544	1.8324 5720	2.1313 5483	182
183	1.5792 1688	1.7039 9096	1.8385 6539	2.1402 3547	183
184	1.5831 6492	1.7089 6094	1.8446 9394	2.1491 5312	184
185	1.5871 2283	1.7139 4541	1.8508 4292	2.1581 0793	185
186	1.5910 9064	1.7189 4441	1.8570 1240	2.1671 0004	186
187	1.5950 6836	1.7239 5800	1.8632 0244	2.1761 2963	187
188	1.5990 5604	1.7289 8621	1.8694 1311	2.1851 9683	188
189	1.6030 5368	1.7340 2909	1.8756 4449	2.1943 0182	189
190	1.6070 6131	1.7390 8667	1.8818 9664	2.2034 4474	190
191	1.6110 7896	1.7441 5901	1.8881 6963	2.2126 2576	191
192	1.6151 0666	1.7492 4614	1.8944 6352	2.2218 4504	192
193	1.6191 4443	1.7543 4811	1.9007 7840	2.2311 0272	193
194	1.6231 9229	1.7594 6496	1.9071 1433	2.2403 9899	194
195	1.6272 5027	1.7645 9673	1.9134 7138	2.2497 3398	195
196	1.6313 1839	1.7697 4347	1.9198 4962	2.2591 0787	196
197	1.6353 9669	1.7749 0522	1.9262 4912	2.2685 2082	197
198	1.6394 8518	1.7800 8203	1.9326 6995	2.2779 7299	198
199	1.6435 8390	1.7852 7393	1.9391 1218	2.2874 6455	199
200	1.6476 9285	1.7904 8098	1.9455 7589	2.2969 9565	200

$(1+i)^n$

n	½%	7/12%	⅝%	⅔%	n
1	1.0050 0000	1.0058 3333	1.0062 5000	1.0066 6667	1
2	1.0100 2500	1.0117 0069	1.0125 3906	1.0133 7778	2
3	1.0150 7513	1.0176 0228	1.0188 6743	1.0201 3363	3
4	1.0201 5050	1.0235 3830	1.0252 3535	1.0269 3452	4
5	1.0252 5125	1.0295 0894	1.0316 4307	1.0337 8075	5
6	1.0303 7751	1.0355 1440	1.0380 9084	1.0406 7262	6
7	1.0355 2940	1.0415 5490	1.0445 7891	1.0476 1044	7
8	1.0407 0704	1.0476 3064	1.0511 0753	1.0545 9451	8
9	1.0459 1058	1.0537 4182	1.0576 7695	1.0616 2514	9
10	1.0511 4013	1.0598 8865	1.0642 8743	1.0687 0264	10
11	1.0563 9583	1.0660 7133	1.0709 3923	1.0758 2732	11
12	1.0616 7781	1.0722 9008	1.0776 3260	1.0829 9951	12
13	1.0669 8620	1.0785 4511	1.0843 6780	1.0902 1950	13
14	1.0723 2113	1.0848 3662	1.0911 4510	1.0974 8763	14
15	1.0776 8274	1.0911 6483	1.0979 6476	1.1048 0422	15
16	1.0830 7115	1.0975 2996	1.1048 2704	1.1121 6958	16
17	1.0884 8651	1.1039 3222	1.1117 3221	1.1195 8404	17
18	1.0939 2894	1.1103 7182	1.1186 8053	1.1270 4794	18
19	1.0993 9858	1.1168 4899	1.1256 7229	1.1345 6159	19
20	1.1048 9558	1.1233 6395	1.1327 0774	1.1421 2533	20
21	1.1104 2006	1.1299 1690	1.1397 8716	1.1497 3950	21
22	1.1159 7216	1.1365 0808	1.1469 1083	1.1574 0443	22
23	1.1215 5202	1.1431 3771	1.1540 7902	1.1651 2046	23
24	1.1271 5978	1.1498 0602	1.1612 9202	1.1728 8793	24
25	1.1327 9558	1.1565 1322	1.1685 5009	1.1807 0718	25
26	1.1384 5955	1.1632 5955	1.1758 5353	1.1885 7857	26
27	1.1441 5185	1.1700 4523	1.1832 0262	1.1965 0242	27
28	1.1498 7261	1.1768 7049	1.1905 9763	1.2044 7911	28
29	1.1556 2197	1.1837 3557	1.1980 3887	1.2125 0897	29
30	1.1614 0008	1.1906 4069	1.2055 2661	1.2205 9236	30
31	1.1672 0708	1.1975 8610	1.2130 6115	1.2287 2964	31
32	1.1730 4312	1.2045 7202	1.2206 4278	1.2369 2117	32
33	1.1789 0833	1.2115 9869	1.2282 7180	1.2451 6731	33
34	1.1848 0288	1.2186 6634	1.2359 4850	1.2534 6843	34
35	1.1907 2689	1.2257 7523	1.2436 7318	1.2618 2489	35
36	1.1966 8052	1.2329 2559	1.2514 4614	1.2702 3705	36
37	1.2026 6393	1.2401 1765	1.2592 6767	1.2787 0530	37
38	1.2086 7725	1.2473 5167	1.2671 3810	1.2872 3000	38
39	1.2147 2063	1.2546 2789	1.2750 5771	1.2958 1153	39
40	1.2207 9424	1.2619 4655	1.2830 2682	1.3044 5028	40
41	1.2268 9821	1.2693 0791	1.2910 4574	1.3131 4661	41
42	1.2330 3270	1.2767 1220	1.2991 1477	1.3219 0092	42
43	1.2391 9786	1.2841 5969	1.3072 3424	1.3307 1360	43
44	1.2453 9385	1.2916 5062	1.3154 0446	1.3395 8502	44
45	1.2516 2082	1.2991 8525	1.3236 2573	1.3485 1559	45
46	1.2578 7892	1.3067 6383	1.3318 9839	1.3575 0569	46
47	1.2641 6832	1.3143 8662	1.3402 2276	1.3665 5573	47
48	1.2704 8916	1.3220 5388	1.3485 9915	1.3756 6610	48
49	1.2768 4161	1.3297 6586	1.3570 2790	1.3848 3721	49
50	1.2832 2581	1.3375 2283	1.3655 0932	1.3940 6946	50

$(1+i)^n$

n	$\frac{1}{2}\%$	$\frac{7}{12}\%$	$\frac{5}{8}\%$	$\frac{2}{3}\%$	n
51	1.2896 4194	1.3453 2504	1.3740 4375	1.4033 6325	51
52	1.2960 9015	1.3531 7277	1.3826 3153	1.4127 1901	52
53	1.3025 7060	1.3610 6628	1.3912 7297	1.4221 3713	53
54	1.3090 8346	1.3690 0583	1.3999 6843	1.4316 1805	54
55	1.3156 2887	1.3769 9170	1.4087 1823	1.4411 6217	55
56	1.3222 0702	1.3850 2415	1.4175 2272	1.4507 6992	56
57	1.3288 1805	1.3931 0346	1.4263 8224	1.4604 4172	57
58	1.3354 6214	1.4012 2990	1.4352 9713	1.4701 7799	58
59	1.3421 3946	1.4094 0374	1.4442 6773	1.4799 7918	59
60	1.3488 5015	1.4176 2526	1.4532 9441	1.4898 4571	60
61	1.3555 9440	1.4258 9474	1.4623 7750	1.4997 7801	51
62	1.3623 7238	1.4342 1246	1.4715 1736	1.5097 7653	62
63	1.3691 8424	1.4425 7870	1.4807 1434	1.5198 4171	63
64	1.3760 3016	1.4509 9374	1.4899 6881	1.5299 7399	64
65	1.3829 1031	1.4594 5787	1.4992 8111	1.5401 7381	65
66	1.3898 2486	1.4679 7138	1.5086 5162	1.5504 4164	66
67	1.3967 7399	1.4765 3454	1.5180 8069	1.5607 7792	67
68	1.4037 5785	1.4851 4766	1.5275 6869	1.5711 8310	68
69	1.4107 7664	1.4938 1102	1.5371 1600	1.5816 5766	69
70	1.4178 3053	1.5025 2492	1.5467 2297	1.5922 0204	70
71	1.4249 1968	1.5112 8965	1.5563 8999	1.6028 1672	71
72	1.4320 4428	1.5201 0550	1.5661 1743	1.6135 0217	72
73	1.4392 0450	1.5289 7279	1.5759 0566	1.6242 5885	73
74	1.4464 0052	1.5378 9179	1.5857 5507	1.6350 8724	74
75	1.4536 3252	1.5468 6283	1.5956 6604	1.6459 8782	75
76	1.4609 0069	1.5558 8620	1.6056 3896	1.6569 6107	76
77	1.4682 0519	1.5649 6220	1.6156 7420	1.6680 0748	77
78	1.4755 4622	1.5740 9115	1.6257 7216	1.6791 2753	78
79	1.4829 2395	1.5832 7334	1.6359 3324	1.6903 2172	79
80	1.4903 3857	1.5925 0910	1.6461 5782	1.7015 9053	80
81	1.4977 9026	1.6017 9874	1.6564 4631	1.7129 3446	81
82	1.5052 7921	1.6111 4257	1.6667 9910	1.7243 5403	82
83	1.5128 0561	1.6205 4090	1.6772 1659	1.7358 4972	83
84	1.5203 6964	1.6299 9405	1.6876 9920	1.7474 2205	84
85	1.5279 7148	1.6395 0235	1.6982 4732	1.7590 7153	85
86	1.5356 1134	1.6490 6612	1.7088 6136	1.7707 9868	86
87	1.5432 8940	1.6586 8567	1.7195 4175	1.7826 0400	87
88	1.5510 0585	1.6683 6134	1.7302 8888	1.7944 8803	88
89	1.5587 6087	1.6780 9344	1.7411 0319	1.8064 5128	89
90	1.5665 5468	1.6878 8232	1.7519 8508	1.8184 9429	90
91	1.5743 8745	1.6977 2830	1.7629 3499	1.8306 1758	91
92	1.5822 5939	1.7076 3172	1.7739 5333	1.8428 2170	92
93	1.5901 7069	1.7175 9290	1.7850 4054	1.8551 0718	93
94	1.5981 2154	1.7276 1219	1.7961 9704	1.8674 7456	94
95	1.6061 1215	1.7376 8993	1.8074 2328	1.8799 2439	95
96	1.6141 4271	1.7478 2646	1.8187 1967	1.8924 5722	96
97	1.6222 1342	1.7580 2211	1.8300 8667	1.9050 7360	97
98	1.6303 2449	1.7682 7724	1.8415 2471	1.9177 7409	98
99	1.6384 7611	1.7785 9219	1.8530 3424	1.9305 5925	99
100	1.6466 6849	1.7889 6731	1.8646 1570	1.9434 2965	100

$(1+i)^n$

n	$\frac{1}{2}\%$	$\frac{7}{12}\%$	$\frac{5}{8}\%$	$\frac{2}{3}\%$	n
101	1.6549 0183	1.7994 0295	1.8762 6955	1.9563 8585	101
102	1.6631 7634	1.8098 9947	1.8879 9624	1.9694 2842	102
103	1.6714 9223	1.8204 5722	1.8997 9621	1.9825 5794	103
104	1.6798 4969	1.8310 7655	1.9116 6994	1.9957 7499	104
105	1.6882 4894	1.8417 5783	1.9236 1788	2.0090 8016	105
106	1.6966 9018	1.8525 0142	1.9356 4049	2.0224 7403	106
107	1.7051 7363	1.8633 0768	1.9477 3824	2.0359 5719	107
108	1.7136 9950	1.8741 7697	1.9599 1161	2.0495 3024	108
109	1.7222 6800	1.8851 0967	1.9721 6105	2.0631 9377	109
110	1.7308 7934	1.8961 0614	1.9844 8706	2.0769 4840	110
111	1.7395 3373	1.9071 6676	1.9968 9010	2.0907 9472	111
112	1.7482 3140	1.9182 9190	2.0093 7067	2.1047 3335	112
113	1.7569 7256	1.9294 8194	2.0219 2923	2.1187 6491	113
114	1.7657 5742	1.9407 3725	2.0345 6629	2.1328 9000	114
115	1.7745 8621	1.9520 5822	2.0472 8233	2.1471 0927	115
116	1.7834 5914	1.9634 4522	2.0600 7785	2.1614 2333	116
117	1.7923 7644	1.9748 9865	2.0729 5333	2.1758 3282	117
118	1.8013 3832	1.9864 1890	2.0859 0929	2.1903 3837	118
119	1.8103 4501	1.9980 0634	2.0989 4622	2.2049 4063	119
120	1.8193 9673	2.0096 6138	2.1120 6464	2.2196 4023	120
121	1.8284 9372	2.0213 8440	2.1252 6504	2.2344 3784	121
122	1.8376 3619	2.0331 7581	2.1385 4795	2.2493 3409	122
123	1.8468 2437	2.0450 3600	2.1519 1387	2.2643 2965	123
124	1.8560 5849	2.0569 6538	2.1653 6333	2.2794 2518	124
125	1.8653 3878	2.0689 6434	2.1788 9685	2.2946 2135	125
126	1.8746 6548	2.0810 3330	2.1925 1496	2.3099 1882	126
127	1.8840 3880	2.0931 7266	2.2062 1818	2.3253 1828	127
128	1.8934 5900	2.1053 8284	2.2200 0704	2.3408 2040	128
129	1.9029 2629	2.1176 6424	2.2338 8209	2.3564 2587	129
130	1.9124 4092	2.1300 1728	2.2478 4385	2.3721 3538	130
131	1.9220 0313	2.1424 4238	2.2618 9287	2.3879 4962	131
132	1.9316 1314	2.1549 3996	2.2760 2970	2.4038 6928	132
133	1.9412 7121	2.1675 1044	2.2902 5489	2.4198 9507	133
134	1.9509 7757	2.1801 5425	2,3045 6898	2.4360 2771	134
135	1.9607 3245	2.1928 7182	2.3189 7254	2.4522 6789	135
136	1.9705 3612	2.2056 6357	2.3334 6612	2.4686 1635	136
137	1.9803 8880	2,2185 2994	2.3480 5028	2.4850 7379	137
138	1,9902 9074	2.2314 7137	2.3627 2559	2.5016 4095	138
139	2.0002 4219	2.2444 8828	2.3774 9263	2.5183 1855	139
140	2.0102 4340	2.2575 8113	2.3923 5196	2.5351 0734	140
141	2.0202 9462	2.2707 5036	2.4073 0416	2.5520 0806	141
142	2.0303 9609	2.2839 9640	2.4223 4981	2.5690 2145	142
143	2.0405 4808	2.2973 1971	2.4374 8950	2.5861 4826	143
144	2.0507 5082	2.3107 2074	2.4527 2380	2.6033 8924	144
145	2.0610 0457	2.3241 9995	2.4680 5333	2.6207 4517	145
146	2.0713 0959	2.3377 5778	2.4834 7866	2.6382 1681	146
147	2.0816 6614	2.3513 9470	2.4990 0040	2.6558 0492	147
148	2.0920 7447	2.3651 1117	2.5146 1916	2.6735 1028	148
149	2.1025 3484	2.3789 0765	2.5303 3553	2.6913 3369	149
150	2.1130 4752	2.3927 8461	2.5461 5012	2.7092 7591	150

$(1+i)^n$

n	½%	$\frac{7}{12}$%	⅝%	⅔%	n
151	2.1236 1276	2.4067 4252	2.5620 6356	2.7273 3775	151
152	2.1342 3082	2.4207 8186	2.5780 7646	2.7455 2000	152
153	2.1449 0197	2.4349 0308	2.5941 8944	2.7638 2347	153
154	2.1556 2648	2.4491 0668	2.6104 0312	2.7822 4896	154
155	2.1664 0462	2.4633 9314	2.6267 1814	2.8007 9729	155
156	2.1772 3664	2.4777 6293	2.6431 3513	2.8194 6927	156
157	2.1881 2282	2.4922 1655	2.6596 5472	2.8382 6573	157
158	2.1990 6344	2.5067 5448	2.6762 7756	2.8571 8750	158
159	2.2100 5875	2.5213 7722	2.6930 0430	2.8762 3542	159
160	2.2211 0905	2.5360 8525	2.7098 3558	2.8954 1032	160
161	2.2322 1459	2.5508 7908	2.7267 7205	2.9147 1306	161
162	2.2433 7566	2.5657 5921	2.7438 1437	2.9341 4448	162
163	2.2545 9254	2.5807 2614	2.7609 6321	2.9537 0544	163
164	2.2658 6551	2.5957 8037	2.7782 1923	2.9733 9681	164
165	2.2771 9483	2.6109 2242	2.7955 8310	2.9932 1945	165
166	2.2885 8081	2.6261 5280	2.8130 5550	3.0131 7425	166
167	2.3000 2371	2.6414 7203	2.8306 3710	3.0332 6208	167
168	2.3115 2383	2.6568 8062	2.8483 2858	3.0534 3883	168
169	2.3230 8145	2.6723 7909	2.8661 3063	3.0738 4038	169
170	2.3346 9686	2.6879 6796	2.8840 4395	3.0943 3265	170
171	2.3463 7034	2.7036 4778	2.9020 6922	3.1149 6154	171
172	2.3581 0219	2.7194 1906	2.9202 0715	3.1357 2795	172
173	2.3698 9270	2.7352 8233	2.9384 5845	3.1566 3280	173
174	2.3817 4217	2.7512 3815	2.9568 2381	3.1776 7702	174
175	2.3936 5088	2.7672 8704	2.9753 0396	3.1988 6153	175
176	2.4056 1913	2.7834 2954	2.9938 9961	3.2201 8728	176
177	2.4176 4723	2.7996 6622	3.0126 1149	3.2416 5519	177
178	2.4297 3546	2.8159 9760	3.0314 4031	3.2632 6623	178
179	2.4418 8414	2.8324 2426	3.0503 8681	3.2850 2134	179
180	2.4540 9356	2.8489 4673	3.0694 5173	3.3069 2148	180
181	2.4663 6403	2.8655 6559	3.0886 3580	3.3289 6762	181
182	2.4786 9585	2.8822 8139	3.1079 3977	3.3511 6074	182
183	2.4910 8933	2.8990 9469	3.1272 6440	3.3735 0181	183
184	2.5035 4478	2.9160 0608	3.1469 1043	3.3959 9182	184
185	2.5160 6250	2.9330 1612	3.1665 7862	3.4186 3177	185
186	2.5286 4281	2.9501 2538	3.1863 6973	3.4414 2265	186
187	2.5412 8603	2.6973 3444	3.2062 8454	3.4643 6546	187
188	2.5539 9246	2.9846 4389	3.2263 2382	3.4874 6123	188
189	2.5667 6242	3.0020 5431	3.2464 8834	3.5107 1097	189
190	2.5795 9623	3.0195 6630	3.2667 7890	3.5341 1571	190
191	2.5924 9421	3.0371 8043	3.2871 9627	3.5576 7649	191
192	2.6054 5668	3.0548 9732	3.3077 4124	3.5813 9433	192
193	2.6184 8397	3.0727 1755	3.3284 1462	3.6052 7029	193
194	2.6315 7639	3.0906 4174	3.3492 1722	3.6293 0543	194
195	2.6447 3427	3.1086 7048	3.3701 4982	3.6535 0080	195
196	2.6579 5794	3.1268 0440	3.3912 1326	3.6778 5747	196
197	2.6712 4773	3.1450 4409	3.4124 0834	3.7023 7652	197
198	2.6846 0397	3.1633 9018	3.4337 3589	3.7270 5903	198
199	2.6980 2699	3.1818 4329	3.4551 9674	3.7519 0609	199
200	2.7115 1712	3.2004 0404	3.4767 9172	3.7769 1880	200

Amount of 1 at Compound Interest

$$(1+i)^n$$

n	$\frac{3}{4}\%$	$\frac{7}{8}\%$	1%	$1\frac{1}{8}\%$	n
1	1.0075 0000	1.0087 5000	1.0100 0000	1.0112 5000	1
2	1.0150 5625	1.0175 7656	1.0201 0000	1.0226 2656	2
3	1.0226 6917	1.0264 8036	1.0303 0100	1.0341 3111	3
4	1.0303 3919	1.0354 6206	1.0406 0401	1.0457 6509	4
5	1.0380 6673	1.0445 2235	1.0510 1005	1.0575 2994	5
6	1.0458 5224	1.0536 6192	1.0615 2015	1.0694 2716	6
7	1.0536 9613	1.0628 8147	1.0721 3535	1.0814 5821	7
8	1.0615 9885	1.0721 8168	1.0828 5671	1.0936 2462	8
9	1.0695 6084	1.0815 6327	1.0936 8527	1.1059 2789	9
10	1.0775 8255	1.0910 2695	1.1046 2213	1.1183 6958	10
11	1.0856 6441	1.1005 7343	1.1156 6835	1.1309 5124	11
12	1.0938 0690	1.1102 0345	1.1268 2503	1.1436 7444	12
13	1.1020 1045	1.1199 1773	1.1380 9328	1.1565 4078	13
14	1.1102 7553	1.1297 1701	1.1494 7421	1.1695 5186	14
15	1.1186 0259	1.1396 0203	1.1609 6896	1.1827 0932	15
16	1.1269 9211	1.1495 7355	1.1725 7864	1.1960 1480	16
17	1.1354 4455	1.1596 3232	1.1843 0443	1.2094 6997	17
18	1.1439 6039	1.1697 7910	1.1961 4748	1.2230 7650	18
19	1.1525 4009	1.1800 1467	1.2081 0895	1.2368 3611	19
20	1.1611 8414	1.1903 3980	1.2201 9004	1.2507 5052	20
21	1.1698 9302	1.2007 5527	1.2323 9194	1.2648 2146	21
22	1.1786 6722	1.2112 6188	1.2447 1586	1.2790 5071	22
23	1.1875 0723	1.2218 6042	1.2571 6302	1.2934 4003	23
24	1.1964 1353	1.2325 5170	1.2697 3465	1.3079 9123	24
25	1.2053 8663	1.2433 3653	1.2824 3200	1.3227 0613	25
26	1.2144 2703	1.2542 1572	1.2952 5631	1.3375 8657	26
27	1.2235 3523	1.2651 9011	1.3082 0888	1.3526 3442	27
28	1.2327 1175	1.2762 6052	1.3212 9097	1.3678 5156	28
29	1.2419 5709	1.2874 2780	1.3345 0388	1.3832 3989	29
30	1.2512 7176	1.2986 9280	1.3478 4892	1.3988 0134	30
31	1.2606 5630	1.3100 5636	1.3613 2740	1.4145 3785	31
32	1.2701 1122	1.3215 1935	1.3749 4068	1.4304 5140	32
33	1.2796 3706	1.3330 8265	1.3886 9009	1.4465 4398	33
34	1.2892 3434	1.3447 4712	1.4025 7699	1.4628 1760	34
35	1.2989 0359	1.3565 1366	1.4166 0276	1.4792 7430	35
36	1.3086 4537	1.3683 8315	1.4307 6878	1.4959 1613	36
37	1.3184 6021	1.3803 5650	1.4450 7647	1.5127 4519	37
38	1.3283 4866	1.3924 3462	1.4595 2724	1.5297 6357	38
39	1.3383 1128	1.4046 1843	1.4741 2251	1.5469 7341	39
40	1.3483 4861	1.4169 0884	1.4888 6373	1.5643 7687	40
41	1.3584 6123	1.4293 0679	1.5037 5237	1.5819 7611	41
42	1.3686 4969	1.4418 1322	1.5187 8989	1.5997 7334	42
43	1.3789 1456	1.4544 2909	1.5339 7779	1.6177 7079	43
44	1.3892 5642	1.4671 5534	1.5493 1757	1.6359 7071	44
45	1.3996 7584	1.4799 9295	1.5648 1075	1.6543 7538	45
46	1.4101 7341	1.4929 4289	1.5804 5885	1.6729 8710	46
47	1.4207 4971	1.5060 0614	1.5962 6344	1.6918 0821	47
48	1.4314 0533	1.5191 8370	1.6122 2608	1.7108 4105	48
49	1.4421 4087	1.5324 7655	1.6283 4834	1.7300 8801	49
50	1.4529 5693	1.5458 8572	1.6446 3182	1.7495 5150	50

Amount of 1 at Compound Interest

$(1+i)^n$

n	$\frac{3}{4}\%$	$\frac{7}{8}\%$	1%	$1\frac{1}{8}\%$	n
51	1.4638 5411	1.5594 1222	1.6610 7814	1.7692 3395	51
52	1.4748 3301	1.5730 5708	1.6776 8892	1.7891 3784	52
53	1.4858 9426	1.5868 2133	1.6944 6581	1.8092 6564	53
54	1.4970 3847	1.6007 0602	1.7114 1047	1.8296 1988	54
55	1.5082 6626	1.6147 1219	1.7285 2457	1.8502 0310	55
56	1.5195 7825	1.6288 4093	1.7458 0982	1.8710 1788	56
57	1.5309 7509	1.6430 9328	1.7632 6792	1.8920 6684	57
58	1.5424 5740	1.6574 7035	1.7809 0060	1.9133 5259	58
59	1.5540 2583	1.6719 7322	1.7987 0960	1.9348 7780	59
60	1.5656 8103	1.6866 0298	1.8166 9670	1.9566 4518	60
61	1.5774 2363	1.7013 6076	1.8348 6367	1.9786 5744	61
62	1.5892 5431	1.7162 4766	1.8532 1230	2.0009 1733	62
63	1.6011 7372	1.7312 6483	1.8717 4443	2.0234 2765	63
64	1.6131 8252	1.7464 1340	1.8904 6187	2.0461 9121	64
65	1.6252 8139	1.7616 9452	1.9093 6649	2.0692 1087	65
66	1.6374 7100	1.7771 0934	1.9284 6015	2.0924 8949	66
67	1.6497 5203	1.7926 5905	1.9477 4475	2.1160 2999	67
68	1.6621 2517	1.8083 4482	1.9672 2220	2.1398 3533	68
69	1.6745 9111	1.8241 6783	1.9868 9442	2.1639 0848	69
70	1.6871 5055	1.8401 2930	2.0067 6337	2.1882 5245	70
71	1.6998 0418	1.8562 3043	2.0268 3100	2.2128 7029	71
72	1.7125 5271	1.8724 7245	2.0470 9931	2.2377 6508	72
73	1.7253 9685	1.8888 5658	2.0675 7031	2.2629 3994	73
74	1.7383 3733	1.9053 8408	2.0882 4601	2.2883 9801	74
75	1.7513 7486	1.9220 5619	2.1091 2847	2.3141 4249	75
76	1.7645 1017	1.9388 7418	2.1302 1975	2.3401 7659	76
77	1.7777 4400	1.9558 3933	2.1515 2195	2.3665 0358	77
78	1.7910 7708	1.9729 5292	2.1730 3717	2.3931 2675	78
79	1.8045 1015	1.9902 1626	2.1947 6754	2.4200 4942	79
80	1.8180 4398	2.0076 3066	2.2167 1522	2.4472 7498	80
81	1.8316 7931	2.0251 9742	2.2388 8237	2.4748 0682	81
82	1.8454 1691	2.0429 1790	2.2612 7119	2.5026 4840	82
83	1.8592 5753	2.0607 9343	2.2838 8390	2.5308 0319	83
84	1.8732 0196	2.0788 2537	2.3067 2274	2.5592 7473	84
85	1.8872 5098	2.0970 1510	2.3297 8997	2.5880 6657	85
86	1.9014 0536	2.1153 6398	2.3530 8787	2.6171 8232	86
87	1.9156 6590	2.1338 7341	2.3766 1875	2.6466 2562	87
88	1.9300 3339	2.1525 4481	2.4003 8494	2.6764 0016	88
89	1.9445 0865	2.1713 7957	2.4243 8879	2.7065 0966	89
90	1.9590 9246	2.1903 7914	2.4486 3267	2.7369 5789	90
91	1.9737 8565	2.2095 4496	2.4731 1900	2.7677 4867	91
92	1.9885 8905	2.2288 7848	2.4978 5019	2.7988 8584	92
93	2.0035 0346	2.2483 8117	2.5228 2869	2.8303 7331	93
94	2.0185 2974	2.2680 5450	2.5480 5698	2.8622 1501	94
95	2.0336 6871	2.2878 9998	2.5735 3755	2.8944 1492	95
96	2.0489 2123	2.3079 1910	2.5992 7293	2.9269 7709	96
97	2.0642 8814	2.3281 1340	2.6252 6565	2.9599 0559	97
98	2.0797 7030	2.3484 8439	2.6515 1831	2.9932 0452	98
99	2.0953 6858	2.3690 3363	2.6780 3349	3.0268 7807	99
100	2.1110 8384	2.3897 6267	2.7048 1383	3.0609 3045	100

Amount of 1 at Compound Interest

$(1+i)^n$

n	¾%	⅞%	1%	1⅛%	n
101	2.1269 1697	2.4106 7309	2.7318 6197	3.0953 6592	101
102	2.1428 6885	2.4317 6648	2.7591 8059	3.1301 8879	102
103	2.1589 4036	2.4530 4444	2.7867 7239	3.1654 0341	103
104	2.1751 3242	2.4745 0858	2.8146 4012	3.2010 1420	104
105	2.1914 4591	2 4961 6053	2.8427 8652	3.2370 2561	105
106	2.2078 8175	2.5180 0193	2.8712 1438	3.2734 4215	106
107	2.2244 4087	2.5400 3445	2.8999 2653	3.3102 6837	107
108	2.2411 2417	2.5622 5975	2.9289 2579	3.3475 0889	108
109	2.2579 3260	2.5846 7953	2.9582 1505	3.3851 6836	109
110	2.2748 6710	2.6072 9547	2.9877 9720	3.4232 5151	110
111	2.2919 2860	2.6301 0931	3.0176 7517	3.4617 6309	111
112	2.3091 1807	2.6531 2276	3.0478 5192	3.5007 0792	112
113	2.3264 3645	2.6763 3759	3.0783 3044	3.5400 9089	113
114	2.3438 8472	2.6997 5554	3.1091 1375	3.5799 1691	114
115	2.3614 6386	2.7233 7840	3.1402 0489	3.6201 9097	115
116	2.3791 7484	2.7472 0796	3.1716 0693	3.6609 1812	116
117	2.3970 1865	2.7712 4603	3.2033 2300	3.7021 0345	117
118	2.4149 9629	2.7954 9444	3.2353 5623	3.7437 5212	118
119	2.4331 0876	2.8199 5501	3.2677 0980	3.7858 6933	119
120	2.4513 5708	2.8446 2962	3.3003 8689	3.8284 6036	120
121	2.4697 4226	2.8695 2013	3.3333 9076	3.8715 3054	121
122	2.4882 6532	2.8946 2843	3.3667 2467	3.9150 8525	122
123	2.5069 2731	2.9199 5643	3.4003 9192	3.9591 2996	123
124	2.5257 2927	2.9455 0605	3.4343 9584	4.0036 7018	124
125	2.5446 7224	2.9712 7922	3.4687 3980	4.0487 1147	125
126	2.5637 5728	2.9972 7792	3.5034 2719	4.0942 5947	126
127	2.5829 8546	3.0235 0410	3.5384 6147	4.1403 1989	127
128	2.6023 5785	3.0499 5976	3.5738 4608	4.1868 9849	128
129	2.6218 7553	3.0766 4691	3.6095 8454	4.2340 0110	129
130	2.6415 3960	3.1035 6757	3.6456 8039	4.2816 3361	130
131	2.6613 5115	3.1307 2378	3.6821 3719	4.3298 0199	131
132	2.6813 1128	3.1581 1762	3.7189 5856	4.3785 1226	132
133	2.7014 2112	3.1857 5115	3.7561 4815	4.4277 7052	133
134	2.7216 8177	3.2136 2647	3.7937 0963	4.4775 8294	134
135	2.7420 9439	3.2417 4570	3.8316 4673	4.5279 5575	135
136	2.7626 6009	3.2701 1098	3.8699 6319	4.5788 9525	136
137	2.7833 8005	3.2987 2445	3.9086 6282	4.6304 0782	137
138	2.8042 5540	3.3275 8829	3.9477 4945	4.6824 9991	138
139	2.8252 8731	3.3567 0468	3.9872 2695	4.7351 7803	139
140	2.8464 7697	3.3860 7585	4.0270 9922	4.7884 4879	140
141	2.8678 2554	3.4157 0401	4.0673 7021	4.8423 1883	141
142	2.8893 3424	3.4455 9142	4.1080 4391	4.8967 9492	142
143	2.9110 0424	3.4757 4035	4.1491 2435	4.9518 8386	143
144	2.9328 3677	3.5061 5308	4.1906 1559	5.0075 9256	144
145	2.9548 3305	3.5368 3192	4.2325 2175	5.0639 2797	145
146	2.9769 9430	3.5677 7919	4.2748 4697	5.1208 9716	146
147	2.9993 2175	3.5989 9726	4.3175 9544	5.1785 0726	147
148	3.0218 1667	3.6304 8849	4.3607 7139	5.2367 6546	148
149	3.0444 8029	3.6622 5526	4.4043 7910	5.2956 7908	149
150	3.0673 1389	3.6943 0000	4.4484 2290	5.3552 5546	150

$(1+i)^n$

n	¾%	⅞%	1%	1⅛%	n
151	3.0903 1875	3.7266 2512	4.4929 0712	5.4155 0209	151
152	3.1134 9614	3.7592 3309	4.5378 3620	5.4764 2649	152
153	3.1368 4736	3.7921 2638	4.5832 1456	5.5380 3629	153
154	3.1603 7372	3.8253 0749	4.6290 4670	5.6003 3919	154
155	3.1840 7652	3.8587 7893	4.6753 3717	5.6633 4301	155
156	3.2079 5709	3.8925 4324	4.7220 9054	5.7270 5562	156
157	3.2320 1677	3.9266 0300	4.7693 1145	5.7914 8499	157
158	3.2562 5690	3.9609 6077	4.8170 0456	5.8566 3920	158
159	3.2806 7882	3.9956 1918	4.8651 7461	5.9225 2639	159
160	3.3052 8391	4.0305 8085	4.9138 2635	5.9891 5481	160
161	3.3300 7354	4.0658 4843	4.9629 6462	6.0565 3280	161
162	3.3550 4910	4.1014 2460	5.0125 9426	6.1246 6880	162
163	3.3802 1196	4.1373 1207	5.0627 2021	6.1935 7132	163
164	3.4055 6355	4.1735 1355	5.1133 4741	6.2632 4900	164
165	3.4311 0528	4.2100 3179	5.1644 8088	6.3337 1055	165
166	3.4568 3857	4.2468 6957	5.2161 2569	6.4049 6479	166
167	3.4827 6486	4.2840 2968	5.2682 8695	6.4770 2065	167
168	3.5088 8560	4.3215 1494	5.3209 6982	6.5498 8713	168
169	3.5352 0224	4.3593 2819	5.3741 7952	6.6235 7336	169
170	3.5617 1625	4.3974 7232	5.4279 2131	6.6980 8856	170
171	3.5884 2913	4.4359 5020	5.4822 0052	6.7734 4206	171
172	3.6153 4234	4.4747 6476	5.5370 2253	6.8496 4328	172
173	3.6424 5741	4.5139 1896	5.5923 9275	6.9267 0177	173
174	3.6697 7584	4.5534 1575	5.6483 1668	7.0046 2716	174
175	3.6972 9916	4.5932 5813	5.7047 9985	7.0834 2922	175
176	3.7250 2891	4.6334 4914	5.7618 4785	7.1631 1780	176
177	3.7529 6662	4.6739 9182	5.8194 6633	7.2437 0287	177
178	3.7811 1387	4.7148 8925	5.8776 6099	7.3251 9453	178
179	3.8094 7223	4.7561 4453	5.9364 3760	7.4076 0297	179
180	3.8380 4327	4.7977 6080	5.9958 0198	7.4909 3850	180
181	3.8668 2859	4.8397 4120	6.0557 6000	7.5752 1156	181
182	3.8958 2981	4.8820 8894	6.1163 1760	7.6604 3269	182
183	3.9250 4853	4.9248 0722	6.1774 8077	7.7466 1256	183
184	3.9544 8639	4.9678 9928	6.2392 5558	7.8337 6195	184
185	3.9841 4504	5.0113 6840	6.3016 4813	7.9218 9177	185
186	4.0140 2613	5.0552 1787	6.3646 6462	8.0110 1305	186
187	4.0441 3133	5.0994 5103	6.4283 1126	8.1011 3695	187
188	4.0744 6231	5.1440 7123	6.4925 9437	8.1922 7474	188
189	4.1050 2078	5.1890 8185	6.5575 2032	8.2844 3783	189
190	4.1358 0843	5.2344 8631	6.6230 9552	8.3776 3776	190
191	4.1668 2700	5.2802 8807	6.6893 2648	8.4718 8618	191
192	4.1980 7820	5.3264 9059	6.7562 1974	8.5671 9490	192
193	4.2295 6379	5.3730 9738	6.8237 8194	8.6635 7584	193
194	4.2612 8551	5.4201 1199	6.8920 1976	8.7610 4107	194
195	4.2932 4516	5.4675 3797	6.9609 3996	8.8596 0278	195
196	4.3254 4449	5.5153 7892	7.0305 4936	8.9592 7332	196
197	4.3578 8533	5.5636 3849	7.1008 5485	9.0600 6514	197
198	4.3905 6947	5.6123 2033	7.1718 6340	9.1619 9087	198
199	4.4234 9874	5.6614 2813	7.2435 8203	9.2650 6327	199
200	4.4566 7498	5.7109 6562	7.3160 1785	9.3692 9523	200

$(1+i)^n$

n	1¼%	1⅜%	1½%	1¾%	n
1	1.0125 0000	1.0137 5000	1.0150 0000	1.0175 0000	1
2	1.0251 5625	1.0276 8906	1.0302 2500	1.0353 0625	2
3	1.0379 7070	1.0418 1979	1.0456 7838	1.0534 2411	3
4	1.0509 4534	1.0561 4481	1.0613 6355	1.0718 5903	4
5	1.0640 8215	1.0706 6680	1.0772 8400	1.0906 1656	5
6	1.0773 8318	1.0853 8847	1.0934 4326	1.1097 0235	6
7	1.0908 5047	1.1003 1256	1.1098 4491	1.1291 2215	7
8	1.1044 8610	1.1154 4186	1.1264 9259	1.1488 8178	8
9	1.1182 9218	1.1307 7918	1.1433 8998	1.1689 8721	9
10	1.1322 7083	1.1463 2740	1.1605 4083	1.1894 4449	10
11	1.1464 2422	1.1620 8940	1.1779 4894	1.2102 5977	11
12	1.1607 5452	1.1780 6813	1.1956 1817	1.2314 3931	12
13	1.1752 6395	1.1942 6656	1.2135 5244	1.2529 8950	13
14	1.1899 5475	1.2106 8773	1.2317 5573	1.2749 1682	14
15	1.2048 2918	1.2273 3469	1.2502 3207	1.2972 2786	15
16	1.2198 8955	1.2442 1054	1.2689 8555	1.3199 2935	16
17	1.2351 3817	1.2613 1843	1.2880 2033	1.3430 2811	17
18	1.2505 7739	1.2786 6156	1.3073 4064	1.3665 3111	18
19	1.2662 0961	1.2962 4316	1.3269 5075	1.3904 4540	19
20	1.2820 3723	1.3140 6650	1.3468 5501	1.4147 7820	20
21	1.2980 6270	1.3321 3492	1.3670 5783	1.4395 3681	21
22	1.3142 8848	1.3504 5177	1.3875 6370	1.4647 2871	22
23	1.3307 1709	1.3690 2048	1.4083 7715	1.4903 6146	23
24	1.3473 5105	1.3878 4451	1.4295 0281	1.5164 4279	24
25	1.3641 9294	1.4069 2738	1.4509 4535	1.5429 8054	25
26	1.3812 4535	1.4262 7263	1.4727 0953	1.5699 8269	26
27	1.3985 1092	1.4458 8388	1.4948 0018	1.5974 5739	27
28	1.4159 9230	1.4657 6478	1.5172 2218	1.6254 1290	28
29	1.4336 9221	1.4859 1905	1.5399 8051	1.6538 5762	29
30	1.4516 1336	1.5063 5043	1.5630 8022	1.6828 0013	30
31	1.4697 5853	1.5270 6275	1.5865 2642	1.7122 4913	31
32	1.4881 3051	1.5480 5986	1.6103 2432	1.7422 1349	32
33	1.5067 3214	1.5693 4569	1.6344 7918	1.7727 0223	33
34	1.5255 6629	1.5909 2419	1.6589 9637	1.8037 2452	34
35	1.5446 3587	1.6127 9940	1.6838 8132	1.8352 8970	35
36	1.5639 4382	1.6349 7539	1.7091 3954	1.8674 0727	36
37	1.5834 9312	1.6574 5630	1.7347 7663	1.9000 8689	37
38	1.6032 8678	1.6802 4633	1.7607 9828	1.9333 3841	38
39	1.6233 2787	1.7033 4971	1.7872 1025	1.9671 7184	39
40	1.6436 1946	1.7267 7077	1.8140 1841	2.0015 9734	40
41	1.6641 6471	1.7505 1387	1.8412 2868	2.0366 2530	41
42	1.6849 6677	1.7745 8343	1.8688 4712	2.0722 6624	42
43	1.7060 2885	1.7989 8396	1.8968 7982	2.1085 3090	43
44	1.7273 5421	1.8237 1999	1.9253 3302	2.1454 3019	44
45	1.7489 4614	1.8487 9614	1.9542 1301	2.1829 7522	45
46	1.7708 0797	1.8742 1708	1.9835 2621	2.2211 7728	46
47	1.7929 4306	1.8999 8757	2.0132 7910	2.2600 4789	47
48	1.8153 5485	1.9261 1240	2.0434 7829	2.2995 9872	48
49	1.8380 4679	1.9525 9644	2.0741 3046	2.3398 4170	49
50	1.8610 2237	1.9794 4464	2.1052 4242	2.3807 8893	50

$(1+i)^n$

n	$1\frac{1}{4}\%$	$1\frac{3}{8}\%$	$1\frac{1}{2}\%$	$1\frac{3}{4}\%$	n
51	1.8842 8515	2.0066 6201	2.1368 2106	2.4224 5274	51
52	1.9078 3872	2.0342 5361	2.1688 7337	2.4648 4566	52
53	1.9316 8670	2.0622 2460	2.2014 0647	2.5079 8046	53
54	1.9558 3279	2.0905 8019	2.2344 2757	2.5518 7012	54
55	1.9802 8070	2.1193 2566	2.2679 4398	2.5965 2785	55
56	2.0050 3420	2.1484 6639	2.3019 6314	2.6419 6708	56
57	2.0300 9713	2.1780 0780	2.3364 9259	2.6882 0151	57
58	2.0554 7335	2.2079 5541	2.3715 3998	2.7352 4503	58
59	2.0811 6676	2.2383 1480	2.4071 1308	2.7831 1182	59
60	2.1071 8135	2.2690 9163	2.4432 1978	2.8318 1628	60
61	2.1335 2111	2.3002 9164	2.4798 6807	2.8813 7306	61
62	2.1601 9013	2.3319 2065	2.5170 6609	2.9317 9709	62
63	2.1871 9250	2.3639 8456	2.5548 2208	2.9831 0354	63
64	2.2145 3241	2.3964 8934	2.5931 4442	3.0343 0785	64
65	2.2422 1407	2.4294 4107	2.6320 4158	3.0884 2574	65
66	2.2702 4174	2.4628 4589	2.6715 2221	3.1424 7319	66
67	2.2986 1976	2.4967 1002	2.7115 9504	3.1974 6647	67
68	2.3273 5251	2.5310 3978	2.7522 6896	3.2534 2213	68
69	2.3564 4442	2.5658 4158	2.7935 5300	3.3103 5702	69
70	2.3858 9997	2.6011 2190	2.8354 5629	3.3682 8827	70
71	2.4157 2372	2.6368 8732	2.8779 8814	3.4272 3331	71
72	2.4459 2027	2.6731 4453	2.9211 5796	3.4872 0990	72
73	2.4764 9427	2.7099 0026	2.9649 7533	3.5482 3607	73
74	2.5074 5045	2.7471 6139	3.0094 4996	3.6103 3020	74
75	2.5387 9358	2.7849 3486	3.0545 9171	3.6735 1098	75
76	2.5705 2850	2.8232 2771	3.1004 1059	3.7377 9742	76
77	2.6026 6011	2.8620 4710	3.1469 1674	3.8032 0888	77
78	2.6351 9336	2.9014 0024	3.1941 2050	3.8697 6503	78
79	2.6681 3327	2.9412 9450	3.2420 3230	3.9374 8592	79
80	2.7014 8494	2.9817 3730	3.2906 6279	4.0063 9192	80
81	2.7352 5350	3.0227 3618	3.3400 2273	4.0765 0378	81
82	2.7694 4417	3.0642 9881	3.3901 2307	4.1478 4260	82
83	2.8040 6222	3.1064 3291	3.4409 7492	4.2204 2984	83
84	2.8391 1300	3.1491 4637	3.4925 8954	4.2942 8737	84
85	2.8746 0191	3.1924 4713	3.5449 7838	4.3694 3740	85
86	2.9105 3444	3.2363 4328	3.5981 5306	4.4459 0255	86
87	2.9469 1612	3.2808 4300	3.6521 2535	4.5237 0584	87
88	2.9837 5257	3.3259 5459	3.7069 0723	4.6028 7070	88
89	3.0210 4948	3.3716 8646	3.7625 1084	4.6834 2093	89
90	3.0588 1260	3.4180 4715	3.8189 4851	4.7653 8080	90
91	3.0970 4775	3.4650 4530	3.8762 3273	4.8487 7496	91
92	3.1357 6085	3.5126 8967	3.9343 7622	4.9336 2853	92
93	3.1749 5786	3.5609 8916	3.9933 9187	5.0199 6703	93
94	3.2146 4483	3.6099 5276	4.0532 9275	5.1078 1645	94
95	3.2548 2789	3.6595 8961	4.1140 9214	5.1972 0324	95
96	3.2955 1324	3.7099 0897	4.1758 0352	5.2881 5429	96
97	3.3367 0716	3.7609 2021	4.2384 4057	5.3806 9699	97
98	3.3784 1600	3.8126 3287	4.3020 1718	5.4748 5919	98
99	3.4206 4620	3.8650 5657	4.3665 4744	5.5706 6923	99
100	3.4634 0427	3.9182 0110	4.4320 4565	5.6681 5594	100

$(1+i)^n$

n	2%	2¼%	2½%	2¾%	n
1	1.0200 0000	1.0225 0000	1.0250 0000	1.0275 0000	1
2	1.0404 0000	1.0455 0625	1.0506 2500	1.0557 5625	2
3	1.0612 0800	1.0690 3014	1.0768 9063	1.0847 8955	3
4	1.0824 3216	1.0930 8332	1.1038 1289	1.1146 2126	4
5	1.1040 8080	1.1176 7769	1.1314 0821	1.1452 7334	5
6	1.1261 6242	1.1428 2544	1.1596 9342	1.1767 6836	6
7	1.1486 8567	1.1685 3901	1.1886 8575	1.2091 2949	7
8	1.1716 5938	1.1948 3114	1.2184 0290	1.2423 8055	8
9	1.1950 9257	1.2217 1484	1.2488 6297	1.2765 4602	9
10	1.2189 9442	1.2492 0343	1.2800 8454	1.3116 5103	10
11	1.2433 7431	1.2773 1050	1.3120 8666	1.3477 2144	11
12	1.2682 4179	1.3060 4999	1.3448 8882	1.3847 8378	12
13	1.2936 0663	1.3354 3611	1.3785 1104	1.4228 6533	13
14	1.3194 7876	1.3654 8343	1.4129 7382	1.4619 9413	14
15	1.3458 6834	1.3962 0680	1.4482 9817	1.5021 9896	15
16	1.3727 8571	1.4276 2146	1.4845 0562	1.5435 0944	16
17	1.4002 4142	1.4597 4294	1.5216 1826	1.5859 5595	17
18	1.4282 4625	1.4925 8716	1.5596 5872	1.6295 6973	18
19	1.4568 1117	1.5261 7037	1.5986 5019	1.6743 8290	19
20	1.4859 4740	1.5605 0920	1.6386 1644	1.7204 2843	20
21	1.5156 6634	1.5956 2066	1.6795 8185	1.7677 4021	21
22	1.5459 7967	1.6315 2212	1.7215 7140	1.8163 5307	22
23	1.5768 9926	1.6682 3137	1.7646 1068	1.8663 0278	23
24	1.6084 3725	1.7057 6658	1.8087 2595	1.9176 2610	24
25	1.6406 0599	1.7441·4632	1.8539 4410	1.9703 6082	25
26	1.6734 1811	1.7833 8962	1.9002 9270	2.0245 4575	26
27	1.7068 8648	1.8235 1588	1.9478 0002	2.0802 2075	27
28	1.7410 2421	1.8645 4499	1.9964 9502	2.1374 2682	28
29	1.7758 4469	1.9064 9725	2.0464 0739	2.1962 0606	29
30	1.8113 6158	1.9493 9344	2.0975 6758	2.2566 0173	30
31	1.8475 8882	1.9932 5479	2.1500 0677	2.3186 5828	31
32	1.8845 4059	2.0381 0303	2.2037 5694	2.3824 2138	32
33	1.9222 3140	2.0839 6034	2.2588 5086	2.4479 3797	33
34	1.9606 7603	2.1308 4945	2.3153 2213	2.5152 5626	34
35	1.9998 8955	2.1787 9356	2.3732 0519	2.5844 2581	35
36	2.0398 8734	2.2278 1642	2.4325 3532	2.6554 9752	36
37	2.0806 8509	2.2779 4229	2.4933 4870	2.7285 2370	37
38	2.1222 9879	2.3291 9599	2.5556 8242	2.8035 5810	38
39	2.1647 4477	2.3816 0290	2.6195 7448	2.8806 5595	39
40	2.2080 3966	2.4351 8897	2.6850 6384	2.9598 7399	40
41	2.2522 0046	2.4899 8072	2.7521 9043	3.0412 7052	41
42	2.2972 4447	2.5460 0528	2.8209 9520	3.1249 0546	42
43	2.3431 8936	2.6032 9040	2.8915 2008	3.2108 4036	43
44	2.3900 5314	2.6618 6444	2.9638 0808	3.2991 3847	44
45	2.4378 5421	2.7217 5639	3.0379 0328	3.3898 6478	45
46	2.4866 1129	2.7829 9590	3.1138 5086	3.4830 8606	46
47	2.5363 4351	2.8456 1331	3.1916 9713	3.5788 7093	47
48	2.5870 7039	2.9096 3961	3.2714 8956	3.6772 8988	48
49	2.6388 1179	2.9751 0650	3.3532 7680	3.7784 1535	49
50	2.6915 8803	3.0420 4640	3.4371 0872	3.8823 2177	50

$(1+i)^n$

n	2%	2¼%	2½%	2¾%	n
51	2.7454 1979	3.1104 9244	3.5230 3644	3.9890 8562	51
52	2.8003 2819	3.1804 7852	3.6111 1235	4.0987 8547	52
53	2.8563 3475	3.2520 3929	3.7013 9016	4.2115 0208	53
54	2.9134 6144	3.3252 1017	3.7939 2491	4.3273 1838	54
55	2.9717 3067	3.4000 2740	3.8887 7303	4.4463 1964	55
56	3.0311 6529	3.4765 2802	3.9859 9236	4.5685 9343	56
57	3.0917 8859	3.5547 4990	4.0856 4217	4.6942 2975	57
58	3.1536 2436	3.6347 3177	4.1877 8322	4.8233 2107	58
59	3.2166 9685	3.7165 1324	4.2924 7780	4.9559 6239	59
60	3.2810 3079	3.8001 3479	4.3997 8975	5.0922 5136	60
61	3.3466 5140	3.8856 3782	4.5097 8449	5.2322 8827	61
62	3.4135 8443	3.9730 6467	4.6225 2910	5.3761 7620	62
63	3.4818 5612	4.0624 5862	4.7380 9233	5.5240 2105	63
64	3.5514 9324	4.1538 6394	4.8565 4464	5.6759 3162	64
65	3.6225 2311	4.2473 2588	4.9779 5826	5.8320 1974	65
66	3.6949 7357	4.3428 9071	5.1024 0721	5.9924 0029	66
67	3.7688 7304	4.4406 0576	5.2299 6739	6.1571 9130	67
68	3.8442 5050	4.5405 1939	5.3607 1658	6.3265 1406	68
69	3.9211 3551	4.6426 8107	5.4947 3449	6.5004 9319	69
70	3.9995 5822	4.7471 4140	5.6321 0286	6.6792 5676	70
71	4.0795 4939	4.8539 5208	5.7729 0543	6.8629 3632	71
72	4.1611 4038	4.9631 6600	5.9172 2806	7.0516 6706	72
73	4.2443 6318	5.0748 3723	6.0651 5876	7.2455 8791	73
74	4.3292 5045	5.1890 2107	6.2167 8773	7.4448 4158	74
75	4.4158 3546	5.3057 7405	6.3722 0743	7.6495 7472	75
76	4.5041 5216	5.4251 5396	6.5315 1261	7.8599 3802	76
77	4.5942 3521	5.5472 1993	6.6948 0043	8.0760 8632	77
78	4.6861 1991	5.6720 3237	6.8621 7044	8.2981 7869	78
79	4.7798 4231	5.7996 5310	7.0337 2470	8.5263 7861	79
80	4.8754 3916	5.9301 4530	7.2095 6782	8.7608 5402	80
81	4.9729 4794	6.0635 7357	7.3898 0701	9.0017 7751	81
82	5.0724 0690	6.2000 0397	7.5745 5219	9.2493 2639	82
83	5.1738 5504	6.3395 0406	7.7639 1599	9.5036 8286	83
84	5.2773 3214	6.4821 4290	7.9580 1389	9.7650 3414	84
85	5.3828 7878	6.6279 9112	8.1569 6424	10.0335 7258	85
86	5.4905 3636	6.7771 2092	8.3608 8834	10.3094 9583	86
87	5.6003 4708	6.9296 0614	8.5699 1055	10.5930 0696	87
88	5.7123 5402	7.0855 2228	8.7841 5832	10.8843 1465	88
89	5.8266 0110	7.2449 4653	9.0037 6228	11.1836 3331	89
90	5.9431 3313	7.4079 5782	9.2288 5633	11.4911 8322	90
91	6.0619 9579	7.5746 3688	9.4595 7774	11.8071 9076	91
92	6.1832 3570	7.7450 6621	9.6960 6718	12.1318 8851	92
93	6.3069 0042	7.9193 3020	9.9384 6886	12.4655 1544	93
94	6.4330 3843	8.0975 1512	10.1869 3058	12.8083 1711	94
95	6.5616 9920	8.2797 0921	10.4416 0385	13.1605 4584	95
96	6.6929 3318	8.4660 0267	10.7026 4395	13.5224 6085	96
97	6.8267 9184	8.6564 8773	10.9702 1004	13.8943 2852	97
98	6.9633 2768	8.8512 5871	11.2444 6530	14.2764 2255	98
99	7.1025 9423	9.0504 1203	11.5255 7693	14.6690 2417	99
100	7.2446 4612	9.2540 4630	11.8137 1635	15.0724 2234	100

Amount of 1 at Compound Interest

$(1+i)^n$

n	3%	3½%	4%	4½%	n
1	1.0300 0000	1.0350 0000	1.0400 0000	1.0450 0000	1
2	1.0609 0000	1.0712 2500	1.0816 0000	1.0920 2500	2
3	1.0927 2700	1.1087 1788	1.1248 6400	1.1411 6613	3
4	1.1255 0881	1.1475 2300	1.1698 5856	1.1925 1860	4
5	1.1592 7407	1.1876 8631	1.2166 5290	1.2461 8194	5
6	1.1940 5230	1.2292 5533	1.2653 1902	1.3022 6012	6
7	1.2298 7387	1.2722 7926	1.3159 3178	1.3608 6183	7
8	1.2667 7008	1.3168 0904	1.3685 6905	1.4221 0061	8
9	1.3047 7318	1.3628 9735	1.4233 1181	1.4860 9514	9
10	1.3439 1638	1.4105 9876	1.4802 4428	1.5529 6942	10
11	1.3842 3387	1.4599 6972	1.5394 5406	1.6228 5305	11
12	1.4257 6089	1.5110 6866	1.6010 3222	1.6958 8143	12
13	1.4685 3371	1.5639 5606	1.6650 7351	1.7721 9610	13
14	1.5125 8972	1.6186 9452	1.7316 7645	1.8519 4492	14
15	1.5579 6742	1.6753 4883	1.8009 4351	1.9352 8244	15
16	1.6047 0644	1.7339 8604	1.8729 8125	2.0223 7015	16
17	1.6528 4763	1.7946 7555	1.9479 0050	2.1133 7681	17
18	1.7024 3306	1.8574 8920	2.0258 1652	2.2084 7877	18
19	1.7535 0605	1.9225 0132	2.1068 4918	2.3078 6031	19
20	1.8061 1123	1.9897 8886	2.1911 2314	2.4117 1402	20
21	1.8602 9457	2.0594 3147	2.2787 6807	2.5202 4116	21
22	1.9161 0341	2.1315 1158	2.3699 1879	2.6336 5201	22
23	1.9735 8651	2.2061 1448	2.4647 1554	2.7521 6635	23
24	2.0327 9411	2.2833 2849	2.5633 0416	2.8760 1383	24
25	2.0937 7793	2.3632 4498	2.6658 3633	3.0054 3446	25
26	2.1565 9127	2.4459 5856	2.7724 6978	3.1406 7901	26
27	2.2212 8901	2.5315 6711	2.8833 6858	3.2820 0956	27
28	2.2879 2768	2.6201 7196	2.9987 0332	3.4296 9999	28
29	2.3565 6551	2.7118 7798	3.1186 5145	3.5840 3649	29
30	2.4272 6247	2.8067 9370	3.2433 9751	3.7453 1813	30
31	2.5000 8035	2.9050 3148	3.3731 3341	3.9138 5745	31
32	2.5750 8276	3.0067 0759	3.5080 5875	4.0899 8104	32
33	2.6523 3524	3.1119 4235	3.6483 8110	4.2740 3018	33
34	2.7319 0530	3.2208 6033	3.7943 1634	4.4663 6154	34
35	2.8138 6245	3.3335 9045	3.9460 8899	4.6673 4781	35
36	2.8982 7833	3.4502 6611	4.1039 3255	4.8773 7846	36
37	2.9852 2668	3.5710 2543	4.2680 8986	5.0968 6049	37
38	3.0747 8348	3.6960 1132	4.4388 1345	5.3262 1921	38
39	3.1670 2698	3.8253 7171	4.6168 6599	5.5658 9908	39
40	3.2620 3779	3.9592 5972	4.8010 2063	5.8163 6454	40
41	3.3598 9893	4.0978 3381	4.9930 6145	6.0781 0094	41
42	3.4606 9589	4.2412 5799	5.1927 8391	6.3516 1548	42
43	3.5645 1677	4.3897 0202	5.4004 9527	6.6374 3818	43
44	3.6714 5227	4.5433 4160	5.6165 1508	6.9361 2290	44
45	3.7815 9584	4.7023 5855	5.8411 7568	7.2482 4843	45
46	3.8950 4372	4.8669 4110	6.0748 2271	7.5744 1961	46
47	4.0118 9503	5.0372 8404	6.3178 1562	7.9152 6849	47
48	4.1322 5188	5.2135 8898	6.5705 2824	8.2714 5557	48
49	4.2562 1944	5.3960 6459	6.8333 4937	8.6436 7107	49
50	4.3839 0602	5.5849 2686	7.1066 8335	9.0326 3627	50

$(1+i)^n$

n	3%	3½%	4%	4½%	n
51	4.5154 2320	5.7803 9930	7.3909 5068	9.4391 0490	51
52	4.6508 8590	5.9827 1327	7.6865 8871	9.8638 6463	52
53	4.7904 1247	6.1921 0824	7.9940 5226	10.3077 3853	53
54	4.9341 2485	6.4088 3202	8.3138 1435	10.7715 8677	54
55	5.0821 4859	6.6331 4114	8.6463 6692	11.2563 0817	55
56	5.2346 1305	6.8653 0108	8.9922 2160	11.7628 4204	56
57	5.3916 5144	7.1055 8662	9.3519 1046	12.2921 6993	57
58	5.5534 0098	7.3542 8215	9.7259 8688	12.8453 1758	58
59	5.7200 0301	7.6116 8203	10.1150 2635	13.4233 5687	59
60	5.8916 0310	7.8780 9090	10.5196 2741	14.0274 0793	60
61	6.0683 5120	8.1538 2408	10.9404 1250	14.6586 4129	61
62	6.2504 0173	8.4392 0793	11.3780 2900	15.3182 8014	62
63	6.4379 1379	8.7345 8020	11.8331 5016	16.0076 0275	63
64	6.6310 5120	9.0402 9051	12.3064 7617	16.7279 4487	64
65	6.8299 8273	9.3567 0068	12.7987 3522	17.4807 0239	65
66	7.0348 8222	9.6841 8520	13.3106 8463	18.2673 3400	66
67	7.2459 2868	10.0231 3168	13.8431 1201	19.0893 6403	67
68	7.4633 0654	10.3739 4129	14.3968 3649	19.9483 8541	68
69	7.6872 0574	10.7370 2924	14.9727 0995	20.8460 6276	69
70	7.9178 2191	11.1128 2526	15.5716 1835	21.7841 3558	70
71	8.1553 5657	11.5017 7414	16.1944 8308	22.7644 2168	71
72	8.4000 1727	11.9043 3624	16.8422 6241	23.7888 2066	72
73	8.6520 1778	12.3209 8801	17.5159 5290	24.8593 1759	73
74	8.9115 7832	12.7522 2259	18.2165 9102	25.9779 8688	74
75	9.1789 2567	13.1985 5038	18.9452 5466	27.1469 9629	75
76	9.4542 9344	13.6604 9964	19.7030 6485	28.3686 1112	76
77	9.7379 2224	14.1386 1713	20.4911 8744	29.6451 9862	77
78	10.0300 5991	14.6334 6873	21.3108 3494	30.9792 3256	78
79	10.3309 6171	15.1456 4013	22.1632 6834	32.3732 9802	79
80	10.6408 9056	15.6757 3754	23.0497 9907	33.8300 9643	80
81	10.9601 1727	16.2243 8835	23.9717 9103	35.3524 5077	81
82	11.2889 2079	16.7922 4195	24.9306 6267	36.9433 1106	82
83	11.6275 8842	17.3799 7041	25.9278 8918	38.6057 6006	83
84	11.9764 1607	17.9882 6938	26.9650 0475	40.3430 1926	84
85	12.3357 0855	18.6178 5881	28.0436 0494	42.1584 5513	85
86	12.7057 7981	19.2694 8387	29.1653 4914	44.0555 8561	86
87	13.0869 5320	19.9439 1580	30.3319 6310	46.0380 8696	87
88	13.4795 6180	20.6419 5285	31.5452 4163	48.1098 0087	88
89	13.8839 4865	21.3644 2120	32.8070 5129	50.2747 4191	89
90	14.3004 6711	22.1121 7595	34.1193 3334	52.5371 0530	90
91	14.7294 8112	22.8861 0210	35.4841 0668	54.9012 7503	91
92	15.1713 6556	23.6871 1568	36.9034 7094	57.3718 3241	92
93	15.6265 0652	24.5161 6473	38.3796 0978	59.9535 6487	93
94	16.0953 0172	25.3742 3049	39.9147 9417	62.6514 7529	94
95	16.5781 6077	26.2623 2856	41.5113 8594	65.4707 9168	95
96	17.0755 0559	27.1815 1006	43.1718 4138	68.4169 7730	96
97	17.5877 7076	28.1328 6291	44.8987 1503	71.4957 4128	97
98	18.1154 0388	29.1175 1311	46.6946 6363	74.7130 4964	98
99	18.6588 6600	30.1366 2607	48.5624 5018	78.0751 3687	99
100	19.2186 3198	31.1914 0798	50.5049 4818	81.5885 1803	100

Amount of 1 at Compound Interest

$$(1+i)^n$$

n	5%	5½%	6%	6½%	n
1	1.0500 0000	1.0550 0000	1.0600 0000	1.0650 0000	1
2	1.1025 0000	1.1130 2500	1.1236 0000	1.1342 2500	2
3	1.1576 2500	1.1742 4138	1.1910 1600	1.2079 4963	3
4	1.2155 0625	1.2388 2465	1.2624 7696	1.2864 6635	4
5	1.2762 8156	1.3069 6001	1.3382 2558	1.3700 8666	5
6	1.3400 9564	1.3788 4281	1.4185 1911	1.4591 4230	6
7	1.4071 0042	1.4546 7916	1.5036 3026	1.5539 8655	7
8	1.4774 5544	1.5346 8651	1.5938 4807	1.6549 9567	8
9	1.5513 2822	1.6190 9427	1.6894 7896	1.7625 7039	9
10	1.6288 9463	1.7081 4446	1.7908 4770	1.8771 3747	10
11	1.7103 3936	1.8020 9240	1.8982 9856	1.9991 5140	11
12	1.7958 5633	1.9012 0749	2.0121 9647	2.1290 9624	12
13	1.8856 4914	2.0057 7390	2.1329 2826	2.2674 8750	13
14	1.9799 3160	2.1160 9146	2.2609 0396	2.4148 7418	14
15	2.0789 2818	2.2324 7649	2.3965 5819	2.5718 4101	15
16	2.1828 7459	2.3552 6270	2.5403 5168	2.7390 1067	16
17	2.2920 1832	2.4848 0215	2.6927 7279	2.9170 4637	17
18	2.4066 1923	2.6214 6627	2.8543 3915	3.1066 5438	18
19	2.5269 5020	2.7656 4691	3.0255 9950	3.3085 8691	19
20	2.6532 9771	2.9177 5749	3.2071 3547	3.5236 4506	20
21	2.7859 6259	3.0782 3415	3.3995 6360	3.7526 8199	21
22	2.9252 6072	3.2475 3703	3.6035 3742	3.9966 0632	22
23	3.0715 2376	3.4261 5157	3.8197 4966	4.2563 8573	23
24	3.2250 9994	3.6145 8990	4.0489 3464	4.5330 5081	24
25	3.3863 5494	3.8133 9235	4.2918 7072	4.8276 9911	25
26	3.5556 7269	4.0231 2893	4.5493 8296	5.1414 9955	26
27	3.7334 5632	4.2444 0102	4.8223 4594	5.4756 9702	27
28	3.9201 2914	4.4778 4307	5.1116 8670	5.8316 1733	28
29	4.1161 3560	4.7241 2444	5.4183 8790	6.2106 7245	29
30	4.3219 4238	4.9839 5129	5.7434 9117	6.6143 6616	30
31	4.5380 3949	5.2580 6861	6.0881 0064	7.0442 9996	31
32	4.7649 4147	5.5472 6238	6.4533 8668	7.5021 7946	32
33	5.0031 8854	5.8523 6181	6.8405 8988	7.9898 2113	33
34	5.2533 4797	6.1742 4171	7.2510 2528	8.5091 5950	34
35	5.5160 1537	6.5138 2501	7.6860 8679	9.0622 5487	35
36	5.7918 1614	6.8720 8538	8.1472 5200	9.6513 0143	36
37	6.0814 0694	7.2500 5008	8.6360 8712	10.2786 3603	37
38	6.3854 7729	7.6488 0283	9.1542 5235	10.9467 4737	38
39	6.7047 5115	8.0694 8699	9.7035 0749	11.6582 8595	39
40	7.0399 8871	8.5133 0877	10.2857 1794	12.4160 7453	40
41	7.3919 8815	8.9815 4076	10.9028 6101	13.2231 1938	41
42	7.7615 8756	9.4755 2550	11.5570 3267	14.0826 2214	42
43	8.1496 6693	9.9966 7940	12.2504 5463	14.9979 9258	43
44	8.5571 5028	10.5464 9677	12.9854 8191	15.9728 6209	44
45	8.9850 0779	11.1265 5409	13.7646 1083	17.0110 9813	45
46	9.4342 5818	11.7385 1456	14.5904 8748	18.1168 1951	46
47	9.9059 7109	12.3841 3287	15.4659 1673	19.2944 1278	47
48	10.4012 6965	13.0652 6017	16.3938 7173	20.5485 4961	48
49	10.9213 3313	13.7838 4948	17.3775 0403	21.8842 0533	49
50	11.4673 9979	14.5419 6120	18.4201 5427	23.3066 7868	50

$(1+i)^n$

n	5%	5½%	6%	6½%	n
51	12.0407 6978	15.3417 6907	19.5253 6353	24.8216 1279	51
52	12.6428 0826	16.1855 6637	20.6968 8534	26.4350 1762	52
53	13.2749 4868	17.0757 7252	21.9386 9846	28.1532 9377	53
54	13.9386 9611	18.0149 4001	23.2550 2037	29.9832 5786	54
55	14.6356 3092	19.0057 6171	24.6503 2159	31.9321 6963	55
56	15.3674 1246	20.0510 7860	26.1293 4089	34.0077 6065	56
57	16.1357 8309	21.1538 8793	27.6971 0134	36.2182 6509	57
58	16.9425 7224	22.3173 5176	29.3589 2742	38.5724 5233	58
59	17.7897 0085	23.5448 0611	31.1204 6307	41.0796 6173	59
60	18.6791 8589	24.8397 7045	32.9876 9085	43.7498 3974	60
61	19.6131 4519	26.2059 5782	34.9669 5230	46.5935 7932	61
62	20.5938 0245	27.6472 8550	37.0649 6944	49.6221 6198	62
63	21.6234 9257	29.1678 8620	39.2888 6761	52.8476 0251	63
64	22.7046 6720	30.7721 1994	41.6461 9967	56.2826 9667	64
65	23.8399 0056	32.4645 8654	44.1449 7165	59.9410 7195	65
66	25.0318 9559	34.2501 3880	46.7936 6994	63.8372 4163	66
67	26.2834 9037	36.1338 9643	49.6012 9014	67.9866 6234	67
68	27.5976 6488	38.1212 6074	52.5773 6755	72.4057 9539	68
69	28.9775 4813	40.2179 3008	55.7320 0960	77.1121 7209	69
70	30.4264 2554	42.4299 1623	59.0759 3018	82.1244 6327	70
71	31.9477 4681	44.7635 6163	62.6204 8599	87.4625 5339	71
72	33.5451 3415	47.2255 5751	66.3777 1515	93.1476 1936	72
73	35.2223 9086	49.8229 6318	70.3603 7806	99.2022 1461	73
74	36.9835 1040	52.5632 2615	74.5820 0074	105.6503 5856	74
75	38.8326 8592	55.4542 0359	79.0569 2079	112.5176 3187	75
76	40.7743 2022	58.5041 8479	83.8003 3603	119.8312 7794	76
77	42.8130 3623	61.7219 1495	88.8283 5620	127.6203 1101	77
78	44.9536 8804	65.1166 2027	94.1580 5757	135.9156 3122	78
79	47.2013 7244	68.6980 3439	99.8075 4102	144.7501 4725	79
80	49.5614 4107	72.4764 2628	105.7959 9348	154.1589 0683	80
81	52.0395 1312	76.4626 2973	112.1437 5309	164.1792 3577	81
82	54.6414 8878	80.6680 7436	118.8723 7828	174.8508 8609	82
83	57.3735 6322	85.1048 1845	126.0047 2097	186.2161 9369	83
84	60.2422 4138	89.7855 8347	133.5650 0423	198.3202 4628	84
85	63.2543 5344	94.7237 9056	141.5789 0449	211.2110 6229	85
86	66.4170 7112	99.9335 9904	150.0736 3875	224.9397 8134	86
87	69.7379 2467	105.4299 4698	159.0780 5708	239.5608 6712	87
88	73.2248 2091	111.2285 9407	168.6227 4050	255.1323 2349	88
89	76.8860 6195	117.3461 6674	178.7401 0493	271.7159 2451	89
90	80.7303 6505	123.8002 0591	189.4645 1123	289.3774 5961	90
91	84.7668 8330	130.6092 1724	200.8323 8190	308.1869 9448	91
92	89.0052 2747	137.7927 2419	212.8823 2482	328.2191 4912	92
93	93.4554 8884	145.3713 2402	225.6552 6431	349.5533 9382	93
94	98.1282 6328	153.3667 4684	239.1945 8017	372.2743 6441	94
95	103.0346 7645	161.8019 1791	253.5462 5498	396.4721 9810	95
96	108.1864 1027	170.7010 2340	268.7590 3028	422.2428 9098	96
97	113.5957 3078	180.0895 7969	284.8845 7209	449.6886 7889	97
98	119.2755 1732	189.9945 0657	301.9776 4642	478.9184 4302	98
99	125.2392 9319	200.4442 0443	320.0963 0520	510.0481 4181	99
100	131.5012 5785	211.4686 3567	339.3020 8351	543.2012 7103	100

$(1+i)^n$

n	7%	7½%	8%	8½%	n
1	1.0700 0000	1.0750 0000	1.0800 0000	1.0850 0000	1
2	1.1449 0000	1.1556 2500	1.1664 0000	1.1772 2500	2
3	1.2250 4300	1.2422 9688	1.2597 1200	1.2772 8913	3
4	1.3107 9601	1.3354 6914	1.3604 8896	1.3858 5870	4
5	1.4025 5173	1.4356 2933	1.4693 2808	1.5036 5669	5
6	1.5007 3035	1.5433 0153	1.5868 7432	1.6314 6751	6
7	1.6057 8148	1.6590 4914	1.7138 2427	1.7701 4225	7
8	1.7181 8618	1.7834 7783	1.8509 3021	1.9206 0434	8
9	1.8384 5921	1.9172 3866	1.9990 0463	2.0838 5571	9
10	1.9671 5136	2.0610 3156	2.1589 2500	2.2609 8344	10
11	2.1048 5195	2.2156 0893	2.3316 3900	2.4531 6703	11
12	2.2521 9159	2.3817 7960	2.5181 7012	2.6616 8623	12
13	2.4098 4500	2.5604 1307	2.7196 2373	2.8879 2956	13
14	2.5785 3415	2.7524 4405	2.9371 9362	3.1334 0357	14
15	2.7590 3154	2.9588 7735	3.1721 6911	3.3997 4288	15
16	2.9521 6375	3.1807 9315	3.4259 4264	3.6887 2102	16
17	3.1588 1521	3.4193 5264	3.7000 1805	4.0022 6231	17
18	3.3799 3228	3.6758 0409	3.9960 1950	4.3424 5461	18
19	3.6165 2754	3.9514 8940	4.3157 0106	4.7115 6325	19
20	3.8696 8446	4.2478 5110	4.6609 5714	5.1120 4612	20
21	4.1405 6237	4.5664 3993	5.0338 3372	5.5465 7005	21
22	4.4304 0174	4.9089 2293	5.4365 4041	6.0180 2850	22
23	4.7405 2986	5.2770 9215	5.8714 6365	6.5295 6092	23
24	5.0723 6695	5.6728 7406	6.3411 8074	7.0845 7360	24
25	5.4274 3264	6.0983 3961	6.8484 7520	7.6867 6236	25
26	5.8073 5292	6.5557 1508	7.3963 5321	8.3401 3716	26
27	6.2138 6763	7.0473 9371	7.9880 6147	9.0490 4881	27
28	6.6488 3836	7.5759 4824	8.6271 0639	9.8182 1796	28
29	7.1142 5705	8.1441 4436	9.3172 7490	10.6527 6649	29
30	7.6122 5504	8.7549 5519	10.0626 5689	11.5582 5164	30
31	8.1451 1290	9.4115 7683	10.8676 6944	12.5407 0303	31
32	8.7152 7080	10.1174 4509	11.7370 8300	13.6066 6279	32
33	9.3253 3975	10.8762 5347	12.6760 4964	14.7632 2913	33
34	9.9781 1354	11.6919 7248	13.6901 3361	16.0181 0360	34
35	10.6765 8148	12.5688 7042	14.7853 4429	17.3796 4241	35
36	11.4239 4219	13.5115 3570	15.9681 7184	18.8569 1201	36
37	12.2236 1814	14.5249 0088	17.2456 2558	20.4597 4953	37
38	13.0792 7141	15.6142 6844	18.6252 7563	22.1988 2824	38
39	13.9948 2041	16.7853 3858	20.1152 9768	24.0857 2865	39
40	14.9744 5784	18.0442 3897	21.7245 2150	26.1330 1558	40
41	16.0226 6989	19.3975 5689	23.4624 8322	28.3543 2190	41
42	17.1442 5678	20.8523 7366	25.3394 8187	30.7644 3927	42
43	18.3443 5475	22.4163 0168	27.3666 4042	33.3794 1660	43
44	19.6284 5959	24.0975 2431	29.5559 7166	36.2166 6702	44
45	21.0024 5176	25.9048 3863	31.9204 4939	39.2950 8371	45
46	22.4726 2338	27.8477 0153	34.4740 8534	42.6351 6583	46
47	24.0457 0702	29.9362 7915	37.2320 1217	46.2591 5492	47
48	25.7289 0651	32.1815 0008	40.2105 7314	50.1911 8309	48
49	27.5299 2997	34.5951 1259	43.4274 1899	54.4574 3365	49
50	29.4570 2506	37.1897 4603	46.9016 1251	59.0863 1551	50

$(1+i)^n$

n	7%	7½%	8%	8½%	n
51	31.5190 1682	39.9789 7698	50.6537 4151	64.1086 5233	51
52	33.7253 4799	42.9774 0026	54.7060 4084	69.5578 8778	52
53	36.0861 2235	46.2007 0528	59.0825 2410	75.4703 0824	53
54	38.6121 5092	49.6657 5817	63.8091 2603	81.8852 8444	54
55	41.3150 0148	53.3906 9004	68.9138 5611	88.8455 3362	55
56	44.2070 5159	57.3949 9179	74.4269 6460	96.3974 0398	56
57	47.3015 4520	61.6996 1617	80.3811 2177	104.5911 8332	57
58	50.6126 5336	66.3270 8739	86.8116 1151	113.4814 3390	58
59	54.1555 3910	71.3016 1894	93.7565 4043	123.1273 5578	59
60	57.9464 2683	76.6492 4036	101.2570 6367	133.5931 8102	60
61	62.0026 7671	82.3979 3339	109.3576 2876	144.9486 0141	61
62	66.3428 6408	88.5777 7839	118.1062 3906	157.2692 3253	62
63	70.9868 6457	95.2211 1177	127.5547 3819	170.6371 1729	63
64	75.9559 4509	102.3626 9515	137.7591 1724	185.1412 7226	64
65	81.2728 6124	110.0398 9729	148.7798 4662	200.8782 8041	65
66	86.9619 6153	118.2928 8959	160.6822 3435	217.9529 3424	66
67	93.0492 9884	127.1648 5631	173.5368 1310	236.4789 3365	67
68	99.5627 4976	136.7022 2053	187.4197 5815	256.5796 4301	68
69	106.5321 4224	146.9548 8707	202.4133 3880	278.3889 1267	69
70	113.9893 9220	157.9765 0360	218.6064 0590	302.0519 7024	70
71	121.9686 4965	169.8247 4137	236.0949 1837	327.7263 8771	71
72	130.5064 5513	182.5615 9697	254.9825 1184	355.5831 3067	72
73	139.6419 0699	196.2537 1675	275.3811 1279	385.8076 9678	73
74	149.4168 4047	210.9727 4550	297.4116 0181	418.6013 5100	74
75	159.8760 1931	226.7957 0141	321.2045 2996	454.1824 6584	75
76	171.0673 4066	243.8053 7902	346.9008 9236	492.7879 7543	76
77	183.0420 5451	262.0907 8245	374.6529 6374	534.6749 5335	77
78	195.8549 9832	281.7475 9113	404.6252 0084	580.1223 2438	78
79	209.5648 4820	302.8786 6046	436.9952 1691	629.4327 2195	79
80	224.2343 8758	325.5945 6000	471.9548 3426	682.9345 0332	80
81	239.9307 9471	350.0141 5200	509.7112 2101	740.9839 3610	81
82	256.7259 5034	376.2652 1340	550.4881 1869	803.9675 7067	82
83	274.6967 6686	404.4851 0440	594.5271 6818	872.3048 1418	83
84	293.9255 4054	434.8214 8723	642.0893 4164	946.4507 2338	84
85	314.5003 2838	467.4330 9878	693.4564 8897	1026.8990 3487	85
86	336.5153 5137	502.4905 8119	748.9330 0808	1114.1854 5283	86
87	360.0714 2596	540.1773 7477	808.8476 4873	1208.8912 1633	87
88	385.2764 2578	580.6906 7788	873.5554 6063	1311.6469 6971	88
89	412.2457 7558	624.2424 7872	943.4398 9748	1423.1369 6214	89
90	441.1029 7988	671.0606 6463	1018.9150 8928	1544.1036 0392	90
91	471.9801 8847	721.3902 1447	1100.4282 9642	1675.3524 1025	91
92	505.0188 0166	775.4944 8056	1188.4625 6013	1817.7573 6512	92
93	540.3701 1778	833.6565 6660	1283.5395 6494	1972.2667 4116	93
94	578.1960 2602	896.1808 0910	1386.2227 3014	2139.9094 1416	94
95	618.6697 4784	963.3943 6978	1497.1205 4855	2321.8017 1436	95
96	661.9766 3019	1035.6489 4751	1616.8901 9244	2519.1548 6008	96
97	708.3149 9430	1113.3226 1858	1746.2414 0783	2733.2830 2319	97
98	757.8970 4390	1196.8218 1497	1885.9407 2046	2965.6120 8016	98
99	810.9498 3698	1286.5834 5109	2036.8159 7809	3217.6891 0698	99
100	867.7163 2557	1383.0772 0993	2199.7612 5634	3491.1926 8107	100

Present Value of 1 per Period at Compound Interest

$$a_{\overline{n}|i} = (1 - v^n)/i$$

n	$\frac{1}{4}\%$	$\frac{7}{24}\%$	$\frac{1}{3}\%$	$\frac{5}{12}\%$	n
1	0.9975 0623	0.9970 9182	0.9966 7774	0.9958 5062	1
2	1.9925 2492	1.9912 8390	1.9900 4426	1.9875 6908	2
3	2.9850 6227	2.9825 8470	2.9801 1056	2.9751 7253	3
4	3.9751 2446	3.9710 0260	3.9668 8760	3.9586 7804	4
5	4.9627 1766	4.9565 4601	4.9503 8631	4.9381 0261	5
6	5.9478 4804	5.9392 2327	5.9306 1759	5.9134 6318	6
7	6.9305 2174	6.9190 4273	6.9075 9228	6.8847 7661	7
8	7.9107 4487	7.8960 1269	7.8813 2121	7.8520 5969	8
9	8.8885 2357	8.8701 4144	8.8518 1516	8.8153 2915	9
10	9.8638 6391	9.8414 3725	9.8190 8487	9.7746 0164	10
11	10.8367 7198	10.8099 0834	10.7831 4107	10.7298 9374	11
12	11.8072 5384	11.7755 6295	11.7439 9442	11.6812 2198	12
13	12.7753 1555	12.7384 0915	12.7016 5557	12.6286 0280	13
14	13.7409 6314	13.6984 5542	13.6561 3512	13.5720 5257	14
15	14.7042 0264	14.6557 0959	14.6074 4364	14.5115 8762	15
16	15.6650 4004	15.6101 7990	15.5555 9167	15.4472 2418	16
17	16.6234 8133	16.5618 7442	16.5005 8970	16.3789 7843	17
18	17.5795 3250	17.5108 0125	17.4424 4821	17.3068 6648	18
19	18.5331 9950	18.4569 6842	18.3811 7762	18.2309 0438	19
20	19.4844 8828	19.4003 8396	19.3167 8832	19.1511 0809	20
21	20.4334 0477	20.3410 5587	20.2492 9069	20.0674 9352	21
22	21.3799 5488	21.2789 9213	21.1786 9504	20.9800 7653	22
23	22.3241 4452	22.2142 0071	22.1050 1167	21.8888 7289	23
24	23.2659 7957	23.1466 8952	23.0282 5083	22.7938 9831	24
25	24.2054 6591	24.0764 6648	23.9484 2275	23.6951 6843	25
26	25.1426 0939	25.0035 3949	24.8655 3763	24.5926 9884	26
27	26.0774 1585	25.9279 1639	25.7796 0561	25.4865 0506	27
28	27.0098 9112	26.8496 0503	26.6906 3682	26.3766 0254	28
29	27.9400 4102	27.7686 1324	27.5986 4135	27.2630 0668	29
30	28.8678 7134	28.6849 4879	28.5036 2925	28.1457 3278	30
31	29.7933 8787	29.5986 1947	29.4056 1055	29.0247 9612	31
32	30.7165 9638	30.5096 3303	30.3045 9523	29.9002 1189	32
33	31.6375 0262	31.4179 9720	31.2005 9325	30.7719 9524	33
34	32.5561 1234	32.3237 1967	32.0936 1454	31.6401 6122	34
35	33.4724 3126	33.2268 0814	32.9836 6898	32.5047 2486	35
36	34.3864 6510	34.1272 7025	33.8707 6642	33.3657 0109	36
37	35.2982 1955	35.0251 1366	34.7549 1670	34.2231 0481	37
38	36.2077 0030	35.9203 4597	35.6361 2960	35.0769 5084	38
39	37.1149 1302	36.8129 7478	36.5144 1488	35.9272 5394	39
40	38.0198 6336	37.7030 0767	37.3897 8228	36.7740 2881	40
41	38.9225 5697	38.5904 5217	38.2622 4147	37.6172 9009	41
42	39.8229 9947	39.4753 1582	39.1318 0213	38.4570 5236	42
43	40.7211 9648	40.3576 0612	39.9984 7388	39.2933 3013	43
44	41.6171 5359	41.2373 3056	40.8622 6633	40.1261 3788	44
45	42.5108 7640	42.1144 9659	41.7231 8903	40.9554 8999	45
46	43.4023 7047	42.9891 1167	42.5812 5153	41.7814 0081	46
47	44.2916 4137	43.8611 8320	43.4364 6332	42.6038 8461	47
48	45.1786 9463	44.7307 1859	44.2888 3387	43.4229 5562	48
49	46.0635 3580	45.5977 2521	45.1383 7263	44.2386 2799	49
50	46.9461 7037	46.4622 1042	45.9850 8900	45.0509 1582	50

$$a_{\overline{n}|i} = (1 - v^n)/i$$

n	$\frac{1}{4}\%$	$\frac{7}{24}\%$	$\frac{1}{3}\%$	$\frac{5}{12}\%$	n
51	47.8266 0386	47.3241 8154	46.8289 9236	45.8598 3317	51
52	48.7048 4176	48.1836 4589	47.6700 9205	46.6653 9401	52
53	49.5808 8953	49.0406 1076	48.5083 9739	47.4676 1228	53
54	50.4547 5265	49.8950 8341	49.3439 1767	48.2665 0184	54
55	51.3264 3656	50.7470 7110	50.1766 6213	49.0620 7651	55
56	52.1959 4669	51.5965 8106	51.0066 3999	49.8543 5003	56
57	53.0632 8847	52.4436 2048	51.8338 6046	50.6433 3612	57
58	53.9284 6730	53.2881 9656	52.6583 3268	51.4290 4840	58
59	54.7914 8858	54.1303 1645	53.4800 6580	52.2115 0046	59
60	55.6523 5769	54.9699 8730	54.2990 6890	52.9907 0584	60
61	56.5110 7999	55.8072 1623	55.1153 5106	53.7666 7800	61
62	57.3676 6083	56.6420 1035	55.9289 2133	54.5394 3035	62
63	58.2221 0557	57.4743 7673	56.7397 8870	55.3089 7627	63
64	59.0744 1952	58.3043 2244	57.5479 6216	56.0753 2905	64
65	59.9246 0800	59.1318 5451	58.3534 5065	56.8385 0194	65
66	60.7726 7631	59.9569 7996	59.1562 6311	57.5985 0814	66
67	61.6186 2974	60.7797 0580	59.9564 0842	58.3553 6078	67
68	62.4624 7355	61.6000 3900	60.7538 9543	59.1090 7296	68
69	63.3042 1302	62.4179 8652	61.5487 3299	59.8596 5770	69
70	64.1438 5339	63.2335 5529	62.3409 2989	60.6071 2798	70
71	64.9813 9989	64.0467 5224	63.1304 9490	61.3514 9672	71
72	65.8168 5774	64.8575 8427	63.9174 3678	62.0927 7680	72
73	66.6502 3216	65.6660 5824	64.7017 6423	62.8309 8103	73
74	67.4815 2834	66.4721 8103	65.4834 8595	63.5661 2216	74
75	68.3107 5146	67.2759 5945	66.2626 1058	64.2982 1292	75
76	69.1379 0670	68.0774 0035	67.0391 4676	65.0272 6596	76
77	69.9629 9920	68.8765 1050	67.8131 0308	65.7532 9388	77
78	70.7860 3411	69.6732 9670	68.5844 8812	66.4763 0924	78
79	71.6070 1657	70.4677 6569	69.3533 1042	67.1963 2453	79
80	72.4259 5169	71.2599 2422	70.1195 7849	67.9133 5221	80
81	73.2428 4458	72.0497 7901	70.8833 0082	68.6274 0467	81
82	74.0577 0033	72.8373 3675	71.6444 8587	69.3384 9426	82
83	74.8705 2402	73.6226 0413	72.4031 4206	70.0466 3326	83
84	75.6813 2072	74.4055 8781	73.1592 7780	70.7518 3393	84
85	76.4900 9548	75.1862 9442	73.9129 0146	71.4541 0846	85
86	77.2968 5335	75.9647 3060	74.6640 2139	72.1534 6898	86
87	78.1015 9935	76.7409 0294	75.4126 4591	72.8499 2759	87
88	78.9043 3850	77.5148 1803	76.1587 8329	73.5434 9633	88
89	79.7050 7581	78.2864 8243	76.9024 4182	74.2341 8720	89
90	80.5038 1627	79.0559 0268	77.6436 2972	74.9220 1212	90
91	81.3005 6486	79.8230 8532	78.3823 5520	75.6069 8300	91
92	82.0953 2654	80.5880 3685	79.1186 2645	76.2891 1168	92
93	82.8881 0628	81.3507 6377	79.8524 5161	76.9684 0995	93
94	83.6789 0900	82.1112 7253	80.5838 3882	77.6448 8955	94
95	84.4677 3966	82.8695 6959	81.3127 9616	78.3185 6218	95
96	85.2546 0315	83.6256 6138	82.0393 3172	78.9894 3950	96
97	86.0395 0439	84.3795 5432	82.7634 5354	79.6575 3308	97
98	86.8224 4827	85.1312 5480	83.4851 6964	80.3228 5450	98
99	87.6034 3967	85.8807 6919	84.2044 8802	80.9854 1524	99
100	88.3824 8346	86.6281 0386	84.9214 1663	81.6452 2677	100

Present Value of 1 per Period at Compound Interest

$$a_{\overline{n}|i} = (1 - v^n)/i$$

n	$\frac{1}{4}\%$	$\frac{7}{24}\%$	$\frac{1}{3}\%$	$\frac{5}{12}\%$	n
101	89.1595 8450	87.3732 6514	85.6359 6344	82.3023 0049	101
102	89.9347 4763	88.1162 5935	86.3481 3635	82.9566 4777	102
103	90.7079 7768	88.8570 9280	87.0579 4323	83.6082 7991	103
104	91.4792 7948	89.5957 7177	87.7653 9195	84.2572 0818	104
105	92.2486 5784	90.3323 0252	88.4704 9034	84.9034 4381	105
106	93.0161 1755	91.0666 9131	89.1732 4621	85.5469 9795	106
107	93.7816 6339	91.7989 4436	89.8736 6735	86.1878 8175	107
108	94.5453 0014	92.5290 6788	90.5717 6150	86.8261 0628	108
109	95.3070 3256	93.2570 6806	91.2675 3641	87.4616 8258	109
110	96.0668 6539	93.9829 5109	91.9609 9977	88.0946 2163	110
111	96.8248 0338	94.7067 2312	92.6521 5927	88.7249 3437	111
112	97.5808 5126	95.4283 9028	93.3410 2255	89.3526 3171	112
113	98.3350 1372	96.1479 5870	94.0275 9726	89.9777 2450	113
114	99.0872 9548	96.8654 3448	94.7118 9098	90.6002 2354	114
115	99.8377 0123	97.5808 2372	95.3939 1131	91.2201 3959	115
116	100.5862 3564	98.2941 3246	96.0736 6578	91.8374 8338	116
117	101.3329 0338	99.0053 6678	96.7511 6194	92.4522 6558	117
118	102.0777 0911	99.7145 3269	97.4264 0727	93.0644 9681	118
119	102.8206 5747	100.4216 3621	98.0994 0927	93.6741 8767	119
120	103.5617 5308	101.1266 8335	98.7701 7538	94.2813 4869	120
121	104.3010 0058	101.8296 8009	99.4387 1304	94.8859 9036	121
122	105.0384 0457	102.5306 3237	100.1050 2964	95.4881 2315	122
123	105.7739 6965	103.2295 4616	100.7691 3256	96.0877 5747	123
124	106.5077 0040	103.9264 2738	101.4310 2916	96.6849 0367	124
125	107.2396 0139	104.6212 8194	102.0907 2677	97.2795 7209	125
126	107.9696 7720	105.3141 1573	102.7482 3269	97.8717 7301	126
127	108.6979 3237	106.0049 3464	103.4035 5420	98.4615 1666	127
128	109.4243 7144	106.6937 4451	104.0566 9857	99.0488 1324	128
129	110.1489 9894	107.3805 5120	104.7076 7303	99.6336 7290	129
130	110.8718 1939	108.0653 6053	105.3564 8478	100.2161 0576	130
131	111.5928 3730	108.7481 7831	106.0031 4101	100.7961 2189	131
132	112.3120 5716	109.4290 1032	106.6476 4888	101.3737 3131	132
133	113.0294 8345	110.1078 6235	107.2900 1552	101.9489 4401	133
134	113.7451 2065	110.7847 4016	107.9302 4806	102.5217 6994	134
135	114.4589 7321	111.4596 4947	108.5683 5358	103.0922 1899	135
136	115.1710 4560	112.1325 9603	109.2043 3915	103.6603 0104	136
137	115.8813 4224	112.8035 8553	109.8382 1181	104.2260 2590	137
138	116.5898 6758	113.4726 2368	110.4699 7859	104.7894 0335	138
139	117.2966 2601	114.1397 1613	111.0996 4646	105.3504 4314	139
140	118.0016 2196	114.8048 6856	111.7272 2242	105.9091 5496	140
141	118.7048 5981	115.4680 8660	112.3527 1341	106.4655 4847	141
142	119.4063 4395	116.1293 7588	112.9761 2636	107.0196 3330	142
143	120.1060 7875	116.7887 4201	113.5974 6817	107.5714 1902	143
144	120.8040 6858	117.4461 9058	114.2167 4572	108.1209 1517	144
145	121.5003 1778	118.1017 2717	114.8339 6586	108.6681 3126	145
146	122.1948 3071	118.7553 5734	115.4491 3545	109.2130 7674	146
147	122.8876 1168	119.4070 8663	116.0622 6128	109.7557 6103	147
148	123.5786 6502	120.0569 2057	116.6733 5015	110.2961 9353	148
149	124.2679 9503	120.7048 6467	117.2824 0882	110.8343 8356	149
150	124.9556 0601	121.3509 2444	117.8894 4404	111.3703 4044	150

$$a_{\overline{n}|i} = (1 - v^n)/i$$

n	1/4%	7/24%	1/3%	5/12%	n
151	125.6415 0226	121.9951 0534	118.4944 6254	111.9040 7343	151
152	126.3256 8804	122.6374 1284	119.0974 7100	112.4355 9176	152
153	127.0081 6762	123.2778 5240	119.6984 7612	112.9649 0463	153
154	127.6889 4525	123.9164 2944	120.2974 8454	113.4920 2117	154
155	128.3680 2519	124.5531 4937	120.8945 0290	114.0169 5051	155
156	129.0454 1166	125.1880 1761	121.4895 3781	114.5397 0171	156
157	129.7211 0889	125.8210 3954	122.0825 9587	115.0602 8383	157
158	130.3951 2109	126.4522 2052	122.6736 8363	115.5787 0585	158
159	131.0674 5246	127.0815 6591	123.2628 0764	116.0949 7674	159
160	131.7381 0719	127.7090 8105	123.8499 7443	116.6091 0543	160
161	132.4070 8946	128.3347 7125	124.4351 9050	117.1211 0081	161
162	133.0744 0346	128.9586 4184	125.0184 6233	117.6309 7172	162
163	133.7400 5332	129.5806 9809	125.5997 9638	118.1387 2699	163
164	134.4040 4321	130.2009 4529	126.1791 9909	118.6443 7539	164
165	135.0663 7727	130.8193 8870	126.7566 7687	119.1479 2566	165
166	135.7270 5962	131.4360 3355	127.3322 3612	119.6493 8641	166
167	136.3860 9439	132.0508 8509	127.9058 8322	120.1487 6662	167
168	137.0434 8567	132.6639 4853	128.4776 2451	120.6460 7460	168
169	137.6992 3758	133.2752 2907	129.0474 6633	121.1413 1907	169
170	138.3533 5419	133.8847 3189	129.6154 1499	121.6345 0858	170
171	139.0058 3959	134.4924 6216	130.1814 7677	122.1256 5166	171
172	139.6566 9785	135.0984 2504	130.7456 5795	122.6147 5680	172
173	140.3059 3302	135.7026 2567	131.3079 6478	123.1018 3246	173
174	140.9535 4914	136.3050 6917	131.8684 0347	123.5868 8705	174
175	141.5995 5027	136.9057 6066	132.4269 8025	124.0699 2898	175
176	142.2439 4042	137.5047 0522	132.9837 0128	124.5509 6658	176
177	142.8867 2361	138.1019 0794	133.5385 7275	125.0300 0817	177
178	143.5279 0385	138.6973 7389	134.0916 0079	125.5070 6204	178
179	144.1674 8514	139.2911 0811	134.6427 9152	125.9821 3643	179
180	144.8054 7146	139.8831 1564	135.1921 5106	126.4552 3956	180
181	145.4418 6679	140.4734 0151	135.7396 8549	126.9263 7961	181
182	146.0766 7510	141.0619 7071	136.2854 0086	127.3955 6471	182
183	146.7099 0035	141.6488 2825	136.8293 0322	127.8628 0299	183
184	147.3415 4649	142.2339 7909	137.3713 9860	128.3281 0253	184
185	147.9716 1744	142.8174 2821	137.9116 9300	128.7914 7136	185
186	148.6001 1715	143.3991 8055	138.4501 9241	129.2529 1749	186
187	149.2270 4952	143.9792 4105	138.9869 0277	129.7124 4891	187
188	149.8524 1848	144.5576 1463	139.5218 3005	130.1700 7357	188
189	150.4762 2791	145.1343 0618	140.0549 8016	130.6257 9936	189
190	151.0984 8170	145.7093 2062	140.5863 5901	131.0796 3418	190
191	151.7191 8375	146.2826 6280	141.1159 7248	131.5315 8586	191
192	152.3383 3790	146.8543 3760	141.6438 2643	131.9816 6223	192
193	152.9559 4803	147.4243 4986	142.1699 2672	132.4298 7106	193
194	153.5720 1799	147.9927 0442	142.6942 7917	132.8762 2010	194
195	154.1865 5161	148.5594 0611	143.2168 8958	133.3207 1707	195
196	154.7995 5272	149.1244 5971	143.7377 6375	133.7633 6965	196
197	155.4110 2516	149.6878 7004	144.2569 0743	134.2041 8550	197
198	156.0209 7273	150.2496 4187	144.7743 2639	134.6431 7224	198
199	156.6293 9923	150.8097 7996	145.2900 2635	135.0803 3746	199
200	157.2363 0846	151.3682 8907	145.8040 1302	135 5156 8872	200

Present Value of 1 per Period at Compound Interest

$$a_{\overline{n}|i} = (1 - v^n)/i$$

n	$\frac{1}{2}\%$	$\frac{7}{12}\%$	$\frac{5}{8}\%$	$\frac{2}{3}\%$	n
1	0.9950 2488	0.9942 0050	0.9937 8882	0.9933 7748	1
2	1.9850 9938	1.9826 3513	1.9814 0504	1.9801 7631	2
3	2.9702 4814	2.9653 3733	2.9628 8699	2.9604 4004	3
4	3.9504 9566	3.9423 4034	3.9382 7279	3.9342 1196	4
5	4.9258 6633	4.9136 7723	4.9076 0029	4.9015 3506	5
6	5.8963 8441	5.8793 8084	5.8709 0712	5.8624 5205	6
7	6.8620 7404	6.8394 8385	6.8282 3068	6.8170 0535	7
8	7.8229 5924	7.7940 1875	7.7796 0813	7.7652 3710	8
9	8.7790 6392	8.7430 1781	8.7250 7640	8.7071 8917	9
10	9.7304 1186	9.6865 1315	9.6646 7220	9.6429 0315	10
11	10.6770 2673	10.6245 3669	10.5984 3200	10.5724 2035	11
12	11.6189 3207	11.5571 2016	11.5263 9205	11.4957 8180	12
13	12.5561 5131	12.4842 9511	12.4485 8837	12.4130 2828	13
14	13.4887 0777	13.4060 9291	13.3650 5676	13.3242 0028	14
15	14.4166 2465	14.3225 4473	14.2758 3281	14.2293 3802	15
16	15.3399 2502	15.2336 8160	15.1809 5186	15.1284 8148	16
17	16.2586 3186	16.1395 3432	16.0804 4905	16.0216 7035	17
18	17.1727 6802	17.0401 3354	16.9743 5931	16.9089 4405	18
19	18.0823 5624	17.9355 0974	17.8627 1733	17.7903 4177	19
20	18.9874 1915	18.8256 9320	18.7455 5759	18.6659 0242	20
21	19.8879 7925	19.7107 1404	19.6229 1438	19.5356 6466	21
22	20.7840 5896	20.5906 0220	20.4948 2174	20.3996 6688	22
23	21.6756 8055	21.4653 8745	21.3613 1353	21.2579 4723	23
24	22.5628 6622	22.3350 9938	22.2224 2338	22.1105 4361	24
25	23.4456 3803	23.1997 6741	23.0781 8473	22.9574 9365	25
26	24.3240 1794	24.0594 2079	23.9286 3079	23.7988 3475	26
27	25.1980 2780	24.9140 8862	24.7737 9457	24.6346 0406	27
28	26.0676 8936	25.7637 9979	25.6137 0889	25.4648 3847	28
29	26.9330 2423	26.6085 8307	26.4484 0635	26.2895 7464	29
30	27.7940 5397	27.4484 6702	27.2779 1935	27.1088 4898	30
31	28.6507 9997	28.2834 8006	28.1022 8010	27.9226 9766	31
32	29.5032 8355	29.1136 5044	28.9215 2060	28.7311 5662	32
33	30.3515 2592	29.9390 0625	29.7356 7265	29.5342 6154	33
34	31.1955 4818	30.7595 7540	30.5447 6785	30.3320 4789	34
35	32.0353 7132	31.5753 8566	31.3488 3761	31.1245 5088	35
36	32.8710 1624	32.3864 6463	32.1479 1315	31.9118 0551	36
37	33.7025 0372	33.1928 3974	32.9420 2550	32.6938 4653	37
38	34.5298 5445	33.9945 3828	33.7312 0546	33.4707 0848	38
39	35.3530 8900	34.7915 8736	34.5154 8369	34.2424 2564	39
40	36.1722 2786	35.5840 1396	35.2948 9062	35.0090 3209	40
41	36.9872 9141	36.3718 4487	36.0694 5652	35.7705 6168	41
42	37.7982 9991	37.1551 0676	36.8392 1145	36.5270 4803	42
43	38.6052 7354	37.9338 2612	37.6041 8529	37.2785 2453	43
44	39.4082 3238	38.7080 2929	38.3644 0774	38.0250 2437	44
45	40.2071 9640	39.4777 4248	39.1199 0831	38.7665 8050	45
46	41.0021 8547	40.2429 9170	39.8707 1634	39.5032 2566	46
47	41.7932 1937	41.0038 0287	40.6168 6096	40.2349 9238	47
48	42.5803 1778	41.7602 0170	41.3583 7114	40.9619 1296	48
49	43.3635 0028	42.5122 1380	42.0952 7566	41.6840 1949	49
50	44.1427 8635	43.2598 6460	42.8276 0314	42.4013 4387	50

$$a_{\overline{n}|i} = (1 - v^n)/i$$

n	$\frac{1}{2}\%$	$\frac{7}{12}\%$	$\frac{5}{8}\%$	$\frac{2}{3}\%$	n
51	44.9181 9537	44.0031 7940	43.5553 8201	43.1139 1775	51
52	45.6897 4664	44.7421 8335	44.2786 4050	43.8217 7260	52
53	46.4574 5934	45.4769 0144	44.9974 0671	44.5249 3967	53
54	47.2213 5258	46.2073 5853	45.7117 0853	45.2234 5000	54
55	47.9814 4535	46.9335 7933	46.4215 7370	45.9173 3444	55
56	48.7377 5657	47.6555 8841	47.1270 2976	46.6066 2362	56
57	49.4903 0505	48.3734 1020	47.8281 0410	47.2913 4796	57
58	50.2391 0950	49.0870 6898	48.5248 2396	47.9715 3771	58
59	50.9841 8855	49.7965 8889	49.2172 1636	48.6472 2289	59
60	51.7255 6075	50.5019 9394	49.9053 0818	49.3184 3334	60
61	52.4632 4453	51.2033 0800	50.5891 2614	49.9851 9868	61
62	53.1972 5824	51.9005 5478	51.2686 9679	50.6475 4835	62
63	53.9276 2014	52.5937 5787	51.9440 4650	51.3055 1161	63
64	54.6543 4839	53.2829 4073	52.6152 0149	51.9591 1749	64
65	55.3774 6109	53.9681 2668	53.2821 8781	52.6083 9486	65
66	56.0969 7621	54.6493 3888	53.9450 3137	53.2533 7238	66
67	56.8129 1165	55.3266 0040	54.6037 5788	53.8940 7852	67
68	57.5252 8522	55.9999 3413	55.2583 9293	54.5305 4158	68
69	58.2341 1465	56.6693 6287	55.9089 6191	55.1627 8965	69
70	58.9394 1756	57.3349 0925	56.5554 9010	55.7908 5064	70
71	59.6412 1151	57.9965 9579	57.1980 0258	56.4147 5229	71
72	60.3395 1394	58.6544 4488	57.8365 2431	57.0345 2215	72
73	61.0343 4222	59.3084 7877	58.4710 8006	57.6501 8756	73
74	61.7257 1366	59.9587 1959	59.1016 9447	58.2617 7572	74
75	62.4136 4543	60.6051 8934	59.7283 9201	58.8693 1363	75
76	63.0981 5466	61.2479 0988	60.3511 9703	59.4728 2811	76
77	63.7792 5836	61.8869 0297	60.9701 3370	60.0723 4581	77
78	64.4569 7350	62.5221 9021	61.5852 2604	60.6678 9319	78
79	65.1313 1691	63.1537 9310	62.1964 9792	61.2594 9654	79
80	65.8023 0538	63.7817 3301	62.8039 7309	61.8471 8200	80
81	66.4699 5561	64.4060 3118	63.4076 7512	62.4309 7549	81
82	67.1342 8419	65.0267 0874	64.0076 2745	63.0109 0281	82
83	67.7953 0765	65.6437 8667	64.6038 5337	63.5869 8954	83
84	68.4530 4244	66.2572 8585	65.1963 7602	64.1592 6114	84
85	69.1075 0491	66.8672 2705	65.7852 1840	64.7277 4285	85
86	69.7587 1135	67.4736 3089	66.3704 0338	65.2924 5979	86
87	70.4066 7796	68.0765 1789	66.9519 5367	65.8534 3687	87
88	71.0514 2086	68.6759 0845	67.5298 9185	66.4106 9888	88
89	71.6929 5608	69.2718 2283	68.1042 4034	66.9642 7041	89
90	72.3312 9958	69.8642 8121	68.6750 2146	67.5141 7590	90
91	72.9664 6725	70.4533 0363	69.2422 5735	68.0604 3964	91
92	73.5984 7487	71.0389 1001	69.8059 7004	68.6030 8574	92
93	74.2273 3818	71.6211 2017	70.3661 8141	69.1421 3815	93
94	74.8530 7282	72.1999 5379	70.9229 1320	69.6776 2068	94
95	75.4756 9434	72.7754 3047	71.4761 8703	70.2095 5696	95
96	76.0952 1825	73.3475 6967	72.0260 2438	70.7379 7049	96
97	76.7116 5995	73.9163 9075	72.5724 4658	71.2628 8460	97
98	77.3250 3478	74.4819 1294	73.1154 7487	71.7843 2245	98
99	77.9353 5799	75.0441 5539	73.6551 3030	72.3023 0707	99
100	78.5426 4477	75.6031 3712	74.1914 3384	72.8168 6132	100

Present Value of 1 per Period at Compound Interest

$$a_{\overline{n}|i} = (1 - v^n)/i$$

n	½%	7/12%	⅝%	⅔%	n
101	79.1469 1021	76.1588 7702	74.7244 0630	73.3280 0792	101
102	79.7481 6937	76.7113 9392	75.2540 6838	73.8357 6944	102
103	80.3464 3718	77.2607 0648	75.7804 4062	74.3401 6830	103
104	80.9417 2854	77.8068 3331	76.3035 4348	74.8412 2677	104
105	81.5340 5825	78.3497 9288	76.8233 9724	75.3389 6697	105
106	82.1234 4104	78.8896 0355	77.3400 2210	75.8334 1088	106
107	82.7098 9158	79.4262 8359	77.8534 3812	76.3245 8032	107
108	83.2934 2446	79.9598 5115	78.3636 6521	76.8124 9699	108
109	83.8740 5419	80.4903 2428	78.8707 2319	77.2971 8242	109
110	84.4517 9522	81.0177 2093	79.3746 3174	77.7786 5801	110
111	85.0266 6191	81.5420 5895	79.8754 1043	78.2569 4503	111
112	85.5986 6856	82.0633 5606	80.3730 7868	78.7320 6458	112
113	86.1678 2942	82.5816 2991	80.8676 5583	79.2040 3764	113
114	86.7341 5862	83.0968 9803	81.3591 6108	79.6728 8505	114
115	87.2976 7027	83.6091 7785	81.8476 1349	80.1386 2751	115
116	87.8583 7838	84.1184 8671	82.3330 3204	80.6012 8559	116
117	88.4162 9690	84.6248 4182	82.8154 3557	81.0608 7970	117
118	88.9714 3970	85.1282 6033	83.2948 4280	81.5174 3015	118
119	89.5238 2059	85.6287 5926	83.7712 7235	81.9709 5708	119
120	90.0734 5333	86.1263 5554	84.2447 4271	82.4214 8052	120
121	90.6203 5157	86.6210 6602	84.7152 7226	82.8690 2036	121
122	91.1645 2892	87.1129 0742	85.1828 7926	83.3135 9636	122
123	91.7059 9893	87.6018 9638	85.6475 8188	83.7552 2815	123
124	92.2447 7505	88.0880 4946	86.1093 9814	84.1939 3523	124
125	92.7808 7070	88.5713 8308	86.5683 4597	84.6297 3696	125
126	93.3142 9920	89.0519 1361	87.0244 4320	85.0626 5259	126
127	93.8450 7384	89.5296 5731	87.4777 0753	85.4927 0122	127
128	94.3732 0780	90.0046 3032	87.9281 5655	85.9199 0185	128
129	94.8987 1422	90.4768 4873	88.3758 0776	86.3442 7334	129
130	95.4216 0619	90.9463 2851	88.8206 7852	86.7658 3442	130
131	95.9418 9671	91.4130 8554	89.2627 8610	87.1846 0371	131
132	96.4595 9872	91.8771 3561	89.7021 4768	87.6005 9969	132
133	96.9747 2509	92.3384 9442	90.1387 8030	88.0138 4072	133
134	97.4872 8865	92.7971 7758	90.5727 0092	88.4243 4507	134
135	97.9973 0214	93.2532 0060	91.0039 2638	88.8321 3084	135
136	98.5047 7825	93.7065 7892	91.4324 7342	89.2372 1604	136
137	99.0097 2960	94.1573 2787	91.8583 5868	89.6396 1856	137
138	99.5121 6875	94.6054 6270	92.2815 9869	90.0393 5616	138
139	100.0121 0821	95.0509 9857	92.7022 0988	90.4364 4649	139
140	100.5095 6041	95.4939 5056	93.1202 0857	90.8309 0709	140
141	101.0045 3772	95.9343 3364	93.5356 1100	91.2227 5536	141
142	101.4970 5246	96.3721 6272	93.9484 3330	91.6120 0861	142
143	101.9871 1688	96.8074 5261	94.3586 9148	91.9986 8402	143
144	102.4747 4316	97.2402 1804	94.7664 0147	92.3827 9867	144
145	102.9599 4344	97.6704 7364	95.1715 7910	92.7643 6952	145
146	103.4427 2979	98.0982 3397	95.5742 4010	93.1434 1340	146
147	103.9231 1422	98.5235 1350	95.9744 0010	93.5199 4706	147
148	104.4011 0868	98.9463 2663	96.3720 7463	93.8939 8712	148
149	104.8767 2505	99.3666 8765	96.7672 7913	94.2655 5010	149
150	105.3499 7518	99.7846 1078	97.1600 2895	94.6346 5239	150

$$a_{\overline{n}|i} = (1 - v^n)/i$$

n	½%	7/12%	⅝%	⅔%	n
151	105.8208 7082	100.2001 1017	97.5503 3933	95.0013 1029	151
152	106.2894 2371	100.6131 9987	97.9382 2542	95.3655 4000	152
153	106.7556 4548	101.0238 9385	98.3237 0228	95.7273 5759	153
154	107.2195 4774	101.4322 0601	98.7067 8488	96.0867 7904	154
155	107.6811 4203	101.8381 5017	99.0874 8808	96.4438 2021	155
156	108.1404 3983	102.2417 4005	99.4658 2666	96.7984 9687	156
157	108.5974 5257	102.6429 8931	99.8418 1532	97.1508 2468	157
158	109.0521 9161	103.0419 1152	100.2154 6864	97.5008 1919	158
159	109.5046 6827	103.4385 2019	100.5868 0113	97.8484 9586	159
160	109.9548 9380	103.8328 2872	100.9558 2721	98.1938 7003	160
161	110.4028 7940	104.2248 5046	101.3225 6120	98.5369 5695	161
162	110.8486 3622	104.6145 9866	101.6870 1734	98.8777 7178	162
163	111.2921 7534	105.0020 8652	102.0492 0978	99.2163 2956	163
164	111.7335 0780	105.3873 2715	102.4091 5258	99.5526 4523	164
165	112.1726 4458	105.7703 3357	102.7668 5971	99.8867 3364	165
166	112.6095 9660	106.1511 1874	103.1223 4505	100.2186 0955	166
167	113.0443 7473	106.5296 9555	103.4756 2241	100.5482 8760	167
168	113.4769 8978	106.9060 7680	103.8267 0550	100.8757 8236	168
169	113.9074 5251	107.2802 7523	104.1756 0795	101.2011 0828	169
170	114.3357 7365	107.6523 0349	104.5223 4330	101.5242 7972	170
171	114.7619 6383	108.0221 7417	104.8669 2502	101.8453 1095	171
172	115.1860 3366	108.3898 9979	105.2093 6648	102.1642 1614	172
173	115.6079 9369	108.7554 9278	105.5496 8098	102.4810 0939	173
174	116.0278 5442	109.1189 6552	105.8878 8172	102.7957 0466	174
175	116.4456 2629	109.4803 3029	106.2239 8183	103.1083 1586	175
176	116.8613 1969	109.8395 9933	106.5579 9436	103.4188 5678	176
177	117.2749 4496	110.1967 8478	106.8899 3229	103.7273 4115	177
178	117.6865 1240	110.5518 9874	107.2198 0848	104.0337 8257	178
179	118.0960 3224	110.9049 5322	107.5476 3576	104.3381 9457	179
180	118.5035 1467	111.2559 6015	107.8734 2684	104.6405 9061	180
181	118.9089 6982	111.6049 3142	108.1971 9438	104.9409 8402	181
182	119.3124 0778	111.9518 7882	108.5189 5094	105.2393 8807	182
183	119.7138 3859	112.2968 1411	108.8387 0900	105.5358 1593	183
184	120.1132 7222	112.6397 4894	109.1564 8100	105.8302 8070	184
185	120.5107 1863	112.9806 9492	109.4722 7925	106.1227 9536	185
186	120.9061 8769	113.3196 6359	109.7861 1603	106.4133 7285	186
187	121.2996 8925	113.6566 6640	110.0980 0351	106.7020 2598	187
188	121.6912 3308	113.9917 1477	110.4079 5379	106.9887 6750	188
189	122.0808 2894	114.3248 2002	110.7159 7893	107.2736 1007	189
190	122.4684 8650	114.6559 9342	111.0220 9086	107.5565 6626	190
191	122.8542 1543	114.9852 4619	111.3263 0147	107.8376 4857	191
192	123.2380 2530	115.3125 8945	111.6286 2258	108.1168 6941	192
193	123.6199 2567	115.6380 3429	111.9290 6592	108.3942 4111	193
194	123.9999 2604	115.9615 9171	112.2276 4315	108.6697 7590	194
195	124.3780 3586	116.2832 7265	112.5243 6586	108.9434 8597	195
196	124.7542 6454	116.6030 8801	112.8192 4558	109.2153 8338	196
197	125.1286 2143	116.9210 4859	113.1122 9374	109.4854 8015	197
198	125.5011 1585	117.2371 6516	113.4035 2173	109.7537 8819	198
199	125.8717 5707	117.5514 4842	113.6929 4085	110.0203 1937	199
200	126.2405 5430	117.8639 0899	113.9805 6234	110.2850 8543	200

Present Value of 1 per Period at Compound Interest

$$a_{\overline{n}|i} = (1 - v^n)/i$$

n	$\frac{3}{4}\%$	$\frac{7}{8}\%$	1%	$1\frac{1}{8}\%$	n
1	0.9925 5583	0.9913 2590	0.9900 9901	0.9888 7515	1
2	1.9777 2291	1.9740 5294	1.9703 9506	1.9667 4923	2
3	2.9555 5624	2.9482 5570	2.9409 8521	2.9337 4460	3
4	3.9261 1041	3.9140 0813	3.9019 6555	3.8899 8230	4
5	4.8894 3961	4.8713 8352	4.8534 3124	4.8355 8200	5
6	5.8455 9763	5.8204 5454	5.7954 7647	5.7706 6205	6
7	6.7946 3785	6.7612 9323	6.7281 9453	6.6953 3948	7
8	7.7366 1325	7.6939 7098	7.6516 7775	7.6097 3002	8
9	8.6715 7642	8.6185 5859	8.5660 1758	8.5139 4810	9
10	9.5995 7958	9.5351 2624	9.4713 0453	9.4081 0690	10
11	10.5206 7452	10.4437 4348	10.3676 2825	10.2923 1832	11
12	11.4349 1267	11.3444 7929	11.2550 7747	11.1666 9302	12
13	12.3423 4508	12.2374 0202	12.1337 4007	12.0313 4044	13
14	13.2430 2242	13.1225 7945	13.0037 0304	12.8863 6880	14
15	14.1369 9495	14.0000 7876	13.8650 5252	13.7318 8509	15
16	15.0243 1261	14.8699 6656	14.7178 7378	14.5679 9514	16
17	15.9050 2492	15.7323 0885	15.5622 5127	15.3948 0360	17
18	16.7791 8107	16.5871 7111	16.3982 6858	16.2124 1395	18
19	17.6468 2984	17.4346 1820	17.2260 0850	17.0209 2850	19
20	18.5080 1969	18.2747 1445	18.0455 5297	17.8204 4845	20
21	19.3627 9870	19.1075 2361	18.8569 8313	18.6110 7387	21
22	20.2112 1459	19.9331 0891	19.6603 7934	19.3929 0371	22
23	21.0533 1473	20.7515 3300	20.4558 2113	20.1660 3580	23
24	21.8891 4614	21.5628 5799	21.2433 8726	20.9305 6693	24
25	22.7187 5547	22.3671 4547	22.0231 5570	21.6865 9276	25
26	23.5421 8905	23.1644 5647	22.7952 0366	22.4342 0792	26
27	24.3594 9286	23.9548 5152	23.5596 0759	23.1735 0598	27
28	25.1707 1251	24.7383 9060	24.3164 4316	23.9045 7946	28
29	25.9758 9331	25.5151 3319	25.0657 8530	24.6275 1986	29
30	26.7750 8021	26.2851 3823	25.8077 0822	25.3424 1766	30
31	27.5683 1783	27.0484 6417	26.5422 8537	26.0493 6233	31
32	28.3556 5045	27.8051 6894	27.2695 8947	26.7484 4236	32
33	29.1371 2203	28.5553 0998	27.9896 9255	27.4397 4522	33
34	29.9127 7621	29.2989 4422	28.7026 6589	28.1233 5745	34
35	30.6826 5629	30.0361 2809	29.4085 8009	28.7993 6460	35
36	31.4468 0525	30.7669 1757	30.1075 0504	29.4678 5127	36
37	32.2052 6576	31.4913 6810	30.7995 0994	30.1289 0114	37
38	32.9580 8016	32.2095 3467	31.4846 6330	30.7825 9692	38
39	33.7052 9048	32.9214 7179	32.1630 3298	31.4290 2044	39
40	34.4469 3844	33.6272 3350	32.8346 8611	32.0682 5260	40
41	35.1830 6545	34.3268 7335	33.4996 8922	32.7003 7340	41
42	35.9137 1260	35.0204 4446	34.1581 0814	33.3254 6195	42
43	36.6389 2070	35.7079 9947	34.8100 0806	33.9435 9649	43
44	37.3587 3022	36.3895 9055	35.4554 5352	34.5548 5438	44
45	38.0731 8136	37.0652 6944	36.0945 0844	35.1593 1212	45
46	38.7823 1401	37.7350 8743	36.7272 3608	35.7570 4536	46
47	39.4861 6774	38.3990 9535	37.3536 9909	36.3481 2891	47
48	40.1847 8189	39.0573 4359	37.9739 5949	36.9326 3674	48
49	40.8781 9542	39.7098 8212	38.5880 7871	37.5106 4202	49
50	41.5664 4707	40.3567 6047	39.1961 1753	38.0822 1708	50

$$a_{\overline{n}|} = (1 - v^n)/i$$

n	$\frac{3}{4}\%$	$\frac{7}{8}\%$	1%	$1\frac{1}{8}\%$	n
51	42.2495 7525	40.9980 2772	39.7981 3617	38.6474 3345	51
52	42.9276 1812	41.6337 3256	40.3941 9423	39.2063 6188	52
53	43.6006 1351	42.2639 2324	40.9843 5072	39.7590 7232	53
54	44.2685 9902	42.8886 4757	41.5686 6408	40.3056 3394	54
55	44.9316 1193	43.5079 5298	42.1471 9216	40.8461 1514	55
56	45.5896 8926	44.1218 8647	42.7199 9224	41.3805 8358	56
57	46.2428 6776	44.7304 9465	43.2871 2102	41.9091 0613	57
58	46.8911 8388	45.3338 2369	43.8486 3468	42.4317 4896	58
59	47.5346 7382	45.9319 1939	44.4045 8879	42.9485 7746	59
60	48.1733 7352	46.5248 2716	44.9550 3841	43.4596 5633	60
61	48.8073 1863	47.1125 9198	45.5000 3803	43.9650 4952	61
62	49.4365 4455	47.6952 5847	46.0396 4161	44.4648 2029	62
63	50.0610 8640	48.2728 7085	46.5739 0258	44.9590 3119	63
64	50.6809 7906	48.8454 7296	47.1028 7385	45.4477 4407	64
65	51.2962 5713	49.4131 0826	47.6266 0777	45.9310 2009	65
66	51.9069 5497	49.9758 1984	48.1451 5621	46.4089 1975	66
67	52.5131 0667	50.5336 5040	48.6585 7050	46.8815 0284	67
68	53.1147 4607	51.0866 4228	49.1669 0149	47.3488 2852	68
69	53.7119 0677	51.6348 3745	49.6701 9949	47.8109 5527	69
70	54.3046 2210	52.1782 7752	50.1685 1435	48.2679 4094	70
71	54.8929 2516	52.7170 0374	50.6618 9539	48.7198 4270	71
72	55.4768 4880	53.2510 5699	51.1503 9148	49.1667 1714	72
73	56.0564 2561	53.7804 7781	51.6340 5097	49.6086 2016	73
74	56.6316 8795	54.3053 0638	52.1129 2175	50.0456 0708	74
75	57.2026 6794	54.8255 8253	52.5870 5124	50.4777 3259	75
76	57.7693 9746	55.3413 4575	53.0564 8637	50.9050 5077	76
77	58.3319 0815	55.8526 3520	53.5212 7364	51.3276 1510	77
78	58.8902 3141	56.3594 8966	53.9814 5905	51.7454 7847	78
79	59.4443 9842	56.8619 4762	54.4370 8817	52.1586 9317	79
80	59.9944 4012	57.3600 4721	54.8882 0611	52.5673 1092	80
81	60.5403 8722	57.8538 2623	55.3348 5753	52.9713 8286	81
82	61.0822 7019	58.3433 2216	55.7770 8666	53.3709 5957	82
83	61.6201 1930	58.8285 7215	56.2149 3729	53.7660 9104	83
84	62.1539 6456	59.3096 1304	56.6484 5276	54.1568 2674	84
85	62.6838 3579	59.7864 8133	57.0776 7600	54.5432 1557	85
86	63.2097 6257	60.2592 1321	57.5026 4951	54.9253 0588	86
87	63.7317 7427	60.7278 4457	57.9234 1535	55.3031 4549	87
88	64.2499 0002	61.1924 1097	58.3400 1520	55.6767 8169	88
89	64.7641 6875	61.6529 4768	58.7524 9030	56.0462 6126	89
90	65.2746 0918	62.1094 8965	59.1608 8148	56.4116 3041	90
91	65.7812 4981	62.5620 7152	59.5652 2919	56.7729 3490	91
92	66.2841 1892	63.0107 2765	59.9655 7346	57.1302 1992	92
93	66.7832 4458	63.4554 9210	60.3619 5392	57.4835 3021	93
94	67.2786 5467	63.8963 9861	60.7544 0982	57.8329 0997	94
95	67.7703 7685	64.3334 8065	61.1429 8002	58.1784 0294	95
96	68.2584 3856	64.7667 7140	61.5277 0299	58.5200 5235	96
97	68.7428 6705	65.1963 0375	61.9086 1682	58.8579 0096	97
98	69.2236 8938	65.6221 1028	62.2857 5923	59.1919 9106	98
99	69.7009 3239	66.0442 2333	62.6591 6755	59.5223 6446	99
100	70.1746 2272	66.4626 7492	63.0288 7877	59.8490 6251	100

Present Value of 1 per Period at Compound Interest

$$a_{\overline{n}|i} = (1 - v^n)/i$$

n	$\frac{3}{4}\%$	$\frac{7}{8}\%$	1%	$1\frac{1}{8}\%$	n
101	70.6447 8682	66.8774 9683	63.3949 2947	60.1721 2609	101
102	71.1114 5094	67.2887 2052	63.7573 5591	60.4915 9564	102
103	71.5746 4113	67.6963 7722	64.1161 9397	60.8075 1114	103
104	72.0343 8325	68.1004 9786	64.4714 7918	61.1199 1213	104
105	72.4907 0298	68.5011 1312	64.8232 4671	61.4288 3770	105
106	72.9436 2579	68.8982 5341	65.1715 3140	61.7343 2653	106
107	73.3931 7696	69.2919 4885	65.5163 6772	62.0364 1684	107
108	73.8393 8160	69.6822 2935	65.8577 8983	62.3351 4644	108
109	74.2822 6461	70.0691 2451	66.1958 3151	62.6305 5273	109
110	74.7218 5073	70.4526 6370	66.5305 2625	62.9226 7266	110
111	75.1581 6450	70.8328 7604	66.8619 0718	63.2115 4280	111
112	75.5912 3027	71.2097 9037	67.1900 0710	63.4971 9931	112
113	76.0210 7223	71.5834 3531	67.5148 5852	63.7796 7793	113
114	76.4477 1437	71.9538 3922	67.8364 9358	64.0590 1402	114
115	76.8711 8052	72.3210 3020	68.1549 4414	64.3352 4255	115
116	77.2914 9431	72.6850 3614	68.4702 4172	64.6083 9807	116
117	77.7086 7922	73.0458 8465	68.7824 1755	64.8785 1478	117
118	78.1227 5853	73.4036 0312	69.0915 0252	65.1456 2648	118
119	78.5337 5536	73.7582 1871	69.3975 2725	65.4097 6660	119
120	78.9416 9267	74.1097 5832	69.7005 2203	65.6709 6821	120
121	79.3465 9322	74.4582 4864	70.0005 1686	65.9292 6399	121
122	79.7484 7962	74.8037 1613	70.2975 4145	66.1846 8627	122
123	80.1473 7432	75.1461 8699	70.5916 2520	66.4372 6702	123
124	80.5432 9957	75.4856 8723	70.8827 9722	66.6870 3784	124
125	80.9362 7749	75.8222 4261	71.1710 8636	66.9340 3000	125
126	81.3263 3001	76.1558 7867	71.4565 2115	67.1782 7442	126
127	81.7134 7892	76.4866 2074	71.7391 2985	67.4198 0165	127
128	82.0977 4583	76.8144 9392	72.0189 4045	67.6586 4193	128
129	82.4791 5219	77.1395 2309	72.2959 8064	67.8948 2514	129
130	82.8577 1929	77.4617 3292	72.5702 7786	68.1283 8086	130
131	83.2334 6828	77.7811 4788	72.8418 5927	68.3593 3830	131
132	83.6064 2013	78.0977 9220	73.1107 5175	68.5877 2638	132
133	83.9765 9566	78.4116 8991	73.3769 8193	68.8135 7368	133
134	84.3440 1554	78.7228 6485	73.6405 7617	69.0369 0846	134
135	84.7087 0029	79.0313 4061	73.9015 6056	69.2577 5867	135
136	85.0706 7026	79.3371 4063	74.1599 6095	69.4761 5196	136
137	85.4299 4567	79.6402 8811	74.4158 0293	69.6921 1566	137
138	85.7865 4657	79.9408 0606	74.6691 1181	69.9056 7680	138
139	86.1404 9288	80.2387 1728	74.9199 1268	70.1168 6210	139
140	86.4918 0434	80.5340 4440	75.1682 3038	70.3256 9800	140
141	86.8405 0059	80.8268 0981	75.4140 8948	70.5322 1063	141
142	87.1866 0108	81.1170 3575	75.6575 1434	70.7364 2584	142
143	87.5301 2514	81.4047 4423	75.8985 2905	70.9383 6918	143
144	87.8710 9195	81.6899 5711	76.1371 5747	71.1380 6594	144
145	88.2095 2055	81.9726 9602	76.3734 2324	71.3355 4110	145
146	88.5454 2982	82.2529 8242	76.6073 4974	71.5308 1939	146
147	88.8788 3854	82.5308 3759	76.8389 6014	71.7239 2523	147
148	89.2097 6530	82.8062 8262	77.0682 7737	71.9148 8280	148
149	89.5382 2858	83.0793 3841	77.2953 2413	72.1037 1599	149
150	89.8642 4673	83.3500 2569	77.5201 2290	72.2904 4845	150

$$a_{\overline{n}|i} = (1 - v^n)/i$$

n	¾%	⅞%	1%	1⅛%	n
151	90.1878 3795	83.6183 6499	77.7426 9594	72.4751 0353	151
152	90.5090 2029	83.8843 7670	77.9630 6529	72.6577 0436	152
153	90.8278 1171	84.1480 8099	78.1812 5276	72.8382 7378	153
154	91.1442 2998	84.4094 9788	78.3972 7996	73.0168 3439	154
155	91.4582 9279	84.6686 4722	78.6111 6828	73.1934 0854	155
156	91.7700 1765	84.9255 4867	78.8229 3889	73.3680 1834	156
157	92.0794 2199	85.1802 2173	79.0326 1276	73.5406 8562	157
158	92.3865 2307	85.4326 8573	79.2402 1065	73.7114 3201	158
159	92.6913 3803	85.6829 5983	79.4457 5312	73 8802 7888	159
160	92.9938 8390	85.9310 6303	79.6492 6052	74.0472 4734	160
161	93.2941 7757	86.1770 1415	79.8507 5299	74.2123 5831	161
162	93.5922 3580	86.4208 3187	80.0502 5048	74.3756 3245	162
163	93.8880 7524	86.6625 3470	80.2477 7275	74.5370 9018	163
164	94.1817 1239	86.9021 4096	80.4433 3936	74.6967 5173	164
165	94.4731 6367	87.1396 6886	80.6369 6966	74.8546 3706	165
166	94.7624 4533	87.3751 3642	80.8286 8284	75.0107 6594	166
167	95.0495 7352	87.6085 6150	81.0184 9786	75.1651 5792	167
168	95.3345 6429	87.8399 6184	81.2064 3352	75.3178 3230	168
169	95.6174 3354	88.0693 5498	81.3925 0844	75.4688 0821	169
170	95.8981 9706	88.2967 5835	81.5767 4103	75.6181 0453	170
171	96.1768 7053	88.5221 8919	81.7591 4953	75.7657 3996	171
172	96.4534 6951	88.7456 6462	81.9397 5201	75.9117 3296	172
173	96.7280 0944	88.9672 0161	82.1185 6635	76.0561 0182	173
174	97.0005 0565	89.1868 1696	82.2956 1025	76.1988 6459	174
175	97.2709 7335	89.4045 2735	82.4709 0123	76.3400 3915	175
176	97.5394 2764	89.6203 4929	82.6444 5667	76.4796 4317	176
177	97.8058 8352	89.8342 9917	82.8162 9373	76.6176 9411	177
178	98.0703 5585	90.0463 9323	82.9864 2944	76.7542 0925	178
179	98.3328 5940	90.2566 4757	83.1548 8063	76.8892 0569	179
180	98.5934 0884	90.4650 7813	83.3216 6399	77.0227 0031	180
181	98.8520 1869	90.6717 0075	83.4867 9603	77.1547 0982	181
182	99.1087 0342	90.8765 3110	83.6502 9310	77.2852 5075	182
183	99.3634 7734	91.0795 8474	83.8121 7138	77.4143 3943	183
184	99.6163 5468	91.2808 7706	83.9724 4691	77.5419 9202	184
185	99.8673 4956	91.4804 2336	84.1311 3556	77.6682 2450	185
186	100.1164 7599	91.6782 3877	84.2882 5303	77.7930 5266	186
187	100.3637 4788	91.8743 3831	84.4438 1488	77.9164 9212	187
188	100.6091 7904	92.0687 3686	84.5978 3651	78.0385 5834	188
189	100.8527 8316	92.2614 4918	84.7503 3318	78.1592 6659	189
190	101.0945 7386	92.4524 8989	84.9013 1998	78.2786 3198	190
191	101.3345 6462	92.6418 7350	85.0508 1186	78.3966 6945	191
192	101.5727 6886	92.8296 1438	85.1988 2363	78.5133 9377	192
193	101.8091 9986	93.0157 2677	85.3453 6993	78.6288 1955	193
194	102.0438 7083	93.2002 2480	85.4904 6528	78.7429 6123	194
195	102.2767 9487	93.3831 2248	85.6341 2404	78.8558 3311	195
196	102.5078 9849	93.5644 3368	85.7763 6043	78.9674 4931	196
197	102.7374 5407	93.7441 7218	85.9171 8855	79.0778 2379	197
198	102.9652 1496	93.9223 5160	86.0566 2232	79.1869 7037	198
199	103.1912 8036	94.0989 8548	86.1946 7557	79.2949 0272	199
200	103.4156 6289	94.2740 8721	86.3313 6195	79.4016 3433	200

Present Value of 1 per Period at Compound Interest

$$a_{\overline{n}|i} = (1 - v^n)/i$$

n	1¼%	1⅜%	1½%	1¾%	n
1	0.9876 5432	0.9864 3650	0.9852 2167	0.9828 0098	1
2	1.9631 1538	1.9594 9346	1.9558 8342	1.9486 9875	2
3	2.9265 3371	2.9193 5237	2.9122 0042	2.8979 8403	3
4	3.8780 5798	3.8661 9222	3.8543 8465	3.8309 4254	4
5	4.8178 3504	4.8001 8962	4.7826 4497	4.7478 5508	5
6	5.7460 0992	5.7215 1874	5.6971 8717	5.6489 9762	6
7	6.6627 2585	6.6303 5140	6.5982 1396	6.5346 4139	7
8	7.5681 2429	7.5268 5712	7.4859 2508	7.4050 5297	8
9	8.4623 4498	8.4112 0308	8.3605 1732	8.2604 9432	9
10	9.3455 2591	9.2835 5421	9.2221 8455	9.1012 2291	10
11	10.2178 0337	10.1440 7320	10.0711 1779	9.9274 9181	11
12	11.0793 1197	10.9929 2054	10.9075 0521	10.7395 4969	12
13	11.9301 8466	11.8302 5454	11.7315 3222	11.5376 4097	13
14	12.7705 5275	12.6562 3136	12.5433 8150	12.3220 0587	14
15	13.6005 4592	13.4710 0504	13.3432 3301	13.0928 8046	15
16	14.4202 9227	14.2747 2754	14.1312 6405	13.8504 9677	16
17	15.2299 1829	15.0675 4874	14.9076 4931	14.5950 8282	17
18	16.0295 4893	15.8496 1651	15.6725 6089	15.3268 6272	18
19	16.8193 0759	16.6210 7671	16.4261 6837	16.0460 5673	19
20	17.5993 1613	17.3820 7320	17.1686 3879	16.7528 8130	20
21	18.3696 9495	18.1327 4792	17.9001 3673	17.4475 4919	21
22	19.1305 6291	18.8732 4086	18.6208 2437	18.1302 6948	22
23	19.8820 3744	19.6036 9012	19.3308 6145	18.8012 4764	23
24	20.6242 3451	20.3242 3193	20.0304 0537	19.4606 8565	24
25	21.3572 6865	21.0350 0067	20.7196 1120	20.1087 8196	25
26	22.0812 5299	21.7361 2890	21.3986 3172	20.7457 3166	26
27	22.7962 9925	22.4277 4737	22.0676 1746	21.3717 2644	27
28	23.5025 1778	23.1099 8508	22.7267 1671	21.9869 5474	28
29	24.2000 1756	23.7829 6925	23.3760 7558	22.5916 0171	29
30	24.8889 0623	24.4468 2540	24.0158 3801	23.1858 4934	30
31	25.5692 9010	25.1016 7734	24.6461 4582	23.7698 7650	31
32	26.2412 7418	25.7476 4719	25.2671 3874	24.3438 5897	32
33	26.9049 6215	26.3848 5543	25.8789 5442	24.9079 6951	33
34	27.5604 5644	27.0134 2089	26.4817 2849	25.4623 7789	34
35	28.2078 5822	27.6334 6080	27.0755 9458	26.0072 5100	35
36	28.8472 6737	28.2450 9080	27.6606 8431	26.5427 5283	36
37	29.4787 8259	28.8484 2496	28.2371 2740	27.0690 4455	37
38	30.1025 0133	29.4435 7579	28.8050 5163	27.5862 8457	38
39	30.7185 1983	30.0306 5430	29.3645 6288	28.0946 2857	39
40	31.3269 3316	30.6097 6996	29.9158 4520	28.5942 2955	40
41	31.9278 3522	31.1810 3079	30.4589 6079	29.0852 3789	41
42	32.5213 1874	31.7445 4332	30.9940 5004	29.5678 0135	42
53	33.1074 7530	32.3004 1264	31.5212 3157	30.0420 6522	43
44	33.6863 9536	32.8487 4243	32.0406 2223	30.5081 7221	44
45	34.2581 6825	33.3896 3495	32.5523 3718	30.9662 6261	45
46	34.8228 8222	33.9231 9108	33.0564 8983	31.4164 7431	46
47	35.3806 2442	34.4495 1031	33.5531 9195	31.8589 4281	47
48	35.9314 8091	34.9686 9081	34.0425 5365	32.2938 0129	48
49	36.4755 3670	35.4808 2941	34.5246 8339	32.7211 8063	49
50	37.0128 7574	35.9860 2161	34.9996 8807	33.1412 0946	50

$$a_{\overline{n}|i} = (1 - v^n)/i$$

n	1¼%	1⅜%	1½%	1¾%	n
51	37.5435 8099	36.4843 6164	35.4676 7298	33.5540 1421	51
52	38.0677 3431	36.9759 4243	35.9287 4185	33.9597 1913	52
53	38.5854 1660	37.4608 5566	36.3829 9690	34.3584 4633	53
54	39.0967 0776	37.9391 9178	36.8305 3882	34.7503 1579	54
55	39.6016 8667	38.4110 3998	37.2714 6681	35.1354 4550	55
56	40.1004 3128	38.8764 8826	37.7058 7863	35.5139 5135	56
57	40.5930 1855	39.3356 2344	38.1338 7058	35.8859 4727	57
58	41.0795 2449	39.7885 3114	38.5555 3751	36.2515 4523	58
59	41.5600 2419	40.2352 9582	38.9709 7292	36.6108 5526	59
60	42.0345 9179	40.6760 0081	39.3802 6889	36.9639 8552	60
61	42.5033 0054	41.1107 2829	39.7835 1614	37.3110 4228	61
62	42.9662 2275	41.5395 5935	40.1808 0408	37.6521 3000	62
63	43.4234 2988	41.9625 7396	40.5722 2077	37.9873 5135	63
64	43.8749 9247	42.3798 5101	40.9578 5298	38.3168 0723	64
65	44.3209 8022	42.7914 6832	41.3377 8618	38.6405 9678	65
66	44.7614 6195	43.1975 0266	41.7121 0461	38.9588 1748	66
67	45.1965 0563	43.5980 2975	42.0808 9125	39.2715 6509	67
68	45.6261 7840	43.9931 2429	42.4442 2783	39.5789 3375	68
69	46.0505 4656	44.3828 5997	42.8021 9490	39.8810 1597	69
70	46.4696 7562	44.7673 0946	43.1548 7183	40.1779 0267	70
71	46.8836 3024	45.1465 4448	43.5023 3678	40.4696 8321	71
72	47.2924 7431	45.5206 3573	43.8446 6677	40.7564 4542	72
73	47.6962 7093	45.8896 5300	44.1819 3771	41.0382 7560	73
74	48.0950 8240	46.2536 6511	44.5142 2434	41.3152 5857	74
75	48.4889 7027	46.6127 3994	44.8416 0034	41.5874 7771	75
76	48.8779 9533	46.9669 4445	45.1641 3826	41.8550 1495	76
77	49.2622 1761	47.3163 4471	45.4819 0962	42.1179 5081	77
78	49.6416 9640	47.6610 0588	45.7949 8485	42.3763 6443	78
79	50.0164 9027	48.0009 9224	46.1034 3335	42.6303 3359	79
80	50.3866 5706	48.3363 6719	46.4073 2349	42.8799 3474	80
81	50.7522 5389	48.6671 9328	46.7067 2265	43.1252 4298	81
82	51.1133 3717	48.9935 3221	47.0016 9720	43.3663 3217	82
83	51.4699 6264	49.3154 4484	47.2923 1251	43.6032 7486	83
84	51.8221 8532	49.6329 9122	47.5786 3301	43.8361 4237	84
85	52.1700 5958	49.9462 3055	47.8607 2218	44.0650 0479	85
86	52.5136 3909	50.2552 2125	48.1386 4254	44.2899 3099	86
87	52.8529 7688	50.5600 2096	48.4124 5571	44.5109 8869	87
88	53.1881 2531	50.8606 8653	48.6822 2237	44.7282 4441	88
89	53.5191 3611	51.1572 7401	48.9480 0234	44.9417 6355	89
90	53.8460 6035	51.4498 3873	49.2098 5452	45.1516 1037	90
91	54.1689 4850	51.7384 3524	49.4678 3696	45.3578 4803	91
92	54.4878 5037	52.0231 1738	49.7220 0686	45.5605 3860	92
93	54.8028 1518	52.3039 3823	49.9724 2055	45.7597 4310	93
94	55.1138 9154	52.5809 5016	50.2191 3355	45.9555 2147	94
95	55.4211 2744	52.8542 0484	50.4622 0054	46.1479 3265	95
96	55.7245 7031	53.1237 5324	50.7016 7541	46.3370 3455	96
97	56.0242 6698	53.3896 4561	50.9376 1124	46.5228 8408	97
98	56.3202 6368	53.6519 3155	51.1700 6034	46.7055 3718	98
99	56.6126 0610	53.9106 5998	51.3990 7422	46.8850 4882	99
100	56.9013 3936	54.1658 7914	51.6247 0367	47.0614 7304	100

Present Value of 1 per Period at Compound Interest

$$a_{\overline{n}|i} = (1 - v^n)/i$$

n	2%	2¼%	2½%	2¾%	n
1	0.9803 9216	0.9779 9511	0.9756 0976	0.9732 3601	1
2	1.9415 6094	1.9344 6955	1.9274 2415	1.9204 2434	2
3	2.8838 8327	2.8698 9687	2.8560 2356	2.8422 6213	3
4	3.8077 2870	3.7847 4021	3.7619 7421	3.7394 2787	4
5	4.7134 5951	4.6794 5253	4.6458 2850	4.6125 8186	5
6	5.6014 3089	5.5544 7680	5.5081 2536	5.4623 6678	6
7	6.4719 9107	6.4102 4626	6.3493 9060	6.2894 0806	7
8	7.3254 8144	7.2471 8461	7.1701 3717	7.0943 1441	8
9	8.1622 3671	8.0657 0622	7.9708 6553	7.8776 7826	9
10	8.9825 8501	8.8662 1635	8.7520 6393	8.6400 7616	10
11	9.7868 4805	9.6491 1134	9.5142 0871	9.3820 6926	11
12	10.5753 4122	10.4147 7882	10.2577 6460	10.1042 0366	12
13	11.3483 7375	11.1635 9787	10.9831 8497	10.8070 1086	13
14	12.1062 4877	11.8959 3924	11.6909 1217	11.4910 0814	14
15	12.8492 6350	12.6121 6551	12.3813 7773	12.1566 9892	15
16	13.5777 0931	13.3126 3131	13.0550 0266	12.8045 7315	16
17	14.2918 7188	13.9976 8343	13.7121 9772	13.4351 0769	17
18	14.9920 3125	14.6676 6106	14.3533 6363	14.0487 6661	18
19	15.6784 6201	15.3228 9590	14.9788 9134	14.6460 0157	19
20	16.3514 3334	15.9637 1237	15.5891 6229	15.2272 5213	20
21	17.0112 0916	16.5904 2775	16.1845 4857	15.7929 4612	21
22	17.6580 4820	17.2033 5232	16.7654 1324	16.3434 9987	22
23	18.2922 0412	17.8027 8955	17.3321 1048	16.8793 1861	23
24	18.9139 2560	18.3890 3624	17.8849 8583	17.4007 9670	24
25	19.5234 5647	18.9623 8263	18.4243 7642	17.9083 1795	25
26	20.1210 3576	19.5231 1260	18.9506 1114	18.4022 5592	26
27	20.7068 9780	20.0715 0376	19.4640 1087	18.8829 7413	27
28	21.2812 7236	20.6078 2764	19.9648 8866	19.3508 2640	28
29	21.8443 8466	21.1323 4977	20.4535 4991	19.8061 5708	29
30	22.3964 5555	21.6453 2985	20.9302 9259	20.2493 0130	30
31	22.9377 0152	22.1470 2186	21.3954 0741	20.6805 8520	31
32	23.4683 3482	22.6376 7419	21.8491 7796	21.1003 2623	32
33	23.9885 6355	23.1175 2977	22.2918 8094	21.5088 3332	33
34	24.4985 9172	23.5868 2618	22.7237 8628	21.9064 0712	34
35	24.9986 1933	24.0457 9577	23.1451 5734	22.2933 4026	35
36	25.4888 4248	24.4946 6579	23.5562 5107	22.6699 1753	36
37	25.9694 5341	24.9336 5848	23.9573 1812	23.0364 1609	37
38	26.4406 4060	25.3629 9118	24.3486 0304	23.3931 0568	38
39	26.9025 8883	25.7828 7646	24.7303 4443	23.7402 4884	39
40	27.3554 7924	26.1935 2221	25.1027 7505	24.0781 0106	40
41	27.7994 8945	26.5951 3174	25.4661 2200	24.4069 1101	41
42	28.2347 9358	26.9879 0390	25.8206 0683	24.7269 2069	42
43	28.6615 6233	27.3720 3316	26.1664 4569	25.0383 6563	43
44	29.0799 6307	27.7477 0969	26.5038 4945	25.3414 7507	44
45	29.4901 5987	28.1151 1950	26.8330 2386	25.6364 7209	45
46	29.8923 1360	28.4744 4450	27.1541 6962	25.9235 7381	46
47	30.2865 8196	28.8258 6259	27.4674 8255	26.2029 9154	47
48	30.6731 1957	29.1695 4777	27.7731 5371	26.4749 3094	48
49	31.0520 7801	29.5056 7019	28.0713 6947	26.7395 9215	49
50	31.4236 0589	29.8343 9627	28.3623 1168	26.9971 6998	50

$$a_{\overline{n}|i} = (1 - v^n)/i$$

n	2%	2¼%	2½%	2¾%	n
51	31.7878 4892	30.1558 8877	28.6461 5774	27.2478 5400	51
52	32.1449 4992	30.4703 0687	28.9230 8072	27.4918 2871	52
53	32.4950 4894	30.7778 0623	29.1932 4948	27.7292 7368	53
54	32.8382 8327	31.0785 3910	29.4568 2876	27.9603 6368	54
55	33.1747 8752	31.3726 5438	29.7139 7928	28.1852 6879	55
56	33.5046 9365	31.6602 9768	29.9648 5784	28.4041 5454	56
57	33.8281 3103	31.9416 1142	30.2096 1740	28.6171 8203	57
58	34.1452 2650	32.2167 3489	30.4484 0722	28.8245 0806	58
59	34.4561 0441	32.4858 0429	30.6813 7290	29.0262 8522	59
60	34.7608 8668	32.7489 5285	30.9086 5649	29.2226 6201	60
61	35.0596 9282	33.0063 1086	31.1303 9657	29.4137 8298	61
62	35.3526 4002	33.2580 0573	31.3467 2836	29.5997 8879	62
63	35.6398 4316	33.5041 6208	31.5577 8377	29.7808 1634	63
64	35.9214 1486	33.7449 0179	31.7636 9148	29.9569 9887	64
65	36.1974 6555	33.9803 4405	31.9645 7705	30.1284 6605	65
66	36.4681 0348	34.2106 0543	32.1605 6298	30.2953 4409	66
67	36.7334 3478	34.4357 9993	32.3517 6876	30.4577 5581	67
68	36.9935 6351	34.6560 3905	32.5383 1099	30.6158 2074	68
69	37.2485 9168	34.8714 3183	32.7203 0340	30.7696 5522	69
70	37.4986 1929	35.0820 8492	32.8978 5698	30.9193 7247	70
71	37.7437 4441	35.2881 0261	33.0710 7998	31.0650 8270	71
72	37.9840 6314	35.4895 8691	33.2400 7803	31.2068 9314	72
73	38.2196 6975	35.6866 3756	33.4049 5417	31.3449 0816	73
74	38.4506 5662	35.8793 5214	33.5658 0895	31.4792 2936	74
75	38.6771 1433	36.0678 2605	33.7227 4044	31.6099 5558	75
76	38.8991 3170	36.2521 5262	33.8758 4433	31.7371 8304	76
77	39.1167 9578	36.4324 2310	34.0252 1398	31.8610 0540	77
78	39.3301 9194	36.6087 2675	34.1709 4047	31.9815 1377	78
79	39.5394 0386	36.7811 5085	34.3131 1265	32.0987 9685	79
80	39.7445 1359	36.9497 8079	34.4518 1722	32.2129 4098	80
81	39.9456 0156	37.1147 0004	34.5871 3875	32.3240 3015	81
82	40.1427 4663	37.2759 9026	34.7191 5976	32.4321 4613	82
83	40.3360 2611	37.4337 3130	34.8479 6074	32.5373 6850	83
84	40.5255 1579	37.5880 0127	34.9736 2023	32.6397 7469	84
85	40.7112 8999	37.7388 7655	35.0962 1486	32.7394 4009	85
86	40.8934 2156	37.8864 3183	35.2158 1938	32.8364 3804	86
87	41.0719 8192	38.0307 4018	35.3325 0671	32.9308 3994	87
88	41.2470 4110	38.1718 7304	35.4463 4801	33.0227 1527	88
89	41.4186 6774	38.3099 0028	35.5574 1269	33.1121 3165	89
90	41.5869 2916	38.4448 9025	35.6657 6848	33.1991 5489	90
91	41.7518 9133	38.5769 0978	35.7714 8144	33.2838 4905	91
92	41.9136 1895	38.7060 2423	35.8746 1604	33.3662 7644	92
93	42.0721 7545	38.8322 9754	35.9752 3516	33.4464 9776	93
94	42.2276 2299	38.9557 9221	36.0734 0016	33.5245 7202	94
95	42.3800 2254	39.0765 6940	36.1691 7089	33.6005 5671	95
96	42.5294 3386	39.1946 8890	36.2626 0574	33.6745 0775	96
97	42.6759 1555	39.3102 0920	36.3537 6170	33.7464 7956	97
98	42.8195 2505	39.4231 8748	36.4426 9434	33.8165 2512	98
99	42.9603 1867	39.5336 7968	36.5294 5790	33.8846 9598	99
100	43.0983 5164	39.6417 4052	36.6141 0526	33.9510 4232	100

$$a_{\overline{n}|i} = (1 - v^n)/i$$

n	3%	3½%	4%	4½%	n
1	0.9708 7379	0.9661 8357	0.9615 3846	0.9569 3780	1
2	1.9134 6970	1.8996 9428	1.8860 9467	1.8726 6775	2
3	2.8286 1135	2.8016 3698	2.7750 9103	2.7489 6435	3
4	3.7170 9840	3.6730 7921	3.6298 9522	3.5875 2570	4
5	4.5797 0719	4.5150 5238	4.4518 2233	4.3899 7674	5
6	5.4171 9144	5.3285 5302	5.2421 3686	5.1578 7248	6
7	6.2302 8296	6.1145 4398	6.0020 5467	5.8927 0094	7
8	7.0196 9219	6.8739 5554	6.7327 4487	6.5958 8607	8
9	7.7861 0892	7.6076 8651	7.4353 3161	7.2687 9050	9
10	8.5302 0284	8.3166 0532	8.1108 9578	7.9127 1818	10
11	9.2526 2411	9.0015 5104	8.7604 7671	8.5289 1692	11
12	9.9540 0399	9.6633 3433	9.3850 7376	9.1185 8078	12
13	10.6349 5533	10.3027 3849	9.9856 4785	9.6828 5242	13
14	11.2960 7314	10.9205 2028	10.5631 2293	10.2228 2528	14
15	11.9379 3509	11.5174 1090	11.1183 8743	10.7395 4573	15
16	12.5611 0203	12.0941 1681	11.6522 9561	11.2340 1505	16
17	13.1661 1847	12.6513 2059	12.1656 6885	11.7071 9143	17
18	13.7535 1308	13.1896 8173	12.6592 9697	12.1599 9180	18
19	14.3237 9911	13.7098 3742	13.1339 3940	12.5932 9359	19
20	14.8774 7486	14.2124 0330	13.5903 2634	13.0079 3645	20
21	15.4150 2414	14.6979 7420	14.0291 5995	13.4047 2388	21
22	15.9369 1664	15.1671 2484	14.4511 1533	13.7844 2476	22
23	16.4436 0839	15.6204 1047	14.8568 4167	14.1477 7489	23
24	16.9355 4212	16.0583 6760	15.2469 6314	14.4954 7837	24
25	17.4131 4769	16.4815 1459	15.6220 7994	14.8282 0896	25
26	17.8768 4242	16.8903 5226	15.9827 6918	15.1466 1145	26
27	18.3270 3147	17.2853 6451	16.3295 8575	15.4513 0282	27
28	18.7641 0823	17.6670 1885	16.6630 6322	15.7428 7351	28
29	19.1884 5459	18.0357 6700	16.9837 1463	16.0218 8853	29
30	19.6004 4135	18.3920 4541	17.2920 3330	16.2888 8854	30
31	20.0004 2849	18.7362 7576	17.5884 9356	16.5443 9095	31
32	20.3887 6553	19.0688 6547	17.8735 5150	16.7888 9086	32
33	20.7657 9178	19.3902 0818	18.1476 4567	17.0228 6207	33
34	21.1318 3668	19.7006 8423	18.4111 9776	17.2467 5796	34
35	21.4872 2007	20.0006 6110	18.6646 1323	17.4610 1240	35
36	21.8322 5250	20.2904 9381	18.9082 8195	17.6660 4058	36
37	22.1672 3544	20.5705 2542	19.1425 7880	17.8622 3979	37
38	22.4924 6159	20.8410 8736	19.3678 6423	18.0499 9023	38
39	22.8082 1513	21.1024 9987	19.5844 8484	18.2296 5572	39
40	23.1147 7197	21.3550 7234	19.7927 7388	18.4015 8442	40
41	23.4123 9997	21.5991 0371	19.9930 5181	18.5661 0949	41
42	23.7013 5920	21.8348 8281	20.1856 2674	18.7235 4975	42
43	23.9819 0213	22.0626 8870	20.3707 9494	18.8742 1029	43
44	24.2542 7392	22.2827 9102	20.5488 4129	19.0183 8305	44
45	24.5187 1254	22.4954 5026	20.7200 3970	19.1563 4742	45
46	24.7754 4907	22.7009 1813	20.8846 5356	19.2883 7074	46
47	25.0247 0783	22.8994 3780	21.0429 3612	19.4147 0884	47
48	25.2667 0664	23.0912 4425	21.1951 3088	19.5356 0654	48
49	25.5016 5693	23.2765 6450	21.3414 7200	19.6512 9813	49
50	25.7297 6401	23.4556 1787	21.4821 8462	19.7620 0778	50

Present Value of 1 per Period at Compound Interest

$$a_{\overline{n}|i} = (1 - v^n)/i$$

n	3%	3½%	4%	4½%	n
51	25.9512 2719	23.6286 1630	21.6174 8521	19.8679 5003	51
52	26.1662 3999	23.7957 6454	21.7475 8193	19.9693 3017	52
53	26.3749 9028	23.9572 6043	21.8726 7493	20.0663 4466	53
54	26.5776 6047	24.1132 9510	21.9929 5667	20.1591 8149	54
55	26.7744 2764	24.2640 5323	22.1086 1218	20.2480 2057	55
56	26.9654 6373	24.4097 1327	22.2189 1940	20.3330 3404	56
57	27.1509 3566	24.5504 4760	22.3267 4943	20.4143 8664	57
58	27.3310 0549	24.6864 2281	22.4295 6676	20.4922 3602	58
59	27.5058 3058	24.8177 9981	22.5284 2957	20.5667 3303	59
60	27.6755 6367	24.9447 3412	22.6234 8997	20.6380 2204	60
61	27.8403 5307	25.0673 7596	22.7148 9421	20.7062 4118	61
62	28.0003 4279	25.1858 7049	22.8027 8289	20.7715 2266	62
63	28.1556 7261	25.3003 5796	22.8872 9124	20.8339 9298	63
64	28.3064 7826	25.4109 7388	22.9685 4927	20.8937 7319	64
65	28.4528 9152	25.5178 4916	23.0466 8199	20.9509 7913	65
66	28.5950 4031	25.6211 1030	23.1218 0961	21.0057 2165	66
67	28.7330 4884	25.7208 7951	23.1940 4770	21.0581 0684	67
68	28.8670 3771	25.8172 7489	23.2635 0740	21.1082 3621	68
69	28.9971 2399	25.9104 1052	23.3302 9558	21.1562 0690	69
70	29.1234 2135	26.0003 9664	23.3945 1498	21.2021 1187	70
71	29.2460 4015	26.0873 3975	23.4562 6440	21.2460 4007	71
72	29.3650 8752	26.1713 4275	23.5156 3885	21.2880 7662	72
73	29.4806 6750	26.2525 0508	23.5727 2966	21.3283 0298	73
74	29.5928 8106	26.3309 2278	23.6276 2468	21.3667 9711	74
75	29.7018 2628	26.4066 8868	23.6804 0834	21.4036 3360	75
76	29.8075 9833	26.4798 9244	23.7311 6187	21.4388 8383	76
77	29.9102 8964	26.5506 2072	23.7799 6333	21.4726 1611	77
78	30.0099 8994	26.6189 5721	23.8268 8782	21.5048 9579	78
79	30.1067 8635	26.6849 8281	23.8720 0752	21.5357 8545	79
80	30.2007 6345	26.7487 7567	23.9153 9185	21.5653 4493	80
81	30.2920 0335	26.8104 1127	23.9571 0754	21.5936 3151	81
82	30.3805 8577	26.8699 6258	23.9972 1879	21.6207 0001	82
83	30.4665 8813	26.9275 0008	24.0357 8730	21.6466 0288	83
84	30.5500 8556	26.9830 9186	24.0728 7240	21.6713 9032	84
85	30.6311 5103	27.0368 0373	24.1085 3116	21.6951 1035	85
86	30.7098 5537	27.0886 9926	24.1428 1842	21.7178 0895	86
87	30.7862 6735	27.1388 3986	24.1757 8694	21.7395 3009	87
88	30.8604 5374	27.1872 8489	24.2074 8745	21.7603 1588	88
89	30.9324 7936	27.2340 9168	24.2379 6870	21.7802 0658	89
90	31.0024 0714	27.2793 1564	24.2672 7759	21.7992 4075	90
91	31.0702 9820	27.3230 1028	24.2954 5923	21.8174 5526	91
92	31.1362 1184	27.3652 2732	24.3225 5695	21.8348 8542	92
93	31.2002 0567	27.4060 1673	24.3486 1245	21.8515 6499	93
94	31.2623 3560	27.4454 2680	24.3736 6582	21.8675 2631	94
95	31.3226 5592	27.4835 0415	24.3977 5559	21.8828 0030	95
96	31.3812 1934	27.5202 9387	24.4209 1884	21.8974 1655	96
97	31.4380 7703	27.5558 3948	24.4431 9119	21.9114 0340	97
98	31.4932 7867	27.5901 8308	24.4646 0692	21.9247 8794	98
99	31.5468 7250	27.6233 6529	24.4851 9896	21.9375 9612	99
100	31.5989 0534	27.6554 2540	24.5049 9900	21.9498 5274	100

Present Value of 1 per Period at Compound Interest

$$a_{\overline{n}|i} = (1 - v^n)/i$$

n	5%	5½%	6%	6½%	n
1	0.9523 8095	0.9478 6730	0.9433 9623	0.9389 6714	1
2	1.8594 1043	1.8463 1971	1.8333 9267	1.8206 2642	2
3	2.7232 4803	2.6979 3338	2.6730 1195	2.6484 7551	3
4	3.5459 5050	3.5051 5012	3.4651 0561	3.4257 9860	4
5	4.3294 7667	4.2702 8448	4.2123 6379	4.1556 7944	5
6	5.0756 9206	4.9955 3031	4.9173 2433	4.8410 1356	6
7	5.7863 7340	5.6829 6712	5.5823 8144	5.4845 1977	7
8	6.4632 1276	6.3345 6599	6.2097 9381	6.0887 5096	8
9	7.1078 2168	6.9521 9525	6.8016 9227	6.6561 0419	9
10	7.7217 3493	7.5376 2583	7.3600 8705	7.1888 3022	10
11	8.3064 1422	8.0925 3633	7.8868 7458	7.6890 4246	11
12	8.8632 5164	8.6185 1785	8.3838 4394	8.1587 2532	12
13	9.3935 7299	9.1170 7853	8.8526 8296	8.5997 4208	13
14	9.8986 4094	9.5896 4790	9.2949 8393	9.0138 4233	14
15	10.3796 5804	10.0375 8094	9.7122 4899	9.4026 6885	15
16	10.8377 6956	10.4621 6203	10.1058 9527	9.7677 6418	16
17	11.2740 6625	10.8646 0856	10.4772 5969	10.1105 7670	17
18	11.6895 8690	11.2460 7447	10.8276 0348	10.4324 6638	18
19	12.0853 2086	11.6076 5352	11.1581 1649	10.7347 1022	19
20	12.4622 1034	11.9503 8249	11.4699 2122	11.0185 0725	20
21	12.8211 5271	12.2752 4406	11.7640 7662	11.2849 8333	21
22	13.1630 0258	12.5831 6973	12.0415 8172	11.5351 9562	22
23	13.4885 7388	12.8750 4240	12.3033 7898	11.7701 3673	23
24	13.7986 4179	13.1516 9895	12.5503 5753	11.9907 3871	24
25	14.0939 4457	13.4139 3266	12.7833 5616	12.1978 7672	25
26	14.3751 8530	13.6624 9541	13.0031 6619	12.3923 7251	26
27	14.6430 3362	13.8980 9991	13.2105 3414	12.5749 9766	27
28	14.8981 2726	14.1214 2172	13.4061 6428	12.7464 7668	28
29	15.1410 7358	14.3331 0116	13.5907 2102	12.9074 8984	29
30	15.3724 5103	14.5337 4517	13.7648 3115	13.0586 7591	30
31	15.5928 1050	14.7239 2907	13.9290 8599	13.2006 3465	31
32	15.8026 7667	14.9041 9817	14.0840 4339	13.3339 2925	32
33	16.0025 4921	15.0750 6936	14.2302 2961	13.4590 8850	33
34	16.1929 0401	15.2370 3257	14.3681 4114	13.5766 0892	34
35	16.3741 9429	15.3905 5220	14.4982 4636	13.6869 5673	35
36	16.5468 5171	15.5360 6843	14.6209 8713	13.7905 6970	36
37	16.7112 8734	15.6739 9851	14.7367 8031	13.8878 5887	37
38	16.8678 9271	15.8047 3793	14.8460 1916	13.9792 1021	38
39	17.0170 4067	15.9286 6154	14.9490 7468	14.0649 8611	39
40	17.1590 8635	16.0461 2469	15.0462 9687	14.1455 2687	40
41	17.2943 6796	16.1574 6416	15.1380 1592	14.2211 5199	41
42	17.4232 0758	16.2629 9920	15.2245 4332	14.2921 6149	42
43	17.5459 1198	16.3630 3242	15.3061 7294	14.3588 3708	43
44	17.6627 7331	16.4578 5063	15.3831 8202	14.4214 4327	44
45	17.7740 6982	16.5477 2572	15.4558 3209	14.4802 2842	45
46	17.8800 6650	16.6329 1537	15.5243 6990	14.5354 2575	46
47	17.9810 1571	16.7136 6386	15.5890 2821	14.5872 5422	47
48	18.0771 5782	16.7902 0271	15.6500 2661	14.6359 1946	48
49	18.1687 2173	16.8627 5139	15.7075 7227	14.6816 1451	49
50	18.2559 2546	16.9315 1790	15.7618 6064	14.7245 2067	50

$$a_{\overline{n}|} = (1 - v^n)/i$$

n	5%	5½%	6%	6½%	n
51	18.3389 7663	16.9966 9943	15.8130 7607	14.7648 0814	51
52	18.4180 7298	17.0584 8287	15.8613 9252	14.8026 3675	52
53	18.4934 0284	17.1170 4538	15.9069 7408	14.8381 5658	53
54	18.5651 4556	17.1725 5486	15.9499 7554	14.8715 0852	54
55	18.6334 7196	17.2251 7048	15.9905 4297	14.9028 2490	55
56	18.6985 4473	17.2750 4311	16.0288 1412	14.9322 2996	56
57	18.7605 1879	17.3223 1575	16.0649 1898	14.9598 4033	57
58	18.8195 4170	17.3671 2393	16.0989 8017	14.9857 6557	58
59	18.8757 5400	17.4095 9614	16.1311 1337	15.0101 0852	59
60	18.9292 8952	17.4498 5416	16.1614 2771	15.0329 6574	60
61	18.9802 7574	17.4880 1343	16.1900 2614	15.0544 2793	61
62	19.0288 3404	17.5241 8334	16.2170 0579	15.0745 8021	62
63	19.0750 8003	17.5584 6762	16.2424 5829	15.0935 0255	63
64	19.1191 2384	17.5909 6457	16.2664 7009	15.1112 7000	64
65	19.1610 7033	17.6217 6737	16.2891 2272	15.1279 5305	65
66	19.2010 1936	17.6509 6433	16.3104 9314	15.1436 1789	66
67	19.2390 6606	17.6786 3917	16.3306 5390	15.1583 2666	67
68	19.2753 0101	17.7048 7125	16.3496 7349	15.1721 3770	68
69	19.3098 1048	17.7297 3579	16.3676 1650	15.1851 0583	69
70	19.3426 7665	17.7533 0406	16.3845 4387	15.1972 8247	70
71	19.3739 7776	17.7756 4366	16.4005 1308	15.2087 1593	71
72	19.4037 8834	17.7968 1864	16.4155 7838	15.2194 5158	72
73	19.4321 7937	17.8168 8970	16.4297 9093	15.2295 3200	73
74	19.4592 1845	17.8359 1441	16.4431 9899	15.2389 9718	74
75	19.4849 6995	17.8539 4731	16.4558 4810	15.2478 8468	75
76	19.5094 9519	17.8710 4010	16.4677 8123	15.2562 2974	76
77	19.5328 5257	17.8872 4180	16.4790 3889	15.2640 6549	77
78	19.5550 9768	17.9025 9887	16.4896 5933	15.2714 2299	78
79	19.5762 8351	17.9171 5532	16.4996 7862	15.2783 3145	79
80	19.5964 6048	17.9309 5291	16.5091 3077	15.2848 1826	80
81	19.6156 7665	17.9440 3120	16.5180 4790	15.2909 0917	81
82	19.6339 7776	17.9564 2768	16.5264 6028	15.2966 2832	82
83	19.6514 0739	17.9681 7789	16.5343 9649	15.3019 9843	83
84	19.6680 0704	17.9793 1554	16.5418 8348	15.3070 4078	84
85	19.6838 1623	17.9898 7255	16.5489 4668	15.3117 7538	85
86	19.6988 7260	17.9998 7919	16.5556 1008	15.3162 2101	86
87	19.7132 1200	18.0093 6416	16.5618 9630	15.3203 9531	87
88	19.7268 6857	18.0183 5466	16.5678 2670	15.3243 1485	88
89	19.7398 7483	18.0268 7645	16.5734 2141	15.3279 9516	89
90	19.7522 6174	18.0349 5398	16.5786 9944	15.3314 5086	90
91	19.7640 5880	18.0426 1041	16.5836 7872	15.3346 9564	91
92	19.7752 9410	18.0498 6769	16.5883 7615	15.3377 4239	92
93	19.7859 9438	18.0567 4662	16.5928 0769	15.3406 0318	93
94	19.7961 8512	18.0632 6694	16.5969 8839	15.3432 8937	94
95	19.8058 9059	18.0694 4734	16.6009 3244	15.3458 1161	95
96	19.8151 3390	18.0753 0553	16.6046 5325	15.3481 7992	96
97	19.8239 3705	18.0808 5833	16.6081 6344	15.3504 0368	97
98	19.8323 2100	18.0861 2164	16.6114 7494	15.3524 9172	98
99	19.8403 0571	18.0911 1055	16.6145 9900	15.3544 5232	99
100	19.8479 1020	18.0958 3939	16.6175 4623	15.3562 9326	100

Present Value of 1 per Period at Compound Interest

$$a_{\overline{n}|i} = (1 - v^n)/i$$

n	7%	7½%	8%	8½%	n
1	0.9345 7944	0.9302 3256	0.9259 2593	0.9216 5899	1
2	1.8080 1817	1.7955 6517	1.7832 6475	1.7711 1427	2
3	2.6243 1604	2.6005 2574	2.5770 9699	2.5540 2237	3
4	3.3872 1126	3.3493 2627	3.3121 2684	3.2755 9666	4
5	4.1001 9744	4.0458 8490	3.9927 1004	3.9406 4208	5
6	4.7665 3966	4.6938 4642	4.6228 7966	4.5535 8717	6
7	5.3892 8940	5.2966 0132	5.2063 7006	5.1185 1352	7
8	5.9712 9851	5.8573 0355	5.7466 3894	5.6391 8297	8
9	6.5152 3225	6.3788 8703	6.2468 8791	6.1190 6264	9
10	7.0235 8154	6.8640 8096	6.7100 8140	6.5613 4806	10
11	7.4986 7434	7.3154 2415	7.1389 6426	6.9689 8439	11
12	7.9426 8630	7.7352 7827	7.5360 7802	7.3446 8607	12
13	8.3576 5074	8.1258 4026	7.9037 7594	7.6909 5490	13
14	8.7454 6799	8.4891 5373	8.2442 3698	8.0100 9668	14
15	9.1079 1401	8.8271 1974	8.5594 7869	8.3042 3658	15
16	9.4466 4860	9.1415 0674	8.8513 6916	8.5753 3325	16
17	9.7632 2299	9.4339 5976	9.1216 3811	8.8251 9194	17
18	10.0590 8691	9.7060 0908	9.3718 8714	9.0554 7644	18
19	10.3355 9524	9.9590 7821	9.6035 9920	9.2677 2022	19
20	10.5940 1425	10.1944 9136	9.8181 4741	9.4633 3661	20
21	10.8355 2733	10.4134 8033	10.0168 0316	9.6436 2821	21
22	11.0612 4050	10.6171 9101	10.2007 4366	9.8097 9559	22
23	11.2721 8738	10.8066 8931	10.3710 5895	9.9629 4524	23
24	11.4693 3400	10.9829 6680	10.5287 5828	10.1040 9700	24
25	11.6535 8318	11.1469 4586	10.6747 7619	10.2341 9078	25
26	11.8257 7867	11.2994 8452	10.8099 7795	10.3540 9288	26
27	11.9867 0904	11.4413 8095	10.9351 6477	10.4646 0174	27
28	12.1371 1125	11.5733 7763	11.0510 7849	10.5664 5321	28
29	12.2776 7407	11.6961 6524	11.1584 0601	10.6603 2554	29
30	12.4090 4118	11.8103 8627	11.2577 8334	10.7468 4382	30
31	12.5318 1419	11.9166 3839	11.3497 9939	10.8265 8416	31
32	12.6465 5532	12.0154 7757	11.4349 9944	10.9000 7757	32
33	12.7537 9002	12.1074 2099	11.5138 8837	10.9678 1343	33
34	12.8540 0936	12.1929 4976	11.5869 3367	11.0302 4279	34
35	12.9476 7230	12.2725 1141	11.6545 6822	11.0877 8137	35
36	13.0352 0776	12.3465 2224	11.7171 9279	11.1408 1233	36
37	13.1170 1660	12.4153 6953	11.7751 7851	11.1896 8878	37
38	13.1934 7345	12.4794 1351	11.8288 6899	11.2347 3620	38
39	13.2649 2846	12.5389 8931	11.8785 8240	11.2762 5457	39
40	13.3317 0884	12.5944 0866	11.9246 1333	11.3145 2034	40
41	13.3941 2041	12.6459 6155	11.9672 3457	11.3497 8833	41
42	13.4524 4898	12.6939 1772	12.0066 9867	11.3822 9339	42
43	13.5069 6167	12.7385 2811	12.0432 3951	11.4122 5197	43
44	13.5579 0810	12.7800 2615	12.0770 7362	11.4398 6357	44
45	13.6055 2159	12.8186 2898	12.1084 0150	11.4653 1205	45
46	13.6500 2018	12.8545 3858	12.1374 0880	11.4887 6686	46
47	13.6916 0764	12.8879 4287	12.1642 6741	11.5103 8420	47
48	13.7304 7443	12.9190 1662	12.1891 3649	11.5303 0802	48
49	13.7667 9853	12.9479 2244	12.2121 6341	11.5486 7099	49
50	13.8007 4629	12.9748 1157	12.2334 8464	11.5655 9538	50

$$a_{\overline{n}|i} = (1 - v^n)/i$$

n	7%	7½%	8%	8½%	n
51	13.8324 7317	12.9998 2472	12.2532 2652	11.5811 9390	51
52	13.8621 2446	13.0230 9276	12.2715 0604	11.5955 7041	52
53	13.8898 3594	13.0447 3745	12.2884 3152	11.6088 2066	53
54	13.9157 3453	13.0648 7205	12.3041 0326	11.6210 3287	54
55	13.9399 3881	13.0836 0191	12.3186 1413	11.6322 8835	55
56	13.9625 5964	13.1010 2503	12.3320 5012	11.6426 6208	56
57	13.9837 0059	13.1172 3258	12.3444 9085	11.6522 2311	57
58	14.0034 5850	13.1323 0938	12.3560 1005	11.6610 3513	58
59	14.0219 2383	13.1463 3431	12.3666 7597	11.6691 5680	59
60	14.0391 8115	13.1593 8075	12.3765 5182	11.6766 4221	60
61	14.0553 0949	13.1715 1698	12.3856 9613	11.6835 4121	61
62	14.0703 8270	13.1828 0649	12.3941 6309	11.6898 9973	62
63	14.0844 6981	13.1933 0836	12.4020 0286	11.6957 6012	63
64	14.0976 3534	13.2030 7755	12.4092 6190	11.7011 6140	64
65	14.1099 3957	13 2121 6516	12.4159 8324	11.7061 3954	65
66	14.1214 3885	13.2206 1875	12.4222 0671	11.7107 2769	66
67	14.1321 8584	13.2284 8256	12.4279 6917	11.7149 5639	67
68	14.1422 2976	13.2357 9773	12.4333 0479	11.7188 5382	68
69	14.1516 1660	13.2426 0254	12.4382 4518	11.7224 4592	69
70	14.1603 8934	13.2489 3260	12.4428 1961	11.7257 5661	70
71	14.1685 8817	13.2548 2102	12.4470 5519	11.7288 0793	71
72	14.1762 5063	13.2602 9862	12.4509 7703	11.7316 2021	72
73	14.1834 1180	13.2653 9407	12.4546 0836	11.7342 1218	73
74	14.1901 0449	13.2701 3402	12.4579 7071	11.7366 0109	74
75	14.1963 5933	13.2745 4327	12.4610 8399	11.7388 0284	75
76	14.2022 0498	13.2786 4490	12.4639 6665	11.7408 3211	76
77	14.2076 6821	13.2824 6038	12.4666 3579	11.7427 0241	77
78	14.2127 7403	13.2860 0965	12.4691 0721	11.7444 2618	78
79	14.2175 4582	13.2893 1130	12.4713 9557	11.7460 1492	79
80	14.2220 0544	13.2923 8261	12.4735 1441	11.7474 7919	80
81	14.2261 7331	13.2952 3964	12.4754 7631	11.7488 2874	81
82	14.2300 6851	13.2978 9734	12.4772 9288	11.7500 7257	82
83	14.2337 0889	13.3003 6962	12.4789 7489	11.7512 1896	83
84	14.2371 1111	13.3026 6941	12.4805 3230	11.7522 7554	84
85	14.2402 9076	13.3048 0875	12.4819 7436	11.7532 4935	85
86	14.2432 6239	13.3067 9884	12.4833 0959	11.7541 4686	86
87	14.2460 3962	13.3086 5008	12.4845 4592	11.7549 7407	87
88	14.2486 3516	13.3103 7217	12.4856 9066	11.7557 3647	88
89	14.2510 6089	13.3119 7411	12.4867 5061	11.7564 3914	89
90	14.2533 2794	13.3134 6429	12.4877 3205	11.7570 8677	90
91	14.2554 4667	13.3148 5050	12.4886 4079	11.7576 8365	91
92	14.2574 2680	13.3161 4000	12.4894 8221	11.7582 3378	92
93	14.2592 7738	13.3173 3954	12.4902 6131	11.7587 4081	93
94	14.2610 0690	13.3184 5538	12.4909 8269	11.7592 0812	94
95	14.2626 2327	13.3194 9338	12.4916 5064	11.7596 3882	95
96	14.2641 3390	13.3204 5896	12.4922 6911	11.7600 3578	96
97	14.2655 4570	13.3213 5717	12.4928 4177	11.7604 0164	97
98	14.2668 6514	13.3221 9272	12.4933 7201	11.7607 3884	98
99	14.2680 9826	13.3229 6997	12.4938 6297	11.7610 4962	99
100	14.2692 5071	13.3236 9290	12.4943 1757	11.7613 3606	100

Amount of 1 per Period at Compound Interest

$$s_{\overline{n}|i} = [(1+i)^n - 1]/i$$

n	$\frac{1}{4}\%$	$\frac{7}{24}\%$	$\frac{1}{3}\%$	$\frac{5}{12}\%$	n
1	1.0000 0000	1.0000 0000	1.0000 0000	1.0000 0000	1
2	2.0025 0000	2.0029 1667	2.0033 3333	2.0041 6667	2
3	3.0075 0625	3.0087 5851	3.0100 1111	3.0125 1736	3
4	4.0150 2502	4.0175 3405	4.0200 4448	4.0250 6952	4
5	5.0250 6258	5.0292 5186	5.0334 4463	5.0418 4064	5
6	6.0376 2523	6.0439 2051	6.0502 2278	6.0628 4831	6
7	7.0527 1930	7.0615 4861	7.0703 9019	7.0881 1018	7
8	8.0703 5110	8.0821 4480	8.0939 5816	8.1176 4397	8
9	9.0905 2697	9.1057 1772	9.1209 3802	9.1514 6749	9
10	10,1132 5329	10.1322 7606	10.1513 4114	10.1895 9860	10
11	11.1385 3642	11.1618 2853	11.1851 7895	11.2320 5526	11
12	12.1663 8277	12.1943 8387	12.2224 6288	12.2788 5549	12
13	13.1967 9872	13.2299 5082	13.2632 0442	13.3300 1739	13
14	14.2297 9072	14.2685 3818	14.3074 1510	14.3855 5913	14
15	15.2653 6520	15.3101 5475	15.3551 0648	15.4454 9896	15
16	16.3035 2861	16.3548 0936	16.4062 9017	16.5098 5520	16
17	17.3442 8743	17.4025 1089	17.4609 7781	17.5786 4627	17
18	18.3876 4815	18.4532 6822	18.5191 8107	18.6518 9063	18
19	19.4336 1727	19.5070 9025	19.5809 1167	19.7296 0684	19
20	20.4822 0131	20.5639 8593	20.6461 8137	20.8118 1353	20
21	21.5334 0682	21.6239 6422	21.7150 0198	21.8985 2942	21
22	22.5872 4033	22.6870 3412	22.7873 8532	22.9897 7330	22
23	23.6437 0843	23.7532 0463	23.8633 4327	24.0855 6402	23
24	24.7028 1770	24.8224 8481	24.9428 8775	25.1859 2054	24
25	25.7645 7475	25.8946 8373	26.0260 3071	26.2908 6187	25
26	26.8289 8619	26.9704 1047	27.1127 8414	27.4004 0713	26
27	27.8960 5865	28.0490 7417	28.2031 6009	28.5145 7549	27
28	28.9657 9880	29.1308 8397	29.2971 7062	29.6333 8622	28
29	30.0382 1330	30.2158 4904	30.3948 2786	30.7568 5867	29
30	31.1133 0883	31.3039 7860	31.4961 4395	31.8850 1224	30
31	32.1910 9210	32.3952 8188	32.6011 3110	33.0178 6646	31
32	33.2715 6983	33.4897 6811	33.7098 0154	34.1554 4090	32
33	34.3547 4876	34.5874 4660	34.8221 6754	35.2977 5524	33
34	35.4406 3563	35.6883 2666	35.9382 4143	36.4448 2922	34
35	36.5292 3722	36.7924 1761	37.0580 3557	37.5966 8268	35
36	37.6205 6031	37.8997 2883	38.1815 6236	38.7533 3552	36
37	38.7146 1171	39.0102 6970	39.3088 3423	39.9148 0775	37
38	39.8113 9824	40.1240 4966	40.4398 6368	41.0811 1945	38
39	40.9109 2673	41.2410 7813	41.5746 6322	42.2522 9078	39
40	42.0132 0405	42.3613 6461	42.7132 4543	43.4283 4199	40
41	43.1182 3706	43.4849 1859	43.8556 2292	44.6092 9342	41
42	44.2260 3265	44.6117 4961	45.0018 0833	45.7951 6548	42
43	45.3365 9774	45.7418 6721	46.1518 1436	46.9859 7866	43
44	46.4499 3923	46,8752 8099	47.3056 5374	48.1817 5358	44
45	47.5660 6408	48.0120 0056	48.4633 3925	49.3825 1088	45
46	48.6849 7924	49.1520 3556	49.6248 8371	50.5882 7134	46
47	49.8066 9169	50.2953 9566	50.7902 9999	51.7990 5581	47
48	50.9312 0842	51.4420 9057	51.9596 0099	53.0148 8521	48
49	52.0585 3644	52.5921 3000	53.1327 9966	54.2357 8056	49
50	53,1886 8278	53.7455 2371	54.3099 0899	55.4617 6298	50

Amount of 1 per Period at Compound Interest

$$s_{\overline{n}|i} = [(1+i)^n - 1]/i$$

n	1/4%	7/24%	1/3%	5/12%	n
51	54.3216 5449	54.9022 8149	55.4909 4202	56.6928 5366	51
52	55.4574 5862	56.0624 1314	56.6759 1183	57.9290 7388	52
53	56.5961 0227	57.2259 2851	57.8648 3154	59.1704 4503	53
54	57.7375 9252	58.3928 3747	59.0577 1431	60.4169 8855	54
55	58.8819 3650	59.5631 4991	60.2545 7336	61.6687 2600	55
56	60.0291 4135	60.7368 7577	61.4554 2194	62.9256 7902	56
57	61.1792 1420	61.9140 2499	62.6602 7334	64.1878 6935	57
58	62.3321 6223	63.0946 0756	63.8691 4092	65.4553 1881	58
59	63.4879 9264	64.2786 3350	65.0820 3806	66.7280 4930	59
60	64.6467 1262	65.4661 1285	66.2989 7818	68.0060 8284	60
61	65.8083 2940	66.6570 5568	67.5199 7478	69.2894 4152	61
62	66.9728 5023	67.8514 7209	68.7450 4136	70.5781 4753	62
63	68.1402 8235	69.0493 7222	69.9741 9150	71.8722 2314	63
64	69.3106 3306	70.2507 6622	71.2074 3880	73.1716 9074	64
65	70.4839 0964	71.4556 6429	72.4447 9693	74.4765 7278	65
66	71.6601 1942	72.6640 7664	73.6862 7959	75.7868 9184	66
67	72.8392 6971	73.8760 1353	74.9319 0052	77.1026 7055	67
68	74.0213 6789	75.0914 8524	76.1816 7352	78.4239 3168	68
69	75.2064 2131	76.3105 0207	77.4356 1243	79.7506 9806	69
70	76.3944 3736	77.5330 7437	78.6937 3114	81.0829 9264	70
71	77.5854 2345	78.7592 1250	79.9560 4358	82.4208 3844	71
72	78.7793 8701	79.9889 2687	81.2225 6372	83.7642 5860	72
73	79.9763 3548	81.2222 2791	82.4933 0560	85.1132 7634	73
74	81.1762 7632	82.4591 2607	83.7682 8329	86.4679 1500	74
75	82.3792 1701	83.6996 3186	85.0475 1090	87.8281 9797	75
76	83.5851 6505	84.9437 5578	86.3310 0260	89.1941 4880	76
77	84.7941 2797	86.1915 0840	87.6187 7261	90.5657 9109	77
78	86.0061 1329	87.4429 0030	88.9108 3519	91.9431 4855	78
79	87.2211 2857	88.6979 4209	90.2072 0464	93.3262 4500	79
80	88.4391 8139	89.9566 4443	91.5078 9532	94.7151 0436	80
81	89.6602 7934	91.2190 1797	92.8129 2164	96.1097 5062	81
82	90.8844 3004	92.4850 7344	94.1222 9804	97.5102 0792	82
83	92.1116 4112	93.7548 2157	95.4360 3904	98.9165 0045	83
84	93.3419 2022	95.0282 7313	96.7541 5917	100.3286 5254	84
85	94.5752 7502	96.3054 3893	98.0766 7303	101.7466 8859	85
86	95.8117 1321	97.5863 2980	99.4035 9527	103.1706 3312	86
87	97.0512 4249	98.8709 5659	100.7349 4059	104.6005 1076	87
88	98.2938 7060	100.1593 3021	102.0707 2373	106.0363 4622	88
89	99.5396 0527	101.4514 6159	103.4109 5947	107.4781 6433	89
90	100.7884 5429	102.7473 6169	104.7556 6267	108.9259 9002	90
91	102.0404 2542	104.0470 4149	106.1048 4821	110.3798 4831	91
92	103.2955 2649	105.3505 1203	107.4585 3104	111.8397 6434	92
93	104.5537 6530	106.6577 8436	108.8167 2614	113.3057 6336	93
94	105.8151 4972	107.9688 6956	110.1794 4856	114.7778 7071	94
95	107.0796 8759	109.2837 7877	111.5467 1339	116.2561 1184	95
96	108.3473 8681	110.6025 2312	112.9185 3577	117.7405 1230	96
97	109.6182 5528	111.9251 1381	114.2949 3089	119.2310 9777	97
98	110.8923 0091	113.2515 6206	115.6759 1399	120.7278 9401	98
99	112.1695 3167	114.5818 7912	117.0615 0037	122.2309 2690	99
100	113.4499 5550	115.9160 7626	118.4517 0537	123.7402 2243	100

Amount of 1 per Period at Compound Interest

$$s_{\overline{n}|i} = [(1+i)^n - 1]/i$$

n	$\frac{1}{4}\%$	$\frac{7}{24}\%$	$\frac{1}{3}\%$	$\frac{5}{12}\%$	n
101	114.7335 8038	117.2541 6482	119.8465 4439	125.2558 0669	101
102	116.0204 1434	118.5961 5613	121.2460 3287	126.7777 0589	102
103	117.3104 6537	119.9420 6159	122.6501 8632	128.3059 4633	103
104	118.6037 4153	121.2918 9260	124.0590 2027	129.8405 5444	104
105	119.9002 5089	122.6456 6062	125.4725 5034	131.3815 5675	105
106	121.2000 0152	124.0033 7713	126.8907 9217	132.9289 7990	106
107	122.5030 0152	125.3650 5365	128.3137 6148	134.4828 5065	107
108	123.8092 5902	126.7307 0172	129.7414 7402	136.0431 9586	108
109	125.1187 8217	128.1003 3294	131.1739 4560	137.6100 4251	109
110	126.4315 7913	129.4739 5891	132.6111 9208	139.1834 1769	110
111	127.7476 5807	130.8515 9129	134.0532 2939	140.7633 4860	111
112	129.0670 2722	132.2332 4176	135.5000 7349	142.3498 6255	112
113	130.3896 9479	133.6189 2205	136.9517 4040	143.9429 8698	113
114	131.7156 6902	135.0086 4391	138.4082 4620	145.5427 4942	114
115	133.0449 5820	136.4024 1912	139.8696 0702	147.1491 7754	115
116	134.3775 7059	137.8002 5951	141.3358 3905	148.7622 9912	116
117	135.7135 1452	139.2021 7693	142.8069 5851	150.3821 4203	117
118	137.0527 9830	140.6081 8328	144.2829 8170	152.0087 3429	118
119	138.3954 3030	142.0182 9048	145.7639 2498	153.6421 0401	119
120	139.7414 1888	143.4325 1049	147.2498 0477	155.2822 7945	120
121	141.0907 7242	144.8508 5532	148.7406 3745	156.9292 8895	121
122	142.4434 9935	146.2733 3698	150.2364 3958	158.5831 6098	122
123	143.7996 0810	147.6999 6754	151.7372 2771	160.2439 2415	123
124	145.1591 0712	149.1307 5912	153.2430 1847	161.9116 0717	124
125	146.5220 0489	150.5657 2383	154.7538 2853	163.5862 3887	125
126	147.8883 0990	152.0048 7386	156.2696 7463	165.2678 4819	126
127	149.2580 3068	153.4482 2141	157.7905 7354	166.9564 6423	127
128	150.6311 7575	154.8957 7872	159.3165 4212	168.6521 1616	128
129	152.0077 5369	156.3475 5807	160.8475 9726	170.3548 3331	129
130	153.3877 7308	157.8035 7178	162.3837 5592	172.0646 4512	130
131	154.7712 4251	159.2638 3220	163.9250 3510	173.7815 8114	131
132	156.1581 7062	160.7283 5171	165.4714 5189	175.5056 7106	132
133	157.5485 6604	162.1971 4274	167.0230 2339	177.2369 4469	133
134	158.9424 3746	163.6702 1774	168.5797 6680	178.9754 3196	134
135	160.3397 9355	165.1475 8920	170.1416 9936	180.7211 6293	135
136	161.7406 4304	166.6292 6967	171.7088 3836	182.4741 6777	136
137	163.1449 9464	168.1152 7171	173.2812 0115	184.2344 7681	137
138	164.5528 5713	169.6056 0792	174.8588 0516	186.0021 2046	138
139	165.9642 3927	171.1002 9094	176.4416 6784	187.7771 2929	139
140	167.3791 4987	172.5993 3346	178.0298 0673	189.5595 3400	140
141	168.7975 9775	174.1027 4818	179.6232 3942	191.3493 6539	141
142	170.2195 9174	175.6105 4786	181.2219 8355	193.1466 5441	142
143	171.6451 4072	177.1227 4529	182.8260 5683	194.9514 3214	143
144	173.0742 5357	178.6393 5330	184.4354 7702	196.7637 2977	144
145	174.5069 3921	180.1603 8475	186.0502 6194	198.5835 7865	145
146	175.9432 0655	181.6858 5254	187.6704 2948	200.4110 1023	146
147	177.3830 6457	183.2157 6961	189.2959 9758	202.2460 5610	147
148	178.8265 2223	184.7501 4893	190.9269 8424	204.0887 4800	148
149	180.2735 8854	186.2890 0353	192.5634 0752	205.9391 1779	149
150	181.7242 7251	187.8323 4646	194.2052 8554	207.7971 9744	150

Amount of 1 per Period at Compound Interest

$$s_{\overline{n}|i} = [(1+i)^n - 1]/i$$

n	$\frac{1}{4}\%$	$\frac{7}{24}\%$	$\frac{1}{3}\%$	$\frac{5}{12}\%$	n
151	183.1785 8319	189.3801 9080	195.8526 3650	209.6630 1910	151
152	184.6365 2965	190.9325 4969	197.5054 7862	211.5366 1501	152
153	186.0981 2097	192.4894 3630	199.1638 3021	213.4180 1758	153
154	187.5633 6627	194.0508 6382	200.8277 0965	215.3072 5932	154
155	189.0322 7469	195.6168 4551	202.4971 3534	217.2043 7290	155
156	190.5048 5538	197.1873 9464	204.1721 2580	219.1093 9112	156
157	191.9811 1752	198.7625 2454	205.8526 9955	221.0223 4691	157
158	193.4610 7031	200.3422 4857	207.5388 7521	222.9432 7336	158
159	194.9447 2298	201.9265 8013	209.2306 7146	224.8722 0366	159
160	196.4320 8479	203.5155 3265	210.9281 0704	226.8091 7118	160
161	197.9231 6500	205.1091 1962	212.6312 0073	228.7542 0939	161
162	199.4179 7292	206.7073 5455	214.3399 7139	230.7073 5193	162
163	200.9165 1785	208.3102 5101	216.0544 3797	232.6686 3256	163
164	202.4188 0914	209.9178 2257	217.7746 1942	234.6380 8520	164
165	203.9248 5617	211.5300 8289	219.5005 3482	236.6157 4389	165
166	205.4346 6831	213.1470 4563	221.2322 0327	238.6016 4282	166
167	206.9482 5498	214.7687 2451	222.9696 4395	240.5958 1633	167
168	208.4656 2562	216.3951 3329	224.7128 7610	242.5982 9890	168
169	209.9867 8968	218.0262 8576	226.4619 1902	244.6091 2515	169
170	211.5117 5665	219.6621 9576	228.2167 9208	246.6283 2983	170
171	213.0405 3605	221.3028 7717	229.9775 1472	248.6559 4788	171
172	214.5731 3739	222.9483 4389	231.7441 0643	250.6920 1433	172
173	216.1095 7023	224.5986 0989	233.5165 8679	252.7365 6439	173
174	217.6498 4415	226.2536 8917	235.2949 7541	254.7896 3340	174
175	219.1939 6876	227.9135 9577	237.0792 9200	256.8512 5688	175
176	220.7419 5369	229.5783 4375	238.8695 5630	258.9214 7045	176
177	222.2938 0857	231.2479 4726	240.6657 8816	261.0003 0991	177
178	223.8495 4309	232.9224 2044	242.4680 0745	263.0878 1120	178
179	225.4091 6695	234.6017 7750	244.2762 3414	265.1840 1041	179
180	226.9726 8987	236.2860 3268	246.0904 8826	267.2889 4379	180
181	228.5401 2159	237.9752 0028	247.9107 8988	269.4026 4772	181
182	230.1114 7190	239.6692 9461	249.7371 5918	271.5251 5875	182
183	231.6867 5058	241.3683 3005	251.5696 1638	273.6565 1358	183
184	233.2659 6745	243.0723 2101	253.4081 8177	275.7967 4905	184
185	234.8491 3237	244.7812 8195	255.2528 7571	277.9459 0218	185
186	236.4362 5520	246.4952 2736	257.1037 1863	280.1040 1010	186
187	238.0273 4584	248.2141 7177	258.9607 3102	282.2711 1014	187
188	239.6224 1420	249.9381 2977	260.8239 3346	284.4472 3977	188
189	241.2214 7024	251.6671 1598	262.6933 4657	286.6324 3660	189
190	242.8245 2392	253.4011 4507	264.5689 9106	288.8267 3842	190
191	244.4315 8523	255.1402 3174	266.4508 8769	291.0301 8316	191
192	246.0426 6419	256.8843 9075	268.3390 5732	293.2428 0892	192
193	247.6577 7085	258.6336 3689	270.2335 2084	295.4646 5396	193
194	249.2769 1528	260.3879 8500	272.1342 9925	297.6957 5669	194
195	250.9001 0756	262.1474 4995	274.0414 1358	299.9361 5568	195
196	252.5273 5783	263.9120 4668	275.9548 8495	302.1858 8966	196
197	254.1586 7623	265.6817 9015	277.8747 3457	304.4449 9753	197
198	255.7940 7292	267.4566 9537	279.8009 8369	306.7135 1835	198
199	257.4335 5810	269.2367 7740	281.7336 5363	308.9914 9135	199
200	259.0771 4200	271.0220 5134	283.6727 6581	311.2789 5589	200

Amount of 1 per Period at Compound Interest

$$s_{\overline{n}|i} = [(1+i)^n - 1]/i$$

n	$\frac{1}{2}\%$	$\frac{7}{12}\%$	$\frac{5}{8}\%$	$\frac{2}{3}\%$	n
1	1.0000 0000	1.0000 0000	1.0000 0000	1.0000 0000	1
2	2.0050 0000	2.0058 3333	2.0062 5000	2.0066 6667	2
3	3.0150 2500	3.0175 3403	3.1087 8906	3.0200 4444	3
4	4.0301 0013	4.0351 3631	4.0376 5649	4.0401 7807	4
5	5.0502 5063	5.0586 7460	5.0628 9185	5.0671 1259	5
6	6.0755 0188	6.0881 8354	6.0945 3492	6.1008 9335	6
7	7.1058 7939	7.1236 9794	7.1326 2576	7.1415 6597	7
8	8.1414 0879	8.1652 5284	8.1772 0468	8.1891 7641	8
9	9.1821 1583	9.2128 8349	9.2283 1220	9.2437 7092	9
10	10.2280 2641	10.2666 2531	10.2859 8916	10.3053 9606	10
11	11.2791 6654	11.3265 1396	11.3502 7659	11.3740 9870	11
12	12.3355 6237	12.3925 8529	12.4212 1582	12.4499 2602	12
13	13.3972 4018	13.4648 7537	13.4988 4842	13.5329 2553	13
14	14.4642 2639	14.5434 2048	14.5832 1622	14.6231 4503	14
15	15.5365 4752	15.6282 5710	15.6743 6132	15.7206 3266	15
16	16.6142 3026	16.7194 2193	16.7723 2608	16.8254 3688	16
17	17.6973 0141	17.8169 5189	17.8771 5312	17.9376 0646	17
18	18.7857 8791	18.9208 8411	18.9888 8532	19.0571 9051	18
19	19.8797 1685	20.0312 5593	20.1075 6586	20.1842 3844	19
20	20.9791 1544	21.1481 0493	21.2332 3814	21.3188 0003	20
21	22.0840 1101	22.2714 6887	22.3659 4588	22.4609 2536	21
22	23.1944 3107	23.4013 8577	23.5057 3304	23.6106 6487	22
23	24.3104 0322	24.5378 9386	24.6526 4387	24.7680 6930	23
24	25.4319 5524	25.6810 3157	25.8067 2290	25.9331 8976	24
25	26.5591 1502	26.8308 3759	26.9680 1492	27.1060 7769	25
26	27.6919 1059	27.9873 5081	28.1365 6501	28.2867 8488	26
27	28.8303 7015	29.1506 1035	29.3124 1854	29.4753 6344	27
28	29.9745 2200	30.3206 5558	30.4956 2116	30.6718 6586	28
29	31.1243 9461	31.4975 2607	31.6862 1879	31.8763 4497	29
30	32.2800 1658	32.6812 6164	32.8842 5766	33.0888 5394	30
31	33.4414 1666	33.8719 0233	34.0897 8427	34.3094 4630	31
32	34.6086 2375	35.0694 8843	35.3028 4542	35.5381 7594	32
33	35.7816 6686	36.2740 6045	36.5234 8820	36.7750 9711	33
34	36.9605 7520	37.4856 5913	37.7517 6000	38.0202 6443	34
35	38.1453 7807	38.7043 2548	38.9877 0850	39.2737 3286	35
36	39.3361 0496	39.9301 0071	40.2313 8168	40.5355 5774	36
37	40.5327 8549	41.1630 2630	41.4828 2782	41.8057 9479	37
38	41.7354 4942	42.4031 4395	42.7420 9549	43.0845 0009	38
39	42.9441 2666	43.6504 9562	44.0092 3359	44.3717 3009	39
40	44.1588 4730	44.9051 2352	45.2842 9130	45.6675 4163	40
41	45.3796 4153	46.1670 7007	46.5673 1812	46.9719 9191	41
42	46.6065 3974	47.4363 7798	47.8583 6386	48.2851 3852	42
43	47.8395 7244	48.7130 9018	49.1574 7863	49.6070 3944	43
44	49.0787 7030	49.9972 4988	50.4647 1287	50.9377 5304	44
45	50.3241 6415	51.2889 0050	51.7801 1733	52.2773 3806	45
46	51.5757 8497	52.5880 8575	53.1037 4306	53.6258 5365	46
47	52.8336 6390	53.8948 4959	54.4356 4146	54.9833 5934	47
48	54.0978 3222	55.2092 3621	55.7758 6421	56.3499 1507	48
49	55.3683 2138	56.5312 9009	57.1244 6337	57.7255 8117	49
50	56.6451 6299	57.8610 5595	58.4814 9126	59.1104 1837	50

$$s_{\overline{n}|i} = [(1+i)^n - 1]/i$$

n	$\frac{1}{2}\%$	$\frac{7}{12}\%$	$\frac{5}{8}\%$	$\frac{2}{3}\%$	n
51	57.9283 8880	59.1985 7877	59.8470 0058	60.5044 8783	51
52	59.2180 3075	60.5439 0381	61.2210 4434	61.9078 5108	52
53	60.5141 2090	61.8970 7659	62.6036 7586	63.3205 7009	53
54	61.8166 9150	63.2581 4287	63.9949 4884	64.7427 0722	54
55	63.1257 7496	64.6271 4870	65.3949 1727	66.1743 2527	55
56	64.4414 0384	66.0041 4040	66.8036 3550	67.6154 8744	56
57	65.7636 1086	67.3891 6455	68.2211 5822	69.0662 5736	57
58	67.0924 2891	68.7822 6801	69.6475 4046	70.5266 9907	58
59	68.4278 9105	70.1834 9791	71.0828 3759	71.9968 7706	59
60	69.7700 3051	71.5929 0165	72.5271 0532	73.4768 5625	60
61	71.1188 8066	73.0105 2691	73.9803 9973	74.9667 0195	61
62	72.4744 7507	74.4364 2165	75.4427 7723	76.4664 7997	62
63	73.8368 4744	75.8706 3411	76.9142 9459	77.9762 5650	63
64	75.2060 3168	77.3132 1281	78.3950 0893	79.4960 9821	64
65	76.5820 6184	78.7642 0655	79.8849 7774	81.0260 7220	65
66	77.9649 7215	80.2236 6442	81.3842 5885	82.5662 4601	66
67	79.3547 9701	81.6916 3579	82.8929 1046	84.1166 8765	67
68	80.7515 7099	83.1681 7034	84.4109 9115	85.6774 6557	68
69	82.1553 2885	84.6533 1800	85.9385 5985	87.2486 4867	69
70	83.5661 0549	86.1471 2902	87.4756 7585	88.8303 0633	70
71	84.9839 3602	87.6496 5394	89.0223 9882	90.4225 0837	71
72	86.4088 5570	89.1609 4359	90.5787 8882	92.0253 2510	72
73	87.8408 9998	90.6810 4909	92.1449 0625	93.6388 2726	73
74	89.2801 0448	92.2100 2188	93.7208 1191	95.2630 8611	74
75	90.7265 0500	93.7479 1367	95.3065 6698	96.8981 7335	75
76	92.1801 3752	95.2947 7650	96.9022 3303	98.5441 6118	76
77	93.6410 3821	96.8506 6270	98.5078 7198	100.2011 2225	77
78	95.1092 4340	98.4156 2490	100.1235 4618	101.8691 2973	78
79	96.5847 8962	99.9897 1604	101.7493 1835	103.5482 5726	79
80	98.0677 1357	101.5729 8938	103.3852 5159	105.2385 7898	80
81	99.5580 5214	103.1654 9849	105.0314 0941	106.9401 6950	81
82	101.0558 4240	104.7672 9723	106.6878 5572	108.6531 0397	82
83	102.5611 2161	106.3784 3980	108.3546 5482	110.3774 5799	83
84	104.0739 2722	107.9989 8070	110.0318 7141	112.1133 0771	84
85	105.5942 9685	109.6289 7475	111.7195 7061	113.8607 2977	85
86	107.1222 6834	111.2684 7710	113.4178 1792	115.6198 0130	86
87	108.6578 7968	112.9175 4322	115.1266 7928	117.3905 9997	87
88	110.2011 6908	114.5762 2889	116.8462 2103	119.1732 0397	88
89	111.7521 7492	116.2445 9022	118.5765 0991	120.9676 9200	89
90	113.3109 3580	117.9226 8367	120.3176 1310	122.7741 4328	90
91	114.8774 9048	119.6105 6599	122.0695 9818	124.5926 3757	91
92	116.4518 7793	121.3082 9429	123.8325 3317	126.4232 5515	92
93	118.0341 3732	123.0159 2601	125.6064 8650	128.2660 7685	93
94	119.6243 0800	124.7335 1891	127.3915 2704	130.1211 8403	94
95	121.2224 2954	126.4611 3110	129.1877 2408	131.9886 5859	95
96	122.8285 4169	128.1988 2103	130.9951 4736	133.8685 8298	96
97	124.4426 8440	129.9466 4749	132.8138 6703	135.7610 4020	97
98	126.0648 9782	131.7046 6960	134.6439 5370	137.6661 1380	98
99	127.6952 2231	133.4729 4684	136.4854 7841	139.5838 8790	99
100	129.3336 9842	135.2515 3903	138.3385 1265	141.5144 4715	100

Amount of 1 per Period at Compound Interest

$$s_{\overline{n}|i} = [(1+i)^n - 1]/i$$

n	$\frac{1}{2}\%$	$\frac{7}{12}\%$	$\frac{5}{8}\%$	$\frac{2}{3}\%$	n
101	130.9803 6692	137.0405 0634	140.2031 2836	143.4578 7680	101
102	132.6352 6875	138.8399 0929	142.0793 9791	145.4142 6264	102
103	134.2984 4509	140.6498 0876	143.9673 9414	147.3836 9106	103
104	135.9699 3732	142.4702 6598	145.8671 9036	149.3662 4900	104
105	137.6497 8701	144.3013 4253	147.7788 6030	151.3620 2399	105
106	139.3380 3594	146.1431 0036	149.7024 7817	153.3711 0415	106
107	141.0347 2612	147.9956 0178	151.6381 1866	155.3935 7818	107
108	142.7398 9975	149.8589 0946	153.5858 5690	157.4295 3537	108
109	144.4535 9925	151.7330 8643	155.5457 6851	159.4790 6560	109
110	146.1758 6725	153.6181 9610	157.5179 2956	161.5422 5937	110
111	147.9067 4658	155.5143 0225	159.5024 1662	163.6192 0777	111
112	149.6462 8032	157.4214 6901	161.4993 0673	165.7100 0249	112
113	151.3945 1172	159.3397 6091	163.5086 7739	167.8147 3584	113
114	153.1514 8428	161.2692 4285	165.5306 0663	169.9335 0074	114
115	154.9172 4170	163.2099 8010	167.5651 7292	172.0663 9075	115
116	156.6918 2791	165.1620 3832	169.6124 5525	174.2135 0002	116
117	158.4752 8704	167.1254 8354	171.6725 3310	176.3749 2335	117
118	160.2676 6348	169.1003 8219	173.7454 8643	178.5507 5618	118
119	162.0690 0180	171.0868 0109	175.8313 9572	180.7410 9455	119
120	163.8793 4681	173.0848 0743	177.9303 4194	182.9400 3518	120
121	165.6987 4354	175.0944 6881	180.0424 0658	185.1656 7542	121
122	167.5272 3726	177.1158 5321	182.1676 7162	187.4001 1325	122
123	169.3648 7344	179.1490 2902	184.3062 1957	189.6494 4734	123
124	171.2116 9781	181.1940 6502	186.4581 3344	191.9137 7699	124
125	173.0677 5630	183.2510 3040	188.6234 9677	194.1932 0217	125
126	174.9330 9508	185.3199 9474	190.8023 9363	196.4878 2352	126
127	176.8077 6056	187.4010 2805	192.9949 0859	198.7977 4234	127
128	178.6917 9936	189.4942 0071	195.2011 2677	201.1230 6062	128
129	180.5852 5836	191.5995 8355	197.4211 3381	203.4638 8103	129
130	182.4881 8465	193.7172 4778	199.6550 1589	205.8203 0690	130
131	184.4006 2557	195.8472 6506	201.9028 5974	208.1924 4228	131
132	186.3226 2870	197.9897 0744	204.1647 5262	210.5803 9189	132
133	188.2542 4184	200.1446 4740	206.4407 8232	212.9842 6117	133
134	190.1955 1305	202.3121 5785	208.7310 3721	215.4041 5625	134
135	192.1464 9062	204.4923 1210	211.0356 0619	217.8401 8396	135
136	194.1072 2307	206.6851 8392	213.3545 7873	220.2924 5185	136
137	196.0777 5919	208.8908 4749	215.6880 4485	222.7610 6820	137
138	198.0581 4798	211.1093 7744	218.0360 9513	225.2461 4198	138
139	200.0484 3872	213.3408 4881	220.3988 2072	227.7477 8293	139
140	202.0486 8092	215.5853 3709	222.7763 1335	230.2661 0148	140
141	204.0589 2432	217.8429 1822	225.1686 6531	232.8012 0883	141
142	206.0792 1894	220.1136 6858	227.5759 6947	235.3532 1688	142
143	208.1096 1504	222.3976 6498	229.9983 1928	237.9222 3833	143
144	210.1501 6311	224.6949 8469	232.4358 0878	240.5083 8659	144
145	212.2009 1393	227.0057 0544	234.8885 3258	243.1117 7583	145
146	214.2619 1850	229.3299 0538	237.3565 8591	245.7325 2100	146
147	216.3332 2809	231.6676 6317	239.8400 6457	248.3707 3781	147
148	218.4148 9423	234.0190 5787	242.3390 6497	251.0265 4273	148
149	220.5069 6870	236.3841 6904	244.8536 8413	253.7000 5301	149
150	222.6095 0354	238.7630 7669	247.3840 1966	256.3913 8670	150

Amount of 1 per Period at Compound Interest

$$s_{\overline{n}|i} = [(1+i)^n - 1]/i$$

n	½%	$\frac{7}{12}$%	⅝%	⅔%	n
151	224.7225 5106	241.1558 6130	249.9301 6978	259.1006 6261	151
152	226.8461 6382	243.5626 0383	252.4922 3334	261.8280 0036	152
153	228.9803 9464	245.9833 8568	255.0703 0980	264.5735 2036	153
154	231.1252 9661	248.4182 8877	257.6644 9923	267.3373 4383	154
155	233.2809 2309	250.8673 9545	260.2749 0235	270.1195 9279	155
156	235.4473 2771	253.3307 8859	262.9016 2049	272.9203 9008	156
157	237.6245 6435	255.8085 5153	265.5447 5562	275.7398 5934	157
158	239.8126 8717	258.3007 6808	268.2044 1035	278.5781 2507	158
159	242.0117 5060	260.8075 2256	270.8806 8791	281.4353 1257	159
160	244.2218 0936	263.3288 9977	273.5736 9221	284.3115 4799	160
161	246.4429 1840	265.8649 8502	276.2835 2779	287.2069 5831	161
162	248.6751 3300	268.4158 6410	279.0102 9983	290.1216 7136	162
163	250.9185 0866	270.9816 2331	281.7541 1421	293.0558 1584	163
164	253.1731 0121	273.5623 4944	284.5150 7742	296.0095 2128	164
165	255.4389 6671	276.1581 2982	287.2932 9666	298.9829 1809	165
166	257.7161 6154	278.7690 5224	290.0888 7976	301.9761 3754	166
167	260.0047 4235	281.3952 0504	292.9019 3526	304.9893 1179	167
168	262.3047 6606	284.0366 7707	295.7325 7235	308.0225 7387	168
169	264.6162 8989	286.6935 5769	298.5809 0093	311.0760 5770	169
170	266.9393 7134	289.3659 3678	301.4470 3156	314.1498 9808	170
171	269.2740 6820	292.0539 0474	304.3310 7551	317.2442 3073	171
172	271.6204 3854	294.7575 5252	307.2331 4473	320.3591 9227	172
173	273.9785 4073	297.4769 7158	310.1533 5189	323.4949 2022	173
174	276.3484 3344	300.2122 5391	313.0918 1033	326.6515 5302	174
175	278.7301 7561	302.9634 9206	316.0486 3415	329.8292 3004	175
176	281.1238 2648	305.7307 7910	319.0239 3811	333.0280 9158	176
177	283.5294 4562	308.5142 0864	322.0178 3773	336.2482 7885	177
178	285.9470 9284	311.3138 7486	325.0304 4921	339.4899 3405	178
179	288.3768 2831	314.1298 7246	328.0618 8952	342.7532 0027	179
180	290.8187 1245	316.9622 9672	331.1122 7633	346.0382 2161	180
181	293.2728 0601	319.8112 4345	334.1817 2806	349.3451 4309	181
182	295.7391 7004	322.6768 0904	337.2703 6386	352.6741 1071	182
183	298.2178 6589	325.5590 9042	340.3783 0363	356.0252 7144	183
184	300.7089 5522	328.4581 8512	343.5056 6803	359.3987 7325	184
185	303.2125 0000	331.3741 9120	346.6525 7845	362.7947 6508	185
186	305.7285 6250	334.3072 0731	349.8191 5707	366.2133 9684	186
187	308.2572 0531	337.2573 3269	353.0055 2680	369.6548 1949	187
188	310.7984 9134	340.2246 6713	356.2118 1134	373.1191 8495	188
189	313.3524 8379	343.2093 1102	359.4381 3516	376.6066 4618	189
190	315.9192 4621	346.2113 6533	362.6846 2351	380.1173 5716	190
191	318.4988 4244	349.2309 3163	365.9514 0241	383.6514 7287	191
192	321.0913 3666	352.2681 1207	369.2385 9867	387.2091 4936	192
193	323.6967 9334	355.3230 0939	372.5463 3991	390.7905 4369	193
194	326.3152 7731	358.3957 2694	375.8747 5454	394.3958 1398	194
195	328.9468 5369	361.4863 6868	379.2239 7175	398.0251 1941	195
196	331.5915 8796	364.5950 3917	382.5941 2158	401.6786 2020	196
197	334.2495 4590	367.7218 4356	385.9853 3484	405.3564 7767	197
198	336.9207 9363	370.8668 8765	389.3977 4318	409.0588 5419	198
199	339.6053 9760	374.0302 7783	392.8314 7907	412.7859 1322	199
200	342.3034 2459	377.2121 2111	396.2866 7582	416.5378 1930	200

Amount of 1 per Period at Compound Interest

$$s_{\overline{n}|i} = [(1+i)^n - 1]/i$$

n	$\frac{3}{4}\%$	$\frac{7}{8}\%$	1%	$1\frac{1}{8}\%$	n
1	1.0000 0000	1.0000 0000	1.0000 0000	1.0000 0000	1
2	2.0075 0000	2.0087 5000	2.0100 0000	2.0112 5000	2
3	3.0225 5625	3.0263 2656	3.0301 0000	3.0338 7656	3
4	4.0452 2542	4.0528 0692	4.0604 0100	4.0680 0767	4
5	5.0755 6461	5.0882 6898	5.1010 0501	5.1137 7276	5
6	6.1136 3135	6.1327 9133	6.1520 1506	6.1713 0270	6
7	7.1594 8358	7.1864 5326	7.2135 3521	7.2407 2986	7
8	8.2131 7971	8.2493 3472	8.2856 7056	8.3221 8807	8
9	9.2747 7856	9.3215 1640	9.3685 2727	9.4158 1269	9
10	10.3443 3940	10.4030 7967	10.4622 1254	10.5217 4058	10
11	11.4219 2194	11.4941 0662	11.5668 3467	11.6401 1016	11
12	12.5075 8636	12.5946 8005	12.6825 0301	12.7710 6140	12
13	13.6013 9325	13.7048 8350	13.8093 2804	13.9147 3584	13
14	14.7034 0370	14.8248 0123	14.9474 2132	15.0712 7662	14
15	15.8136 7923	15.9545 1824	16.0968 9554	16.2408 2848	15
16	16.9322 8183	17.0941 2028	17.2578 6449	17.4235 3780	16
17	18.0592 7394	18.2436 9383	18.4304 4314	18.6195 5260	17
18	19.1947 1849	19.4033 2615	19.6147 4757	19.8290 2257	18
19	20.3386 7888	20.5731 0526	20.8108 9504	21.0520 9907	19
20	21.4912 1897	21.7531 1993	22.0190 0399	22.2889 3519	20
21	22.6524 0312	22.9434 5973	23.2391 9403	23.5396 8571	21
22	23.8222 9614	24.1442 1500	24.4715 8598	24.8045 0717	22
23	25.0009 6336	25.3554 7688	25.7163 0183	26.0835 5788	23
24	26.1884 7059	26.5773 3730	26.9734 6485	27.3769 9790	24
25	27.3848 8412	27.8098 8900	28.2431 9950	28.6849 8913	25
26	28.5902 7075	29.0532 2553	29.5256 3150	30.0076 9526	26
27	29.8046 9778	30.3074 4126	30.8208 8781	31.3452 8183	27
28	31.0282 3301	31.5726 3137	32.1290 9669	32.6979 1625	28
29	32.2609 4476	32.8488 9189	33.4503 8766	34.0657 6781	29
30	33.5029 0184	34.1363 1970	34.7848 9153	35.4490 0769	30
31	34.7541 7361	35.4350 1249	36.1327 4045	36.8478 0903	31
32	36.0148 2991	36.7450 6885	37.4940 6785	38.2623 4688	32
33	37.2849 4113	38.0665 8820	38.8690 0853	39.6927 9829	33
34	38.5645 7819	39.3996 7085	40.2576 9862	41.1393 4227	34
35	39.8538 1253	40.7444 1797	41.6602 7560	42.6021 5987	35
36	41.1527 1612	42.1009 3163	43.0768 7836	44.0814 3417	36
37	42.4613 6149	43.4693 1478	44.5076 4714	45.5773 5030	37
38	43.7798 2170	44.8496 7128	45.9527 2361	47.0900 9549	38
39	45.1081 7037	46.2421 0591	47.4122 5085	48.6198 5906	39
40	46.4464 8164	47.6467 2433	48.8863 7336	50.1668 3248	40
41	47.7948 3026	49.0636 3317	50.3752 3709	51.7312 0934	41
42	49.1532 9148	50.4929 3996	51.8789 8946	53.3131 8545	42
43	50.5219 4117	51.9347 5319	53.3977 7936	54.9129 5879	43
44	51.9008 5573	53.3891 8228	54.9317 5715	56.5307 2957	44
45	53.2901 1215	54.8563 3762	56.4810 7472	58.1667 0028	45
46	54.6897 8799	56.3363 3058	58.0458 8547	59.8210 7566	46
47	56.0999 6140	57.8292 7347	59.6263 4432	61.4940 6276	47
48	57.5207 1111	59.3352 7961	61.2226 0777	63.1858 7097	48
49	58.9521 1644	60.8544 6331	62.8348 3385	64.8967 1201	49
50	60.3942 5732	62.3869 3986	64.4631 8218	66.6268 0002	50

$$s_{\overline{n}|i} = [(1+i)^n - 1]/i$$

n	$\frac{3}{4}$%	$\frac{7}{8}$%	1%	$1\frac{1}{8}$%	n
51	61.8472 1424	63.9328 2559	66.1078 1401	68.3763 5152	51
52	63.3110 6835	65.4922 3781	67.7688 9215	70.1455 8548	52
53	64.7859 0136	67.0652 9489	69.4465 8107	71.9347 2332	53
54	66.2717 9562	68.6521 1622	71.1410 4688	73.7439 8895	54
55	67.7688 3409	70.2528 2224	72.8524 5735	75.5736 0883	55
56	69.2771 0035	71.8675 3443	74.5809 8192	77.4238 1193	56
57	70.7966 7860	73.4963 7536	76.3267 9174	79.2948 2981	57
58	72.3276 5369	75.1394 6864	78.0900 5966	81.1868 9665	58
59	73.8701 1109	76.7969 3900	79.8709 6025	83.1002 4923	59
60	75.4241 3693	78.4689 1221	81.6696 6986	85.0351 2704	60
61	76.9898 1795	80.1555 1519	83.4863 6655	86.9917 7222	61
62	78.5672 4159	81.8568 7595	85.3212 3022	88.9704 2966	62
63	80.1564 9590	83.5731 2362	87.1744 4252	90.9713 4699	63
64	81.7576 6962	85.3043 8845	89.0461 8695	92.9947 7464	64
65	83.3708 5214	87.0508 0185	90.9366 4882	95.0409 6586	65
66	84.9961 3353	88.8124 9636	92.8460 1531	97.1101 7672	66
67	86.6336 0453	90.5896 0571	94.7744 7546	99.2026 6621	67
68	88.2833 5657	92.3822 6476	96.7222 2021	101.3186 9621	68
69	89.9454 8174	94.1906 0957	98.6894 4242	103.4585 3154	69
70	91.6200 7285	96.0147 7741	100.6763 3684	105.6224 4002	70
71	93.3072 2340	97.8549 0671	102.6831 0021	107.8106 9247	71
72	95.0070 2758	99.7111 3714	104.7099 3121	110.0235 6276	72
73	96.7195 8028	101.5836 0959	106.7570 3052	112.2613 2784	73
74	98.4449 7714	103.4724 6618	108.8246 0083	114.5242 6778	74
75	100.1833 1446	105.3778 5025	110.9128 4684	116.8126 6579	75
76	101.9346 8932	107.2999 0644	113.0219 7530	119.1268 0828	76
77	103.6991 9949	109.2387 8063	115.1521 9506	121.4669 8487	77
78	105.4769 4349	111.1946 1996	117.3037 1701	123.8334 8845	78
79	107.2680 2056	113.1675 7288	119.4767 5418	126.2266 1520	79
80	109.0725 3072	115.1577 8914	121.6715 2172	128.6466 6462	80
81	110.8905 7470	117.1654 1980	123.8882 3694	131.0939 3960	81
82	112.7222 5401	119.1906 1722	126.1271 1931	133.5687 4642	82
83	114.5676 7091	121.2335 3512	128.3883 9050	136.0713 9481	83
84	116.4269 2845	123.2943 2855	130.6722 7440	138.6021 9801	84
85	118.3001 3041	125.3731 5393	132.9789 9715	141.1614 7273	85
86	120.1873 8139	127.4701 6903	135.3087 8712	143.7495 3930	86
87	122.0887 8675	129.5855 3301	137.6618 7499	146.3667 2162	87
88	124.0044 5265	131.7194 0642	140.0384 9374	149.0133 4724	88
89	125.9344 8604	133.8719 5123	142.4388 7868	151.6897 4739	89
90	127.8789 9469	136.0433 3080	144.8632 6746	154.3962 5705	90
91	129.8380 8715	138.2337 0994	147.3119 0014	157.1332 1494	91
92	131.8118 7280	140.4432 5491	149.7850 1914	159.9009 6361	92
93	133.8004 6185	142.6721 3339	152.2828 6933	162.6998 4945	93
94	135.8039 6531	144.9205 1455	154.8056 9803	165.5302 2276	94
95	137.8224 9505	147.1885 6906	157.3537 5501	168.3924 3776	95
96	139.8561 6377	149.4764 6903	159.9272 9256	171.2868 5269	96
97	141.9050 8499	151.7843 8813	162.5265 6548	174.2138 2978	97
98	143.9693 7313	154.1125 0153	165.1518 3114	177.1737 3537	98
99	146.0491 4343	156.4609 8592	167.8033 4945	180.1669 3989	99
100	148.1445 1201	158.8300 1955	170.4813 8294	183.1938 1796	100

Amount of 1 per Period at Compound Interest

$$s_{\overline{n}|i} = [(1+i)^n - 1]/i$$

n	$\frac{3}{4}\%$	$\frac{7}{8}\%$	1%	$1\frac{1}{8}\%$	n
101	150.2555 9585	161 2197 8222	173.1861 9677	186.2547 4842	101
102	152.3825 1281	163.6304 5532	175.9180 5874	189.3501 1434	102
103	154.5253 8166	166.0622 2180	178.6772 3933	192.4803 0312	103
104	156.6843 2202	168.5122 6624	181.4640 1172	195.6457 0653	104
105	158.8594 5444	170.9897 7482	184.2786 5184	198.8467 2073	105
106	161.0509 0035	173.4859 3535	187.1214 3836	202.0837 4634	106
107	163.2587 8210	176.0039 3728	189.9926 5274	205.3571 8849	107
108	165.4832 2296	178.5439 7174	192.8925 7927	208.6674 5686	108
109	167.7243 4714	181.1062 3149	195.8215 0506	212.0149 6575	109
110	169.9822 7974	183.6909 1101	198.7797 2011	215.4001 3411	110
111	172.2571 4684	186.2982 0648	201.7675 1731	218.8233 8562	111
112	174.5490 7544	188.9283 1579	204.7851 9248	222.2851 4871	112
113	176.8581 9351	191.5814 3855	207.8330 4441	225.7858 5663	113
114	179.1846 2996	194.2577 7614	210.9113 7485	229.3259 4752	114
115	181.5285 1468	196.9575 3168	214.0204 8860	232.9058 6443	115
116	183.8899 7854	199.6809 1009	217.1606 9349	236.5260 5540	116
117	186.2691 5338	202.4281 1805	220.3323 0042	240.1869 7352	117
118	188.6661 7203	205.1993 6408	223.5356 2343	243.8890 7698	118
119	191.0811 6832	207.9948 5852	226.7709 7966	247.6328 2909	119
120	193.5142 7708	210.8148 1353	230.0386 8946	251.4186 9842	120
121	195.9656 3416	213.6594 4315	233.3390 7635	255.2471 5878	121
122	198.4353 7642	216.5289 6328	236.6724 6712	259.1186 8931	122
123	200.9236 4174	219.4235 9170	240.0391 9179	263.0337 7457	123
124	203.4305 6905	222.3435 4813	243.4395 8370	266.9929 0453	124
125	205.9562 9832	225.2890 5418	246.8739 7954	270.9965 7471	125
126	208.5009 7056	228.2603 3340	250.3427 1934	275.0452 8617	126
127	211.0647 2784	231.2576 1132	253.8461 4653	279.1395 4564	127
128	213.6477 1330	234.2811 1542	257.3846 0800	283.2798 6553	128
129	216.2500 7115	237.3310 7518	260.9584 5408	287.4667 6402	129
130	218.8719 4668	240.4077 2209	264.5680 3862	291.7007 6511	130
131	221.5134 8628	243.5112 8965	268.2137 1900	295.9823 9872	131
132	224.1748 3743	246.6420 1344	271.8958 5619	300.3122 0071	132
133	226.8561 4871	249.8001 3106	275.6148 1475	304.6907 1296	133
134	229.5575 6982	252.9858 8220	279.3709 6290	309.1184 8349	134
135	232.2792 5160	256.1995 0867	283.1646 7253	313.5960 6643	135
136	235.0213 4598	259.4412 5437	286.9963 1926	318.1240 2217	136
137	237.7840 0608	262.7113 6535	290.8662 8245	322.7029 1742	137
138	240.5673 8612	266.0100 8980	294.7749 4527	327.3333 2524	138
139	243.3716 4152	269.3376 7808	298.7226 9473	332.0158 2515	139
140	246.1969 2883	272.6943 8276	302.7099 2167	336.7510 0318	140
141	249.0434 0580	276.0804 5861	306.7370 2089	341.5394 5197	141
142	251.9112 3134	279.4961 6263	310.8043 9110	346.3817 7081	142
143	254.8005 6558	282.9417 5405	314.9124 3501	351.2785 6573	143
144	257.7115 6982	286.4174 9440	319.0615 5936	356.2304 4959	144
145	260.6444 0659	289.9236 4747	323.2521 7495	361.2380 4215	145
146	263.5992 3964	293.4604 7939	327.4846 9670	366.3019 7012	146
147	266.5762 3394	297.0282 5858	331.7595 4367	371.4228 6729	147
148	269.5755 5569	300.6272 5585	336.0771 3911	376.6013 7454	148
149	272.5973 7236	304.2577 4433	340.4379 1050	381.8381 4001	149
150	275.6418 5265	307.9199 9960	344.8422 8960	387.1338 1908	150

$$s_{\overline{n}|i} = [(1+i)^n - 1]/i$$

n	$\frac{3}{4}\%$	$\frac{7}{8}\%$	1%	$1\frac{1}{8}\%$	n
151	278.7091 6655	311.6142 9959	349.2907 1250	392.4890 7455	151
152	281.7994 8530	315.3409 2472	353.7836 1962	397.9045 7664	152
153	284.9129 8144	319.1001 5781	358.3214 5582	403.3810 0312	153
154	288.0498 2880	322.8922 8419	362.9046 7038	408.9190 3941	154
155	291.2102 0251	326.7175 9167	367.5337 1708	414.5193 7860	155
156	294.3942 7903	330.5763 7060	372.2090 5425	420.1827 2161	156
157	297.6022 3613	334.4689 1384	376.9311 4480	425.9097 7723	157
158	300.8342 5290	338.3955 1684	381.7004 5624	431.7012 6222	158
159	304.0905 0979	342.3564 7761	386.5174 6081	437.5579 0142	159
160	307.3711 8862	346.3520 9679	391.3826 3541	443.4804 2781	160
161	310.6764 7253	350.3826 7764	396.2964 6177	449.4695 8263	161
162	314.0065 4608	354.4485 2607	401.2594 2639	455.5261 1543	162
163	317.3615 9517	358.5499 5067	406.2720 2065	461.6507 8423	163
164	320.7418 0714	362.6872 6274	411.3347 4086	467.8443 5555	164
165	324.1473 7069	366.8607 7629	416.4480 8826	474.1076 0455	165
166	327.5784 7597	371.0708 0808	421.6125 6915	480.4413 1510	166
167	331.0353 1454	375.3176 7765	426.8286 9484	486.8462 7990	167
168	334.5180 7940	379.6017 0733	432.0969 8179	493.3233 0055	168
169	338.0269 6499	383.9232 2227	437.4179 5161	499.8731 8768	169
170	341.5621 6723	388.2825 5046	442.7921 3112	506.4967 6104	170
171	345.1238 8349	392.6800 2278	448.2200 5243	513.1948 4960	171
172	348.7123 1261	397.1159 7298	453.7022 5296	519.9682 9166	172
173	352.3276 5496	401.5907 3774	459.2392 7549	526.8179 3494	173
174	355.9701 1237	406.1046 5670	464.8316 6824	533.7446 3671	174
175	359.6398 8821	410.6580 7245	470.4799 8492	540.7492 6387	175
176	363.3371 8737	415.2513 3058	476.1847 8477	547.8326 9309	176
177	367.0622 1628	419.8847 7972	481.9466 3262	554.9958 1089	177
178	370.8151 8290	424.5587 7154	487.7660 9895	562.2395 1376	178
179	374.5962 9677	429.2736 6080	493.6437 5994	569.5647 0829	179
180	378.4057 6900	434.0298 0533	499.5801 9754	576.9723 1126	180
181	382.2438 1226	438.8275 6612	505.5759 9951	584.4632 4976	181
182	386.1106 4086	443.6673 0733	511.6317 5951	592.0384 6132	182
183	390.0064 7066	448.5493 9627	517.7480 7710	599.6988 9401	183
184	393.9315 1919	453.4742 0348	523.9255 5787	607.4455 0657	184
185	397.8860 0559	458.4421 0276	530.1648 1345	615.2792 6852	185
186	401.8701 5063	463.4534 7116	536.4664 6159	623.2011 6029	186
187	405.8841 7676	468.5086 8904	542.8311 2620	631.2121 7334	187
188	409.9283 0808	473.6081 4007	549.2594 3746	639.3133 1029	188
189	414.0027 7039	478.7522 1129	555.7520 3184	647.5055 8503	189
190	418.1077 9117	483.9412 9314	562.3095 5216	655.7900 2286	190
191	422.2435 9961	489.1757 7946	568.9326 4768	664.1676 6062	191
192	426.4104 2660	494.4560 6753	575.6219 7415	672.6395 4680	192
193	430.6085 0480	499.7825 5812	582.3781 9390	681.2067 4170	193
194	434.8380 6859	505.1556 5550	589.2019 7584	689.8703 1755	194
195	439.0993 5410	510.5757 6749	596.0939 9559	698.6313 5862	195
196	443.3925 9926	516.0433 0545	603.0549 3555	707.4909 6140	196
197	447.7180 4375	521.5586 8437	610.0854 8490	716.4502 3472	197
198	452.0759 2908	527.1223 2286	617.1863 3975	725.5102 9986	198
199	456.4664 9855	532.7346 4319	624.3582 0315	734.6722 9073	199
200	460.8899 9729	538.3960 7131	631.6017 8518	743.9373 5401	200

Amount of 1 per Period at Compound Interest

$$s_{\overline{n}|i} = [(1+i)^n - 1]/i$$

n	$1\frac{1}{4}\%$	$1\frac{3}{8}\%$	$1\frac{1}{2}\%$	$1\frac{3}{4}\%$	n
1	1.0000 0000	1.0000 0000	1.0000 0000	1.0000 0000	1
2	2.0125 0000	2.0137 5000	2.0150 0000	2.0175 0000	2
3	3.0376 5625	3.0414 3906	3.0452 2500	3.0528 0625	3
4	4.0756 2695	4.0832 5885	4.0909 0338	4.1062 3036	4
5	5.1265 7229	5.1394 0366	5.1522 6693	5.1780 8938	5
6	6.1906 5444	6.2100 7046	6.2295 5093	6.2687 0596	6
7	7.2680 3762	7.2954 5893	7.3229 9419	7.3784 0831	7
8	8.3588 8809	8.3957 7149	8.4328 3911	8.5075 3045	8
9	9.4633 7420	9.5112 1335	9.5593 3169	9.6564 1224	9
10	10.5816 6637	10.6419 9253	10.7027 2167	10.8253 9945	10
11	11.7139 3720	11.7883 1993	11.8632 6249	12.0148 4394	11
12	12.8603 6142	12.9504 0933	13.0412 1143	13.2251 0371	12
13	14.0211 1594	14.1284 7745	14.2368 2960	14.4565 4303	13
14	15.1963 7988	15.3227 4402	15.4503 8205	15.7095 3253	14
15	16.3863 3463	16.5334 3175	16.6821 3778	16.9844 4935	15
16	17.5911 6382	17.7607 6644	17.9323 6984	18.2816 7721	16
17	18.8110 5336	19.0049 7697	19.2013 5539	19.6016 0656	17
18	20.0461 9153	20.2662 9541	20.4893 7572	20.9446 3468	18
19	21.2967 6893	21.5449 5697	21.7967 1636	22.3111 6578	19
20	22.5629 7854	22.8412 0013	23.1236 6710	23.7016 1119	20
21	23.8450 1577	24.1552 6663	24.4705 2211	25.1163 8938	21
22	25.1430 7847	25.4874 0155	25.8375 7994	26.5559 2620	22
23	26.4573 6695	26.8378 5332	27.2251 4364	28.0206 5490	23
24	27.7880 8403	28.2068 7380	28.6335 2080	29.5110 1637	24
25	29.1354 3508	29.5947 1832	30.0630 2361	31.0274 5915	25
26	30.4996 2802	31.0016 4569	31.5139 6896	32.5704 3969	26
27	31.8808 7337	32.4279 1832	32.9866 7850	34.1404 2238	27
28	33.2793 8429	33.8738 0220	34.4814 7867	35.7378 7977	28
29	34.6953 7659	35.3395 6698	35.9987 0085	37.3632 9267	29
30	36.1290 6880	36.8254 8602	37.5386 8137	39.0171 5029	30
31	37.5806 8216	38.3318 3646	39.1017 6159	40.6999 5042	31
32	39.0504 4069	39.8588 9921	40.6882 8801	42.4121 9955	32
33	40.5385 7120	41.4069 5907	42.2986 1233	44.1544 1305	33
34	42.0453 0334	42.9763 0476	43.9330 9152	45.9271 1527	34
35	43.5708 6963	44.5672 2895	45.5920 8789	47.7308 3979	35
36	45.1155 0550	46.1800 2835	47.2759 6921	49.5661 2949	36
37	46.6794 4932	47.8150 0374	48.9851 0874	51.4335 3675	37
38	48.2926 4243	49.4724 6004	50.7198 8538	53.3336 2365	38
39	49.8862 2921	51.1527 0636	52.4806 8366	55.2669 6206	39
40	51.4895 5708	52.8560 5608	54.2678 9391	57.2341 3390	40
41	53.1331 7654	54.5828 2685	56.0819 1232	59.2357 3124	41
42	54.7973 4125	56.3333 4072	57.9231 4100	61.2723 5654	42
43	56.4823 0801	58.1079 2415	59.7919 8812	63.3446 2278	43
44	58.1883 3687	59.9069 0811	61.6888 6794	65.4531 5367	44
45	59.9156 9108	61.7306 2810	63.6142 0096	67.5985 8386	45
46	61.6646 3721	63.5794 2423	65.5684 1398	69.7815 5908	46
47	63.4354 4518	65.4536 4131	67.5519 4018	72.0027 3637	47
48	65.2283 8824	67.3536 2888	69.5652 1929	74.2627 8425	48
49	67.0437 4310	69.2797 4128	71.6086 9758	76.5623 8298	49
50	68.8817 8989	71.2323 3772	73.6828 2804	78.9022 2468	50

Amount of 1 per Period at Compound Interest

$$s_{\overline{n}|i} = [(1+i)^n - 1]/i$$

n	$1\frac{1}{4}\%$	$1\frac{3}{8}\%$	$1\frac{1}{2}\%$	$1\frac{3}{4}\%$	n
51	70.7428 1226	73.2117 8237	75.7880 7046	81.2830 1361	51
52	72.6270 9741	75.2184 4437	77.9248 9152	83.7054 6635	52
53	74.5349 3613	77.2526 9798	80.0937 6489	86.1703 1201	53
54	76.4666 2283	79.3149 2258	82.2951 7136	88.6782 9247	54
55	78.4224 5562	81.4055 0277	84.5295 9893	91.2301 6259	55
56	80.4027 3631	83.5248 2843	86.7975 4292	93.8266 9043	56
57	82.4077 7052	85.6732 9482	89.0995 0606	96.4686 5752	57
58	84.4378 6765	87.8513 0262	91.4359 9865	99.1568 5902	58
59	86.4933 4099	90.0592 5804	93.8075 3863	101.8921 0405	59
60	88.5745 0776	92.2975 7283	96.2146 5171	104.6752 1588	60
61	90.6816 8910	94.5666 6446	98.6578 7149	107.5070 3215	61
62	92.8152 1022	96.8669 5610	101.1377 3956	110.3884 0522	62
63	94.9754 0034	99.1988 7674	103.6548 0565	113.3202 0231	63
64	97.1625 9285	101.5628 6130	106.2096 2774	116.3033 0585	64
65	99.3771 2526	103.9593 5064	108.8027 7215	119.3386 1370	65
66	101.6193 3933	106.3887 9171	111.4348 1374	122.4270 3944	66
67	103.8895 8107	108.8516 3760	114.1063 3594	125.5695 1263	67
68	106.1882 0083	111.3483 4761	116.8179 3098	128.7669 7910	68
69	108.5155 5334	113.8793 8739	119.5701 9995	132.0204 0124	69
70	110.8719 9776	116.4452 2897	122.3637 5295	135.3307 5826	70
71	113.2578 9773	119.0463 5087	125.1992 0924	138.6990 4653	71
72	115.6736 2145	121.6832 3819	128.0771 9738	142.1262 7984	72
73	118.1195 4172	124.3563 8272	130.9983 5534	145.6134 8974	73
74	120.5960 3599	127.0662 8298	133.9633 3067	149.1617 2581	74
75	123.1034 8644	129.8134 4437	136.9727 8063	152.7720 5601	75
76	125.6422 8002	132.5983 7923	140.0273 7234	156.4455 6699	76
77	128.2128 0852	135.4216 0695	143.1277 8292	160.1833 6441	77
78	130.8154 6863	138.2836 5404	146.2746 9967	163.9865 7329	78
79	133.4506 6199	141.1850 5429	149.4688 2016	167.8563 3832	79
80	136.1187 9526	144.1263 4878	152.7108 5247	171.7938 2424	80
81	138.8202 8020	147.1080 8608	156.0015 1525	175.8002 1617	81
82	141.5555 3370	150.1308 2226	159.3415 3798	179.8767 1995	82
83	144.3249 7787	153.1951 2107	162.7316 6105	184.0245 6255	83
84	147.1290 4010	156.3015 5398	166.1726 3597	188.2449 9239	84
85	149.9681 5310	159.4507 0035	169.6652 2551	192.5392 7976	85
86	152.8427 5501	162.6431 4748	173.2102 0389	196.9087 1716	86
87	155.7532 8945	165.8794 9076	176.8083 5695	201.3546 1971	87
88	158.7002 0557	169.1603 3375	180.4604 8230	205.8783 2555	88
89	161.6839 5814	172.4862 8834	184.1673 8954	210.4811 9625	89
90	164.7050 0762	175.8579 7481	187.9299 0038	215.1646 1718	90
91	167.7638 2021	179.2760 2196	191.7488 4889	219.9299 9798	91
92	170.8608 6796	182.7410 6726	195.6250 8162	224.7787 7295	92
93	173.9966 2881	186.2537 5694	199.5594 5784	229.7124 0148	93
94	177.1715 8667	189.8147 4610	203.5528 4971	234.7323 6850	94
95	180.3862 3151	193.4246 9886	207.6061 4246	239.8401 8495	95
96	183.6410 5940	197.0842 8847	211.7202 3459	245.0373 8819	96
97	186.9365 7264	200.7941 9743	215.8960 3811	250.3255 4248	97
98	190.2732 7980	204.5551 1765	220.1344 7868	255.7062 3947	98
99	193.6516 9580	208.3677 5051	224.4364 9586	261.1810 9866	99
100	197.0723 4200	212.2328 0708	228.8030 4330	266.7517 6789	100

Amount of 1 per Period at Compound Interest

$$s_{\overline{n}|i} = [(1+i)^n - 1]/i$$

n	2%	2¼%	2½%	2¾%	n
1	1.0000 0000	1.0000 0000	1.0000 0000	1.0000 0000	1
2	2.0200 0000	2.0225 0000	2.0250 0000	2.0275 0000	2
3	3.0604 0000	3.0680 0625	3.0756 2500	3.0832 5625	3
4	4.1216 0800	4.1370 3639	4.1525 1563	4.1680 4580	4
5	5.2040 4016	5.2301 1971	5.2563 2852	5.2826 6706	5
6	6.3081 2096	6.3477 9740	6.3877 3673	6.4279 4040	6
7	7.4342 8338	7.4906 2284	7.5474 3015	7.6047 0876	7
8	8.5829 6905	8.6591 6186	8.7361 1590	8.8138 3825	8
9	9.7546 2843	9.8539 9300	9.9545 1880	10.0562 1880	9
10	10.9497 2100	11.0757 0784	11.2033 8177	11.3327 6482	10
11	12.1687 1542	12.3249 1127	12.4834 6631	12.6444 1585	11
12	13.4120 8973	13.6022 2177	13.7955 5297	13.9921 3729	12
13	14.6803 3152	14.9082 7176	15.1404 4179	15.3769 2107	13
14	15.9739 3815	16.2437 0788	16.5189 5284	16.7997 8639	14
15	17.2934 1692	17.6091 9130	17.9319 2666	18.2617 8052	15
16	18.6392 8525	19.0053 9811	19.3802 2483	19.7639 7948	16
17	20.0120 7096	20.4330 1957	20.8647 3045	21.3074 8892	17
18	21.4123 1238	21.8927 6251	22.3863 4871	22.8934 4487	18
19	22.8405 5863	23.3853 4966	23.9460 0743	24.5230 1460	19
20	24.2973 6980	24.9115 2003	25.5446 5761	26.1973 9750	20
21	25.7833 1719	26.4720 2923	27.1832 7405	27.9178 2593	21
22	27.2989 8354	28.0676 4989	28.8628 5590	29.6855 6615	22
23	28.8449 6321	29.6991 7201	30.5844 2730	31.5019 1921	23
24	30.4218 6247	31.3674 0338	32.3490 3798	33.3682 2199	24
25	32.0302 9972	33.0731 6996	34.1577 6393	35.2858 4810	25
26	33.6709 0572	34.8173 1628	36.0117 0803	37.2562 0892	26
27	35.3443 2383	36.6007 0590	37.9120 0073	39.2807 5467	27
28	37.0512 1031	38.4242 2178	39.8598 0075	41.3609 7542	28
29	38.7922 3451	40.2887 6677	41.8562 9577	43.4984 0224	29
30	40.5680 7921	42.1952 6402	43.9027 0316	45.6946 0830	30
31	42.3794 4079	44.1446 5746	46.0002 7074	47.9512 1003	31
32	44.2270 2961	46.1379 1226	48.1502 7751	50.2698 6831	32
33	46.1115 7020	48.1760 1528	50.3540 3445	52.6522 8969	33
34	48.0338 0160	50.2599 7563	52.6128 8531	55.1002 2765	34
35	49.9944 7763	52.3908 2508	54.9282 0744	57.6154 8391	35
36	51.9943 6719	54.5696 1864	57.3014 1263	60.1999 0972	36
37	54.0342 5453	56.7974 3506	59.7339 4794	62.8554 0724	37
38	56.1149 3962	59.0753 7735	62.2272 9664	65.5839 3094	38
39	58.2372 3841	61.4045 7334	64.7829 7906	68.3874 8904	39
40	60.4019 8318	63.7861 7624	67.4025 5354	71.2681 4499	40
41	62.6100 2284	66.2213 6521	70.0876 1737	74.2280 1898	41
42	64.8622 2330	68.7113 4592	72.8398 0781	77.2692 8950	42
43	67.1594 6777	71.2573 5121	75.6608 0300	80.3941 9496	43
44	69.5026 5712	73.8606 4161	78.5523 2308	83.6050 3532	44
45	71.8927 1027	76.5225 0605	81.5161 3116	86.9041 7379	45
46	74.3305 6447	79.2442 6243	84.5540 3443	90.2940 3857	46
47	76.8171 7576	82.0272 5834	87.6678 8530	93.7771 2463	47
48	79.3535 1927	84.8728 7165	90.8595 8243	97.3559 9556	48
49	81.9405 8966	87.7825 1126	94.1310 7199	101.0332 8544	49
50	84.5794 0145	90.7576 1776	97.4843 4879	104.8117 0079	50

Amount of 1 per Period at Compound Interest

$$s_{\overline{n}|i} = [(1+i)^n - 1]/i$$

n	2%	2¼%	2½%	2¾%	n
51	87.2709 8948	93.7996 6416	100.9214 5751	108.6940 2256	51
52	90.0164 0927	96.9101 5661	104.4444 9395	112.6831 0818	52
53	92.8167 3746	100.0906 3513	108.0556 0629	116.7818 9365	53
54	95.6730 7221	103.3426 7442	111.7569 9645	120.9933 9573	54
55	98.5865 3365	106.6678 8460	115.5509 2136	125.3207 1411	55
56	101.5582 6432	110.0679 1200	119.4396 9440	129.7670 3375	56
57	104.5894 2961	113.5444 4002	123.4256 8676	134.3356 2718	57
58	107.6812 1820	117.0991 8992	127.5113 2893	139.0298 5692	58
59	110.8348 4257	120.7339 2169	131.6991 1215	143.8531 7799	59
60	114.0515 3942	124.4504 3493	135.9915 8995	148.8091 4038	60
61	117.3325 7021	128.2505 6972	140.3913 7970	153.9013 9174	61
62	120.6792 2161	132.1362 0754	144.9011 6419	159.1336 8002	62
63	124.0928 0604	136.1092 7221	149.5236 9330	164.5098 5622	63
64	127.5746 6216	140.1717 3083	154.2617 8563	170.0338 7726	64
65	131.1261 5541	144.3255 9477	159.1183 3027	175.7098 0889	65
66	134.7486 7852	148.5729 2066	164.0962 8853	181.5418 2863	66
67	138.4436 5209	152.9158 1137	169.1986 9574	187.5342 2892	67
68	142.2125 2513	157.3564 1713	174.4286 6314	193.6914 2021	68
69	146.0567 7563	161.8969 3651	179.7893 7971	200.0179 3427	69
70	149.9779 1114	166.5396 1758	185.2841 1421	206.5184 2746	70
71	153.9774 6937	171.2867 5898	190.9162 1706	213.1976 8422	71
72	158.0570 1875	176.1407 1106	196.6891 2249	220.0606 2054	72
73	162.2181 5913	181.1038 7705	202.6063 5055	227.1122 8760	73
74	166.4625 2231	186.1787 1429	208.6715 0931	234.3578 7551	74
75	170.7917 7276	191.3677 3536	214.8882 9705	241.8027 1709	75
76	175.2076 0821	196.6735 0941	221.2605 0447	249.4522 9181	76
77	179.7117 6038	202.0986 6337	227.7920 1709	257.3122 2983	77
78	184.3059 9558	207.6458 8329	234.4868 1751	265.3883 1615	78
79	188.9921 1549	213.3179 1567	241.3489 8795	273.6864 9485	79
80	193.7719 5780	219.1175 6877	248.3827 1265	282.2128 7345	80
81	198.6473 9696	225.0477 1407	255.5922 8047	290.9737 2747	81
82	203.6203 4490	231.1112 8763	262.9820 8748	299.9755 0498	82
83	208.6927 5180	237.3112 9160	270.5566 3966	309.2248 3137	83
84	213.8666 0683	243.6507 9567	278.3205 5566	318.7285 1423	84
85	219.1439 3897	250.1329 3857	286.2785 6955	328.4935 4837	85
86	224.5268 1775	256.7609 2969	294.4355 3379	338.5271 2095	86
87	230.0173 5411	263.5380 5060	302.7964 2213	348.8366 1678	87
88	235.6177 0119	270.4676 5674	311.3663 3268	359.4296 2374	88
89	241.3300 5521	277.5531 7902	320.1504 9100	370.3139 3839	89
90	247.1566 5632	284.7981 2555	329.1542 5328	381.4975 7170	90
91	253.0997 8944	292.2060 8337	338.3831 0961	392.9887 5492	91
92	259.1617 8523	299.7807 2025	347.8426 8735	404.7959 4568	92
93	265.3450 2094	307.5257 8645	357.5387 5453	416.9278 3418	93
94	271.6519 2135	315.4451 1665	367.4772 2339	429.3933 4962	94
95	278.0849 5978	323.5426 3177	377.6641 5398	442.2016 6674	95
96	284.6466 5898	331.8223 4099	388.1057 5783	455.3622 1257	96
97	291.3395 9216	340.2883 4366	398.8084 0177	468.8846 7342	97
98	298.1663 8400	348.9448 3139	409.7786 1182	482.7790 0194	98
99	305.1297 1168	357.7960 9010	421.0230 7711	497.0554 2449	99
100	312.2323 0591	366.8465 0213	432.5486 5404	511.7244 4867	100

Amount of 1 per Period at Compound Interest

$$s_{\overline{n}|i} = [(1+i)^n - 1]/i$$

n	3%	$3\frac{1}{2}$%	4%	$4\frac{1}{2}$%	n
1	1.0000 0000	1.0000 0000	1.0000 0000	1.0000 0000	1
2	2.0300 0000	2.0350 0000	2.0400 0000	2.0450 0000	2
3	3.0909 0000	3.1062 2500	3.1216 0000	3.1370 2500	3
4	4.1836 2700	4.2149 4288	4.2464 6400	4.2781 9113	4
5	5.3091 3581	5.3624 6588	5.4163 2256	5.4707 0973	5
6	6.4684 0988	6.5501 5218	6.6329 7546	6.7168 9166	6
7	7.6624 6218	7.7794 0751	7.8982 9448	8.0191 5179	7
8	8.8923 3605	9.0516 8677	9.2142 2626	9.3800 1362	8
9	10.1591 0613	10.3684 9581	10.5827 9531	10.8021 1423	9
10	11.4638 7931	11.7313 9316	12.0061 0712	12.2882 0937	10
11	12.8077 9569	13.1419 9192	13.4863 5141	13.8411 7879	11
12	14.1920 2956	14.6019 6164	15.0258 0546	15.4640 3184	12
13	15.6177 9045	16.1130 3030	16.6268 3768	17.1599 1327	13
14	17.0863 2416	17.6769 8636	18.2919 1119	18.9321 0937	14
15	18.5989 1389	19.2956 8088	20.0235 8764	20.7840 5429	15
16	20.1568 8130	20.9710 2971	21.8245 3114	22.7193 3673	16
17	21.7615 8774	22.7050 1575	23.6975 1239	24.7417 0689	17
18	23.4144 3537	24.4996 9130	25.6454 1288	26.8550 8370	18
19	25.1168 6844	26.3571 8050	27.6712 2940	29.0635 6246	19
20	26.8703 7449	28.2796 8181	29.7780 7858	31.3714 2277	20
21	28.6764 8572	30.2694 7068	31.9692 0172	33.7831 3680	21
22	30.5367 8030	32.3289 0215	34.2479 6979	36.3033 7795	22
23	32.4528 8370	34.4604 1373	36.6178 8858	38.9370 2996	23
24	34.4264 7022	36.6665 2821	39.0826 0412	41.6891 9631	24
25	36.4592 6432	38.9498 5669	41.6459 0829	44.5652 1015	25
26	38.5530 4225	41.3131 0168	44.3117 4462	47.5706 4460	26
27	40.7096 3352	43.7590 6024	47.0842 1440	50.7113 2361	27
28	42.9309 2252	46.2906 2734	49.9675 8298	53.9933 3317	28
29	45.2188 5020	48.9107 9930	52.9662 8630	57.4230 3316	29
30	47.5754 1571	51.6226 7728	56.0849 3775	61.0070 6966	30
31	50.0026 7818	54.4294 7098	59.3283 3526	64.7523 8779	31
32	52.5027 5852	57.3345 0247	62.7014 6867	68.6662 4524	32
33	55.0778 4128	60.3412 1005	66.2095 2742	72.7562 2628	33
34	57.7301 7652	63.4531 5240	69.8579 0851	77.0302 5646	34
35	60.4620 8181	66.6740 1274	73.6522 2486	81.4966 1800	35
36	63.2759 4427	70.0076 0318	77.5983 1385	86.1639 6581	36
37	66.1742 2259	73.4578 6930	81.7022 4640	91.0413 4427	37
38	69.1594 4927	77.0288 9472	85.9703 3626	96.1382 0476	38
39	72.2342 3275	80.7249 0604	90.4091 4971	101.4644 2398	39
40	75.4012 5973	84.5502 7775	95.0255 1570	107.0303 2306	40
41	78.6632 9753	88.5095 3747	99.8265 3633	112.8466 8760	41
42	82.0231 9645	92.6073 7128	104.8195 9778	118.9247 8854	42
43	85.4838 9234	96.8486 2928	110.0123 8169	125.2764 0402	43
44	89.0484 0911	101.2383 3130	115.4128 7696	131.9138 4220	44
45	92.7198 6139	105.7816 7290	121.0293 9204	138.8499 6510	45
46	96.5014 5723	110.4840 3145	126.8705 6772	146.0982 1353	46
47	100.3965 0095	115.3509 7255	132.9453 9043	153.6726 3314	47
48	104.4083 9598	120.3882 5659	139.2632 0604	161.5879 0163	48
49	108.5406 4785	125.6018 4557	145.8337 3429	169.8593 5720	49
50	112.7968 6729	130.9979 1016	152.6670 8366	178.5030 2828	50

$$s_{\overline{n}|i} = [(1+i)^n - 1]/i$$

n	3%	3½%	4%	4½%	n
51	117.1807 7331	136.5828 3702	159.7737 6700	187.5356 6455	51
52	121.6961 9651	142.3632 3631	167.1647 1768	196.9747 6946	52
53	126.3470 8240	148.3459 4958	174.8513 0639	206.8386 3408	53
54	131.1374 9488	154.5380 5782	182.8453 5865	217.1463 7262	54
55	136.0716 1972	160.9468 8984	191.1591 7299	227.9179 5938	55
56	141.1537 6831	167.5800 3099	199.8055 3991	239.1742 6756	56
57	146.3883 8136	174.4453 3207	208.7977 6151	250.9371 0960	57
58	151.7800 3280	181.5509 1869	218.1496 7197	263.2292 7953	58
59	157.3334 3379	188.9052 0085	227.8756 5885	276.0745 9711	59
60	163.0534 3680	196.5168 8288	237.9906 8520	289.4979 5398	60
61	168.9450 3991	204.3949 7378	248.5103 1261	303.5253 6190	61
62	175.0133 9110	212.5487 9786	259.4507 2511	318.1840 0319	62
63	181.2637 9284	220.9880 0579	270.8287 5412	333.5022 8333	63
64	187.7017 0662	229.7225 8599	282.6619 0428	349.5098 8608	64
65	194.3327 5782	238.7628 7650	294.9683 8045	366.2378 3096	65
66	201.1627 4055	248.1195 7718	307.7671 1567	383.7185 3335	66
67	208.1976 2277	257.8037 6238	321.0778 0030	401.9858 6735	67
68	215.4435 5145	267.8268 9406	334.9209 1231	421.0752 3138	68
69	222.9068 5800	278.2008 3535	349.3177 4880	441.0236 1679	69
70	230.5940 6374	288.9378 6459	364.2904 5876	461.8696 7955	70
71	238.5118 8565	300.0506 8985	379.8620 7711	483.6538 1513	71
72	246.6672 4222	311.5524 6400	396.0565 6019	506.4182 3681	72
73	255.0672 5949	323.4568 0024	412.8988 2260	530.2070 5747	73
74	263.7192 7727	335.7777 8824	430.4147 7550	555.0663 7505	74
75	272.6308 5559	348.5300 1083	448.6313 6652	581.0443 6193	75
76	281.8097 8126	361.7285 6121	467.5766 2118	608.1913 5822	76
77	291.2640 7469	375.3890 6085	487.2796 8603	636.5599 6934	77
78	301.0019 9693	389.5276 7798	507.7708 7347	666.2051 6796	78
79	311.0320 5684	404.1611 4671	529.0817 0841	697.1844 0052	79
80	321.3630 1855	419.3067 8685	551.2449 7675	729.5576 9854	80
81	332.0039 0910	434.9825 2439	574.2947 7582	763.3877 9497	81
82	342.9640 2638	451.2069 1274	598.2665 6685	798.7402 4575	82
83	354.2529 4717	467.9991 5469	623.1972 2952	835.6835 5680	83
84	365.8805 3558	485.3791 2510	649.1251 1870	874.2893 1686	84
85	377.8569 5165	503.3673 9448	676.0901 2345	914.6323 3612	85
86	390.1926 6020	521.9852 5329	704.1337 2839	956.7907 9125	86
87	402.8984 4001	541.2547 3715	733.2990 7753	1000.8463 7685	87
88	415.9853 9321	561.1986 5295	763.6310 4063	1046.8844 6381	88
89	429.4649 5500	581.8406 0581	795.1762 8225	1094.9942 6468	89
90	443.3489 0365	603.2050 2701	827.9833 3354	1145.2690 0659	90
91	457.6493 7076	625.3172 0295	862.1026 6688	1197.8061 1189	91
92	472.3788 5189	648.2033 0506	897.5867 7356	1252.7073 8692	92
93	487.5502 1744	671.8904 2073	934.4902 4450	1310.0792 1933	93
94	503.1767 2397	696.4065 8546	972.8698 5428	1370.0327 8420	94
95	519.2720 2569	721.7808 1595	1012.7846 4845	1432.6842 5949	95
96	535.8501 8645	748.0431 4451	1054.2960 3439	1498.1550 5117	96
97	552.9256 9205	775.2246 5457	1097.4678 7577	1566.5720 2847	97
98	570.5134 6281	803.3575 1748	1142.3665 9080	1638.0677 6976	98
99	588.6288 6669	832.4750 3059	1189.0612 5443	1712.7808 1939	99
100	607.2877 3270	862.6116 5666	1237.6237 0461	1790.8559 5627	100

Amount of 1 per Period at Compound Interest

$$s_{\overline{n}|i} = [(1+i)^n - 1]/i$$

n	5%	5½%	6%	6½%	n
1	1.0000 0000	1.0000 0000	1.0000 0000	1.0000 0000	1
2	2.0500 0000	2.0550 0000	2.0600 0000	2.0650 0000	2
3	3.1525 0000	3.1680 2500	3.1836 0000	3.1992 2500	3
4	4.3101 2500	4.3422 6638	4.3746 1600	4.4071 7463	4
5	5.5256 3125	5.5810 9103	5.6370 9296	5.6936 4098	5
6	6.8019 1281	6.8880 5103	6.9753 1854	7.0637 2764	6
7	8.1420 0845	8.2668 9384	8.3938 3765	8.5228 6994	7
8	9.5491 0888	9.7215 7300	9.8974 6791	10.0768 5648	8
9	11.0265 6432	11.2562 5951	11.4913 1598	11.7318 5215	9
10	12.5778 9254	12.8753 5379	13.1807 9494	13.4944 2254	10
11	14.2067 8716	14.5834 9825	14.9716 4264	15.3715 6001	11
12	15.9171 2652	16.3855 9065	16.8699 4120	17.3707 1141	12
13	17.7129 8285	18.2867 9814	18.8821 3767	19.4998 0765	13
14	19.5986 3199	20.2925 7203	21.0150 6593	21.7672 9515	14
15	21.5785 6359	22.4086 6350	23.2759 6988	24.1821 6933	15
16	23.6574 9177	24.6411 3999	25.6725 2808	26.7540 1034	16
17	25.8403 6636	26.9964 0269	28.2128 7976	29.4930 2101	17
18	28.1323 8467	29.4812 0483	30.9056 5255	32.4100 6738	18
19	30.5390 0391	32.1026 7110	33.7599 9170	35.5167 2176	19
20	33.0659 5410	34.8683 1801	36.7855 9120	38.8253 0867	20
21	35.7192 5181	37.7860 7550	39.9927 2668	42.3489 5373	21
22	38.5052 1440	40.8643 0965	43.3922 9028	46.1016 3573	22
23	41.4304 7512	44.1118 4669	46.9958 2769	50.0982 4205	23
24	44.5019 9887	47.5379 9825	50.8155 7735	54.3546 2778	24
25	47.7270 9882	51.1525 8816	54.8645 1200	58.8876 7859	25
26	51.1134 5376	54.9659 8051	59.1563 8272	63.7153 7769	26
27	54.6691 2645	58.9891 0943	63.7057 6568	68.8568 7725	27
28	58.4025 8277	63.2335 1045	68.5281 1162	74.3325 7427	28
29	62.3227 1191	67.7113 5353	73.6397 9832	80.1641 9159	29
30	66.4388 4750	72.4354 7797	79.0581 8622	86.3748 6405	30
31	70.7607 8988	77.4194 2926	84.8016 7739	92.9892 3021	31
32	75.2988 2937	82.6774 9787	90.8897 7803	100.0335 3017	32
33	80.0637 7084	88.2247 6025	97.3431 6471	107.5357 0963	33
34	85.0669 5938	94.0771 2207	104.1837 5460	115.5255 3076	34
35	90.3203 0735	100.2513 6378	111.4347 7987	124.0346 9026	35
36	95.8363 2272	106.7651 8879	119.1208 6666	133.0969 4513	36
37	101.6281 3886	113.6372 7417	127.2681 1866	142.7482 4656	37
38	107.7095 4580	120.8873 2425	135.9042 0578	153.0268 8259	38
39	114.0950 2309	128.5361 2708	145.0584 5813	163.9736 2995	39
40	120.7997 7424	136.6056 1407	154.7619 6562	175.6319 1590	40
41	127.8397 6295	145.1189 2285	165.0476 8356	188.0479 9044	41
42	135.2317 5110	154.1004 6360	175.9505 4457	201.2711 0981	42
43	142.9933 3866	163.5759 8910	187.5075 7724	215.3537 3195	43
44	151.1430 0559	173.5726 6850	199.7580 3188	230.3517 2453	44
45	159.7001 5587	184.1191 6527	212.7435 1379	246.3245 8662	45
46	168.6851 6366	195.2457 1936	226.5081 2462	263.3356 8475	46
47	178.1194 2185	206.9842 3392	241.0986 1210	281.4525 0426	47
48	188.0253 9294	219.3683 6679	256.5645 2882	300.7469 1704	48
49	198.4266 6259	232.4336 2696	272.9584 0055	321.2954 6665	49
50	209.3479 9572	246.2174 7645	290.3359 0458	343.1796 7198	50

Amount of 1 per Period at Compound Interest

$$s_{\overline{n}|i} = [(1+i)^n - 1]/i$$

n	5%	5½%	6%	6½%	n
51	220.8153 9550	260.7594 3765	308.7560 5886	366.4863 5066	51
52	232.8561 6528	276.1012 0672	328.2814 2239	391.3079 6345	52
53	245.4989 7354	292.2867 7309	348.9783 0773	417.7429 8108	53
54	258.7739 2222	309.3625 4561	370.9170 0620	445.8962 7485	54
55	272.7126 1833	327.3774 8562	394.1720 2657	475.8795 3271	55
56	287.3482 4924	346.3832 4733	418.8223 4816	507.8117 0234	56
57	302.7156 6171	366.4343 2593	444.9516 8905	541.8194 6299	57
58	318.8514 4479	387.5882 1386	472.6487 9040	578.0377 2808	58
59	335.7940 1703	409.9055 6562	502.0077 1782	616.6101 8041	59
60	353.5837 1788	433.4503 7173	533.1281 8089	657.6898 4214	60
61	372.2629 0378	458.2901 4217	566.1158 7174	701.4396 8187	61
62	391.8760 4897	484.4960 9999	601.0828 2405	748.0332 6120	62
63	412.4698 5141	512.1433 8549	638.1477 9349	797.6554 2317	63
64	434.0933 4398	541.3112 7170	677.4366 6110	850.5030 2568	64
65	456.7980 1118	572.0833 9164	719.0828 6076	906.7857 2235	65
66	480.6379 1174	604.5479 7818	763.2278 3241	966.7267 9430	66
67	505.6698 0733	638.7981 1698	810.0215 0236	1030.5640 3593	67
68	531.9532 9770	674.9320 1341	859.6227 9250	1098.5506 9827	68
69	559.5509 6258	713.0532 7415	912.2001 6005	1170.9564 9365	69
70	588.5285 1071	753.2712 0423	967.9321 6965	1248.0686 6574	70
71	618.9549 3625	795.7011 2046	1027.0080 9983	1330.1931 2901	71
72	650.9026 8306	840.4646 8209	1089.6285 8582	1417.6556 8240	72
73	684.4478 1721	887.6902 3960	1156.0063 0097	1510.8033 0176	73
74	719.6702 0807	937.5132 0278	1226.3666 7903	1610.0055 1637	74
75	756.6537 1848	990.0764 2893	1300.9486 7977	1715.6558 7493	75
76	795.4864 0440	1045.5306 3252	1380.0056 0055	1828.1735 0681	76
77	836.2607 2462	1104.0348 1731	1463.8059 3659	1948.0047 8475	77
78	879.0737 6085	1165.7567 3226	1552.6342 9278	2075.6250 9576	78
79	924.0274 4889	1230.8733 5254	1646.7923 5035	2211.5407 2698	79
80	971.2288 2134	1299.5713 8693	1746.5998 9137	2356.2908 7423	80
81	1020.7902 6240	1372.0478 1321	1852.3958 8485	2510.4497 8106	81
82	1072.8297 7552	1448.5104 4294	1964.5396 3794	2674.6290 1683	82
83	1127.4712 6430	1529.1785 1730	2083.4120 1622	2849.4799 0292	83
84	1184.8448 2752	1614.2833 3575	2209.4167 3719	3035.6960 9661	84
85	1245.0870 6889	1704.0689 1921	2342.9817 4142	3234.0163 4289	85
86	1308.3414 2234	1798.7927 0977	2484.5606 4591	3445.2274 0518	86
87	1374.7584 9345	1898.7263 0881	2634.6342 8466	3670.1671 8652	87
88	1444.4964 1812	2004.1562 5579	2793.7123 4174	3909.7280 5364	88
89	1517.7212 3903	2115.3848 4986	2962.3350 8225	4164.8603 7713	89
90	1594.6073 0098	2232.7310 1660	3141.0751 8718	4436.5763 0164	90
91	1675.3376 6603	2356.5312 2252	3330.5396 9841	4725.9537 6125	91
92	1760.1045 4933	2487.1404 3976	3531.3720 8032	5034.1407 5573	92
93	1849.1097 7680	2624.9331 6394	3744.2544 0514	5362.3599 0485	93
94	1942.5652 6564	2770.3044 8796	3969.9096 6944	5711.9132 9867	94
95	2040.6935 2892	2923.6712 3480	4209.1042 4961	6084.1876 6308	95
96	2143.7282 0537	3085.4731 5271	4462.6505 0459	6480.6598 6118	96
97	2251.9146 1564	3256.1741 7611	4731.4095 3486	6902.9027 5216	97
98	2365.5103 4642	3436.2637 5580	5016.2941 0696	7352.5914 3105	98
99	2484.7858 6374	3626.2582 6237	5318.2717 5337	7831.5098 7406	99
100	2610.0251 5693	3826.7024 6680	5638.3680 5857	8341.5580 1588	100

Amount of 1 per Period at Compound Interest

$$s_{\overline{n}|i} = [(1+i)^n - 1]/i$$

n	7%	7½%	8%	8½%	n
1	1.0000 0000	1.0000 0000	1.0000 0000	1.0000 0000	1
2	2.0700 0000	2.0750 0000	2.0800 0000	2.0850 0000	2
3	3.2149 0000	3.2306 2500	3.2464 0000	3.2622 2500	3
4	4.4399 4300	4.4729 2188	4.5061 1200	4.5395 1413	4
5	5.7507 3901	5.8083 9102	5.8666 0096	5.9253 7283	5
6	7.1532 9074	7.2440 2034	7.3359 2904	7.4290 2952	6
7	8.6540 2109	8.7873 2187	8.9228 0336	9.0604 9702	7
8	10.2598 0257	10.4463 7101	10.6366 2763	10.8306 3927	8
9	11.9779 8875	12.2298 4883	12.4875 5784	12.7512 4361	9
10	13.8164 4796	14.1470 8750	14.4865 6247	14.8350 9932	10
11	15.7835 9932	16.2081 1906	16.6454 8746	17.0960 8276	11
12	17.8884 5127	18.4237 2799	18.9771 2646	19.5492 4979	12
13	20.1406 4286	20.8055 0759	21.4952 9658	22.2109 3603	13
14	22.5504 8786	23.3659 2066	24.2149 2030	25.0988 6559	14
15	25.1290 2201	26.1183 6470	27.1521 1393	28.2322 6916	15
16	27.8880 5355	29.0772 4206	30.3242 8304	31.6320 1204	16
17	30.8402 1730	32.2580 3521	33.7502 2569	35.3207 3306	17
18	33.9990 3251	35.6773 8785	37.4502 4374	39.3229 9538	18
19	37.3789 6479	39.3531 9194	41.4462 6324	43.6654 4998	19
20	40.9954 9232	43.3046 8134	45.7619 6430	48.3770 1323	20
21	44.8651 7678	47.5525 3244	50.4229 2144	53.4890 5936	21
22	49.0057 3916	52.1189 7237	55.4567 5516	59.0356 2940	22
23	53.4361 4090	57.0278 9530	60.8932 9557	65.0536 5790	23
24	58.1766 7076	62.3049 8744	66.7647 5922	71.5832 1882	24
25	63.2490 3772	67.9778 6150	73.1059 3995	78.6677 9242	25
26	68.6764 7036	74.0762 0112	79.9544 1515	86.3545 5478	26
27	74.4838 2328	80.6319 1620	87.3507 6836	94.6946 9193	27
28	80.6976 9091	87.6793 0991	95.3388 2983	103.7437 4075	28
29	87.3465 2927	95.2552 5816	103.9659 3622	113.5619 5871	29
30	94.4607 8632	103.3994 0252	113.2832 1111	124.2147 2520	30
31	102.0730 4137	112.1543 5771	123.3458 6800	135.7729 7684	31
32	110.2181 5426	121.5659 3454	134.2135 3744	148.3136 7987	32
33	118.9334 2506	131.6833 7963	145.9506 2044	161.9203 4266	33
34	128.2587 6481	142.5596 3310	158.6266 7007	176.6835 7179	34
35	138.2368 7835	154.2516 0558	172.3168 0368	192.7016 7539	35
36	148.9134 5984	166.8204 7600	187.1021 4797	210.0813 1780	36
37	160.3374 0202	180.3320 1170	203.0703 1981	228.9382 2981	37
38	172.5610 2017	194.8569 1258	220.3159 4540	249.3979 7935	38
39	185.6402 9158	210.4711 8102	238.9412 2103	271.5968 0759	39
40	199.6351 1199	227.2565 1960	259.0565 1871	295.6825 3624	40
41	214.6095 6983	245.3007 5857	280.7810 4021	321.8155 5182	41
42	230.6322 3972	264.6983 1546	304.2435 2342	350.1698 7372	42
43	247.7764 9650	285.5506 8912	329.5830 0530	380.9343 1299	43
44	266.1208 5125	307.9669 9080	356.9496 4572	414.3137 2959	44
45	285.7493 1084	332.0645 1511	386.5056 1738	450.5303 9661	45
46	306.7517 6260	357.9693 5375	418.4260 6677	489.8254 8032	46
47	329.2243 8598	385.8170 5528	452.9001 5211	532.4606 4615	47
48	353.2700 9300	415.7533 3442	490.1321 6428	578.7198 0107	48
49	378.9989 9951	447.9348 3451	530.3427 3742	628.9109 8416	49
50	406.5289 2947	482.5299 4709	573.7701 5642	683.3684 1782	50

$$s_{\overline{n}|i} = [(1+i)^n - 1]/i$$

n	7%	7½%	8%	8½%	n
51	435.9859 5454	519.7196 9313	620.6717 6893	742.4547 3333	51
52	467.5049 7135	559.6986 7011	671.3255 1044	806.5633 8566	52
53	501.2303 1935	602.6760 7037	726.0315 5128	876.1212 7345	53
54	537.3164 4170	648.8767 7565	785.1140 7538	951.5915 8169	54
55	575.9285 9262	698.5425 3382	848.9232 0141	1033.4768 6613	55
56	617.2435 9410	751.9332 2386	917.8370 5752	1122.3223 9975	56
57	661.4506 4569	809.3282 1564	992.2640 2213	1218.7198 0373	57
58	708.7521 9089	871.0278 3182	1072.6451 4390	1323.3109 8705	58
59	759.3648 4425	937.3549 1920	1159.4567 5541	1436.7924 2095	59
60	813.5203 8335	1008.6565 3814	1253.2132 9584	1559.9197 7673	60
61	871.4668 1019	1085.3057 7851	1354.4703 5951	1693.5129 5775	61
62	933.4694 8690	1167.7037 1189	1463.8279 8827	1838.4615 5916	62
63	999.8123 5098	1256.2814 9029	1581.9342 2733	1995.7307 9169	63
64	1070.7992 1555	1351.5026 0206	1709.4889 6552	2166.3679 0898	64
65	1146.7551 6064	1453.8652 9721	1847.2480 8276	2351.5091 8125	65
66	1228.0280 2188	1563.9051 9450	1996.0279 2938	2552.3874 6165	66
67	1314.9899 8341	1682.1980 8409	2156.7101 6373	2770.3403 9589	67
68	1408.0392 8225	1809.3629 4040	2330.2469 7683	3006.8193 2954	68
69	1507.6020 3201	1946.0651 6093	2517.6667 3497	3263.3989 7255	69
70	1614.1341 7425	2093.0200 4800	2720.0800 7377	3541.7878 8522	70
71	1728.1235 6645	2250.9965 5160	2938.6864 7967	3843.8398 5546	71
72	1850.0922 1610	2420.8212 9296	3174.7813 9805	4171.5662 4318	72
73	1980.5986 7123	2603.3828 8994	3429.7639 0989	4527.1493 7385	73
74	2120.2405 7821	2799.6366 0668	3705.1450 2268	4912.9570 7063	74
75	2269.6574 1869	3010.6093 5218	4002.5566 2449	5331.5584 2163	75
76	2429.5334 3800	3237.4050 5360	4323.7611 5445	5785.7408 8747	76
77	2600.6007 7866	3481.2104 3262	4670.6620 4681	6278.5288 6290	77
78	2783.6428 3316	3743.3012 1506	5045.3150 1056	6813.2038 1625	78
79	2979.4978 3148	4025.0488 0619	5449.9402 1140	7393.3261 4063	79
80	3189.0626 7969	4327.9274 6666	5886.9354 2831	8022.7588 6259	80
81	3413.2970 6727	4653.5220 2666	6358.8902 6258	8705.6933 6591	81
82	3653.2278 6198	5003.5361 7866	6868.6014 8358	9446.6773 0201	82
83	3909.9538 1231	5379.8013 9206	7419.0896 0227	10250.6448 7268	83
84	4184.6505 7918	5784.2864 9646	8013.6167 7045	11122.9496 8686	84
85	4478.5761 1972	6219.1079 8369	8655.7061 1209	12069.4004 1024	85
86	4793.0764 4810	6686.5410 8247	9349.1626 0105	13096.2994 4511	86
87	5129.5917 9946	7189.0316 6366	10098.0956 0914	14210.4848 9794	87
88	5489.6632 2543	7729.2090 3843	10906.9432 5787	15419.3761 1427	88
89	5874.9396 5121	8309.8997 1631	11780.4987 1850	16731.0230 8398	89
90	6287.1854 2679	8934.1421 9504	12723.9386 1598	18154.1600 4612	90
91	6728.2884 0667	9605.2028 5966	13742.8537 0526	19698.2636 5004	91
92	7200.2685 9513	10326.5930 7414	14843.2820 0168	21373.6160 6029	92
93	7705.2873 9679	11102.0875 5470	16031.7445 6181	23191.3734 2542	93
94	8245.6575 1457	11935.7441 2130	17315.2841 2676	25163.6401 6658	94
95	8823.8535 4059	12831.9249 3040	18701.5068 5690	27303.5495 8074	95
96	9442.5232 8843	13795.3193 0018	20198.6274 0545	29625.3512 9510	96
97	10104.4999 1862	14830.9682 4769	21815.5175 9788	32144.5061 5518	97
98	10812.8149 1292	15944.2908 6627	23561.7590 0572	34877.7891 7837	98
99	11570.7119 5683	17141.1126 8124	25447.6997 2617	37843.4012 5853	99
100	12381.6617 9381	18427.6961 3233	27484.5157 0427	41061.0903 6551	100

$i = \frac{1}{4}\%$

n	$(1+i)^n$	$(1+i)^{-n}$	$s_{\overline{n}\rceil i}$	$a_{\overline{n}\rceil i}$	n
201	1.6518 1209	0.6053 9574	260.7248 3485	157.8417 0420	201
202	1.6559 4162	0.6038 8602	262.3766 4694	158.4455 9022	202
203	1.6600 8147	0.6023 8007	264.0325 8855	159.0479 7030	203
204	1.6642 3168	0.6008 7788	265.6926 7003	159.6488 4818	204
205	1.6683 9225	0.5993 7943	267.3569 0170	160.2482 2761	205
206	1.6725 6323	0.5978 8472	269.0252 9396	160.8461 1233	206
207	1.6767 4464	0.5963 9373	270.6978 5719	161.4425 0606	207
208	1.6809 3650	0.5949 0647	272.3746 0183	162.0374 1253	208
209	1.6851 3885	0.5934 2291	274.0555 3834	162.6308 3544	209
210	1.6893 5169	0.5919 4305	275.7406 7718	163.2227 7850	210
211	1.6935 7507	0.5904 6689	277.4300 2888	163.8132 4538	211
212	1.6978 0901	0.5889 9440	279.1236 0395	164.4022 3978	212
213	1.7020 5353	0.5875 2559	280.8214 1296	164.9897 6537	213
214	1.7063 0867	0.5860 6044	282.5234 6649	165.5758 2581	214
215	1.7105 7444	0.5845 9894	284.2297 7516	166.1604 2474	215
216	1.7148 5087	0.5831 4109	285.9403 4960	166.7435 6583	216
217	1.7191 3800	0.5816 8687	287.6552 0047	167.3252 5270	217
218	1.7234 3585	0.5802 3628	289.3743 3847	167.9054 8898	218
219	1.7277 4444	0.5787 8930	291.0977 7432	168.4842 7828	219
220	1.7320 6380	0.5773 4594	292.8255 1875	169.0616 2422	220
221	1.7363 9396	0.5759 0617	294.5575 8255	169.6375 3039	221
222	1.7407 3494	0.5744 7000	296.2939 7651	170.2120 0039	222
223	1.7450 8678	0.5730 3741	298.0347 1145	170.7850 3780	223
224	1.7494 4950	0.5716 0838	299.7797 9823	171.3566 4618	224
225	1.7538 2312	0.5701 8293	301.5292 4772	171.9268 2911	225
226	1.7582 0768	0.5687 6102	303.2830 7084	172.4955 9013	226
227	1.7626 0320	0.5673 4267	305.0412 7852	173.0629 3280	227
228	1.7670 0970	0.5659 2785	306.8038 8171	173.6288 6065	228
229	1.7714 2723	0.5645 1656	308.5708 9142	174.1933 7721	229
230	1.7758 5580	0.5631 0879	310.3423 1865	174.7564 8599	230
231	1.7802 9544	0.5617 0452	312.1181 7444	175.3181 9052	231
232	1.7847 4617	0.5603 0376	313.8984 6988	175.8784 9428	232
233	1.7892 0804	0.5589 0650	315.6832 1605	176.4374 0078	233
234	1.7936 8106	0.5575 1272	317.4724 2409	176.9949 1350	234
235	1.7981 6526	0.5561 2241	319.2661 0515	177.5510 3591	235
236	1.8026 6068	0.5547 3557	321.0642 7042	178.1057 7148	236
237	1.8071 6733	0.5533 5219	322.8669 3109	178.6591 2367	237
238	1.8116 8525	0.5519 7226	324.6740 9842	179.2110 9593	238
239	1.8162 1446	0.5505 9577	326.4857 8367	179.7616 9170	239
240	1.8207 5500	0.5492 2271	328.3019 9813	180.3109 1441	240
241	1.8253 0688	0.5478 5308	330.1227 5312	180.8587 6749	241
242	1.8298 7015	0.5464 8686	331.9480 6000	181.4052 5436	242
243	1.8344 4483	0.5451 2405	333.7779 3015	181.9503 7841	243
244	1.8390 3094	0.5437 6464	335.6123 7498	182.4941 4305	244
245	1.8436 2851	0.5424 0862	337.4514 0592	183.0365 5168	245
246	1.8482 3759	0.5410 5598	339.2950 3443	183.5776 0766	246
247	1.8528 5818	0.5397 0671	341.1432 7202	184.1173 1437	247
248	1.8574 9033	0.5383 6081	342.9961 3020	184.6556 7518	248
249	1.8621 3405	0.5370 1827	344.8536 2052	185.1926 9345	249
250	1.8667 8939	0.5356 7907	346.7157 5457	185.7283 7252	250

$i = \frac{7}{24}\%$

n	$(1+i)^n$	$(1+i)^{-n}$	$s_{\overline{n}\mid i}$	$a_{\overline{n}\mid i}$	n
201	1.7957 0322	0.5568 8490	272.8125 3234	151.9251 7862	201
202	1.8009 4069	0.5552 6537	274.6082 3556	152.4804 4399	202
203	1.8061 9343	0.5536 5056	276.4091 7624	153.0340 9455	203
204	1.8114 6149	0.5520 4044	278.2153 6967	153.5861 3499	204
205	1.8167 4492	0.5504 3500	280.0268 3117	154.1365 6999	205
206	1.8220 4376	0.5488 3424	281.8435 7609	154.6854 0423	206
207	1.8273 5806	0.5472 3813	283.6656 1986	155.2326 4235	207
208	1.8326 8785	0.5456 4666	285.4929 7791	155.7782 8901	208
209	1.8380 3319	0.5440 5982	287.3256 6577	156.3223 4883	209
210	1.8433 9412	0.5424 7759	289.1636 9896	156.8648 2642	210
211	1.8487 7069	0.5408 9996	291.0070 9308	157.4057 2638	211
212	1.8541 6294	0.5393 2693	292.8558 6377	157.9450 5331	212
213	1.8595 7091	0.5377 5847	294.7100 2670	158.4828 1177	213
214	1.8649 9466	0.5361 9456	296.5695 9761	159.0190 0634	214
215	1.8704 3423	0.5346 3521	298.4345 9227	159.5536 4155	215
216	1.8758 8966	0.5330 8039	300.3050 2650	160.0867 2195	216
217	1.8813 6101	0.5315 3010	302.1809 1616	160.6182 5204	217
218	1.8868 4831	0.5299 8431	304.1622 7717	161.1482 3635	218
219	1.8923 5162	0.5284 4302	305.9491 2548	161.6766 7937	219
220	1.8978 7097	0.5269 0621	307.8414 7709	162.2035 8558	220
221	1.9034 0643	0.5253 7387	309.7393 4807	162.7289 5945	221
222	1.9089 5803	0.5238 4598	311.6427 5450	163.2528 0543	222
223	1.9145 2583	0.5223 2254	313.5517 1253	163.7751 2798	223
224	1.9201 0986	0.5208 0353	315.4662 3836	164.2959 3151	224
225	1.9257 1018	0.5192 8894	317.3863 4822	164.8152 2045	225
226	1.9313 2684	0.5177 7875	319.3120 5841	165.3329 9920	226
227	1.9369 5987	0.5162 7296	321.2433 8524	165.8492 7216	227
228	1.9426 0934	0.5147 7154	323.1803 4512	166.3640 4370	228
229	1.9482 7528	0.5132 7449	325.1229 5446	166.8773 1819	229
230	1.9539 5775	0.5117 8179	327.0712 2974	167.3890 9998	230
231	1.9596 5680	0.5102 9344	329.0251 8749	167.8993 9341	231
232	1.9653 7246	0.5088 0941	330.9848 4429	168.4082 0282	232
233	1.9711 0480	0.5073 2970	332.9502 1675	168.9155 3252	233
234	1.9768 5385	0.5058 5429	334.9213 2155	169.4213 8681	234
235	1.9826 1968	0.5043 8317	336.8981 7541	169.9257 6998	235
236	1.9884 0232	0.5029 1633	338.8807 9508	170.4286 8631	236
237	1.9942 0183	0.5014 5376	340.8691 9740	170.9301 4007	237
238	2.0000 1825	0.4999 9544	342.8633 9923	171.4301 3551	238
239	2.0058 5163	0.4985 4136	344.8634 1748	171.9286 7687	239
240	2.0117 0203	0.4970 9151	346.8692 6911	172.4257 6838	240
241	2.0175 6950	0.4956 4588	348.8809 7115	172.9214 1425	241
242	2.0234 5408	0.4942 0445	350.8985 4065	173.4156 1870	242
243	2.0293 5582	0.4927 6721	352.9219 9472	173.9083 8590	243
244	2.0352 7477	0.4913 3415	354.9513 5054	174.3997 2005	244
245	2.0412 1099	0.4899 0526	356.9866 2531	174.8896 2531	245
246	2.0471 6452	0.4884 8052	359.0278 3630	175.3781 0584	246
247	2.0531 3542	0.4870 5993	361.0750 0083	175.8651 6577	247
248	2.0591 2373	0.4856 4347	363.1281 3624	176.3508 0924	248
249	2.0651 2951	0.4842 3113	365.1872 5998	176.8350 4038	249
250	2.0711 5280	0.4828 2290	367.2523 8948	177.3178 6327	250

$i = \frac{1}{3}\%$

n	$(1+i)^n$	$(1+i)^{-n}$	$s_{\overline{n}\mid i}$	$a_{\overline{n}\mid i}$	n
201	1.9520 6114	0.5122 7904	285.6183 4165	146.3162 8896	201
202	1.9585 6801	0.5105 7711	287.5704 0279	146.8268 6607	202
203	1.9650 9657	0.5088 8084	289.5289 7080	147.3357 4691	203
204	1.9716 4689	0.5071 9021	291.4940 6737	147.8429 3712	204
205	1.9782 1905	0.5055 0519	293.4657 1426	148.3484 4232	205
206	1.9848 1311	0.5038 2577	295.4439 3331	148.8522 6809	206
207	1.9914 2915	0.5021 5193	297.4287 4642	149.3544 2002	207
208	1.9980 6725	0.5004 8365	299.4201 7557	149.8549 0368	208
209	2.0047 2748	0.4988 2092	301.4182 4283	150.3537 2459	209
210	2.0114 0990	0.4971 6371	303.4229 7030	150.8508 8830	210
211	2.0181 1460	0.4955 1200	305.4343 8020	151.3464 0030	211
212	2.0248 4165	0.4938 6578	307.4524 9480	151.8402 6608	212
213	2.0315 9112	0.4922 2503	309.4773 3645	152.3324 9111	213
214	2.0383 6309	0.4905 8973	311.5089 2757	152.8230 8084	214
215	2.0451 5764	0.4889 5986	313.5472 9067	153.3120 4070	215
216	2.0519 7483	0.4873 3541	315.5924 4830	153.7993 7612	216
217	2.0588 1474	0.4857 1636	317.6444 2313	154.2850 9247	217
218	2.0656 7746	0.4841 0268	319.7032 3787	154.7691 9516	218
219	2.0725 6305	0.4824 9437	321.7689 1533	155.2516 8953	219
220	2.0794 7159	0.4808 9140	323.8414 7838	155.7325 8092	220
221	2.0864 0317	0.4792 9375	325.9209 4998	156.2118 7467	221
222	2.0933 5784	0.4777 0141	328.0073 5315	156.6895 7609	222
223	2.1003 3570	0.4761 1437	330.1007 1099	157.1656 9045	223
224	2.1073 3682	0.4745 3259	332.2010 4669	157.6402 2304	224
225	2.1143 6128	0.4729 5607	334.3083 8351	158.1131 7911	225
226	2.1214 0915	0.4713 8479	336.4227 4479	158.5845 6390	226
227	2.1284 8051	0.4698 1872	338.5441 5394	159.0543 8262	227
228	2.1355 7545	0.4682 5786	340.6726 3446	159.5226 4049	228
229	2.1426 9403	0.4667 0219	342.8082 0990	159.9893 4268	229
230	2.1498 3635	0.4651 5169	344.9509 0394	160.4544 9436	230
231	2.1570 0247	0.4636 0633	347.1007 4028	160.9181 0070	231
232	2.1641 9248	0.4620 6611	349.2577 4275	161.3801 6681	232
233	2.1714 0645	0.4605 3101	351.4219 3523	161.8406 9781	233
234	2.1786 4447	0.4590 0100	353.5933 4168	162.2996 9882	234
235	2.1859 0662	0.4574 7608	355.7719 8615	162.7571 7490	235
236	2.1931 9298	0.4559 5623	357.9578 9277	163.2131 3113	236
237	2.2005 0362	0.4544 4142	360.1510 8575	163.6675 7256	237
238	2.2078 3863	0.4529 3165	362.3515 8937	164.1205 0421	238
239	2.2151 9809	0.4514 2690	364.5594 2800	164.5719 3110	239
240	2.2225 8209	0.4499 2714	366.7746 2609	165.0218 5824	240
241	2.2299 9069	0.4484 3236	368.9972 0818	165.4702 9061	241
242	2.2374 2400	0.4469 4256	371.2271 9887	165.9172 3316	242
243	2.2448 8208	0.4454 5770	373.4646 2287	166.3626 9086	243
244	2.2523 6502	0.4439 7777	375.7095 0494	166.8066 6863	244
245	2.2598 7290	0.4425 0276	377.9618 6996	167.2491 7139	245
246	2.2674 0581	0.4410 3265	380.2217 4286	167.6902 0405	246
247	2.2749 6383	0.4395 6743	382.4891 4867	168.1297 7148	247
248	2.2825 4704	0.4381 0707	384.7641 1250	168.5678 7855	248
249	2.2901 5553	0.4366 5157	387.0466 5954	169.0045 3011	249
250	2.2977 8938	0.4352 0090	389.3368 1507	169.4397 3101	250

$i = \frac{5}{12}\%$

n	$(1+i)^n$	$(1+i)^{-n}$	$s_{\overline{n}\vert}$	$a_{\overline{n}\vert}$	n
201	2.3065 6646	0.4335 4484	313.5759 5154	135.9492 3725	201
202	2.3161 7716	0.4317 4590	315.8825 1801	136.3809 8315	202
203	2.3258 2790	0.4299 5443	318.1986 9516	136.8109 3758	203
204	2.3355 1885	0.4281 7038	320.5245 2306	137.2391 0796	204
205	2.3452 5017	0.4263 9374	322.8600 4191	137.6655 0170	205
206	2.3550 2205	0.4246 2447	325.2052 9208	138.0901 2618	206
207	2.3648 3464	0.4228 6255	327.5603 1413	138.5129 8872	207
208	2.3746 8812	0.4211 0793	329.9251 4877	138.9340 9665	208
209	2.3845 8265	0.4193 6059	332.2998 3689	139.3534 5725	209
210	2.3945 1841	0.4176 2051	334.6844 1955	139.7710 7776	210
211	2.4044 9557	0.4158 8764	337.0789 3796	140.1869 6540	211
212	2.4145 1431	0.4141 6197	339.4834 3354	140.6011 2737	212
213	2.4245 7478	0.4124 4345	341.8979 4784	141.0135 7083	213
214	2.4346 7718	0.4107 3207	344.3225 2263	141.4243 0290	214
215	2.4448 2167	0.4090 2779	346.7571 9980	141.8333 3069	215
216	2.4550 0842	0.4073 3058	349.2020 2147	142.2406 6127	216
217	2.4652 3762	0.4056 4041	351.6570 2989	142.6463 0167	217
218	2.4755 0945	0.4039 5725	354.1222 6752	143.0502 5893	218
219	2.4858 2407	0.4022 8108	356.5977 7696	143.4525 4001	219
220	2.4961 8167	0.4006 1187	359.0836 0103	143.8531 5188	220
221	2.5065 8243	0.3989 4958	361.5797 8271	144.2521 0146	221
222	2.5170 2652	0.3972 9418	364.0863 6513	144.6493 9564	222
223	2.5275 1413	0.3956 4566	366.6033 9166	145.0450 4130	223
224	2.5380 4544	0.3940 0398	369.1309 0579	145.4390 4528	224
225	2.5486 2063	0.3923 6911	371.6689 5123	145.8314 1439	225
226	2.5592 3988	0.3907 4102	374.2175 7186	146.2221 5541	226
227	2.5699 0338	0.3891 1969	376.7768 1174	146.6112 7509	227
228	2.5806 1131	0.3875 0508	379.3467 1512	146.9987 8018	228
229	2.5913 6386	0.3858 9718	381.9273 2644	147.3846 7735	229
230	2.6021 6121	0.3842 9594	384.5186 9030	147.7689 7330	230
231	2.6130 0355	0.3827 0136	387.1208 5151	148.1516 7465	231
232	2.6238 9106	0.3811 1338	389.7338 5505	148.5327 8804	232
233	2.6348 2394	0.3795 3200	392.3577 4612	148.9123 2004	233
234	2.6458 0238	0.3779 5718	394.9925 7006	149.2902 7722	234
235	2.6568 2655	0.3763 8889	397.6383 7243	149.6666 6611	235
236	2.6678 9666	0.3748 2711	400.2951 9899	150.0414 9322	236
237	2.6790 1290	0.3732 7181	402.9630 9565	150.4147 6503	237
238	2.6901 7545	0.3717 2297	405.6421 0855	150.7864 8800	238
239	2.7013 8452	0.3701 8055	408.3322 8400	151.1566 6855	239
240	2.7126 4029	0.3686 4453	411.0336 6852	151.5253 1307	240
241	2.7239 4295	0.3671 1488	413.7463 0880	151.8924 2796	241
242	2.7352 9272	0.3655 9159	416.4702 5175	152.2580 1954	242
243	2.7466 8977	0.3640 7461	419.2055 4447	152.6220 9415	243
244	2.7581 3431	0.3625 6392	421.9522 3424	152.9846 5808	244
245	2.7696 2654	0.3610 5951	424.7103 6855	153.3457 1759	245
246	2.7811 6665	0.3595 6134	427.4799 9508	153.7052 7892	246
247	2.7927 5484	0.3580 6938	430.2611 6173	154.0633 4831	247
248	2.8043 9132	0.3565 8362	433.0539 1657	154.4199 3192	248
249	2.8160 7628	0.3551 0402	435.8583 0789	154.7750 3594	249
250	2.8278 0993	0.3536 3056	438.6743 8417	155.1286 6650	250

$i = \frac{1}{2}\%$

n	$(1+i)^n$	$(1+i)^{-n}$	$s_{\overline{n}\vert i}$	$a_{\overline{n}\vert i}$	n
201	2.7250 7471	0.3669 6242	345.0149 4171	126.6075 1671	**201**
202	2.7387 0008	0.3651 3673	347.7400 1642	126.9726 5345	**202**
203	2.7523 9358	0.3633 2013	350.4787 1650	127.3359 7358	**203**
204	2.7661 5555	0.3615 1257	353.2311 1008	127.6974 8615	**204**
205	2.7799 8633	0.3597 1400	355.9972 6563	128.0572 0015	**205**
206	2.7938 8626	0.3579 2438	358.7772 5196	128.4151 2452	**206**
207	2.8078 5569	0.3561 4366	361.5711 3822	128.7712 6818	**207**
208	2.8218 9497	0.3543 7180	364.3789 9391	129.1256 3998	**208**
209	2.8360 0444	0.3526 0876	367.2008 8888	129.4782 4874	**209**
210	2.8501 8447	0.3508 5448	370.0368 9333	129.8291 0322	**210**
211	2.8644 3539	0.3491 0894	372.8870 7779	130.1782 1216	**211**
212	2.8787 5757	0.3473 7208	375.7515 1318	130.5255 8424	**212**
213	2.8931 5135	0.3456 4386	378.6302 7075	130.8712 2810	**213**
214	2.9076 1711	0.3439 2424	381.5234 2210	131.2151 5234	**214**
215	2.9221 5520	0.3422 1317	384.4310 3921	131.5573 6551	**215**
216	2.9367 6597	0.3405 1062	387.3531 9441	131.8978 7613	**216**
217	2.9514 4980	0.3388 1654	390.2899 6038	132.2366 9267	**217**
218	2.9662 0705	0.3371 3088	393.2414 1018	132.5738 2355	**218**
219	2.9810 3809	0.3354 5361	396.2076 1723	132.9092 7716	**219**
220	2.9959 4328	0.3337 8469	399.1886 5532	133.2430 6186	**220**
221	3.0109 2299	0.3321 2407	402.1845 9859	133.5751 8593	**221**
222	3.0259 7761	0.3304 7171	405.1955 2159	133.9056 5764	**222**
223	3.0411 0750	0.3288 2757	408.2214 9920	134.2344 8521	**223**
224	3.0563 1303	0.3271 9162	411.2626 0669	134.5616 7683	**224**
225	3.0715 9460	0.3255 6380	414.3189 1973	134.8872 4062	**225**
226	3.0869 5257	0.3239 4408	417.3905 1432	135.2111 8470	**226**
227	3.1023 8733	0.3223 3241	420.4774 6690	135.5335 1712	**227**
228	3.1178 9927	0.3207 2877	423.5798 5423	135.8542 4589	**228**
229	3.1334 8877	0.3191 3310	426.6977 5350	136.1733 7899	**229**
230	3.1491 5621	0.3175 4538	429.8312 4227	136.4909 2437	**230**
231	3.1649 0199	0.3159 6555	432.9803 9848	136.8068 8992	**231**
232	3.1807 2650	0.3143 9358	436.1453 0047	137.1212 8350	**232**
233	3.1966 3013	0.3128 2944	439.3260 2697	137.4341 1294	**233**
234	3.2126 1329	0.3112 7307	442.5226 5711	137.7453 8601	**234**
235	3.2286 7635	0.3097 2445	445.7352 7039	138.0551 1045	**235**
236	3.2448 1973	0.3081 8353	448.9639 4675	138.3632 9398	**236**
237	3.2610 4383	0.3066 5028	452.2087 6648	138.6699 4426	**237**
238	3.2773 4905	0.3051 2466	455.4698 1031	138.9750 6892	**238**
239	3.2937 3580	0.3036 0662	458.7471 5936	139.2786 7554	**239**
240	3.3102 0448	0.3020 9614	462.0408 9516	139.5807 7168	**240**
241	3.3267 5550	0.3005 9318	465.3510 9964	139.8813 6486	**241**
242	3.3433 8928	0.2990 9769	468.6778 5514	140.1804 6255	**242**
243	3.3601 0622	0.2976 0964	472.0212 4441	140.4780 7218	**243**
244	3.3769 0675	0.2961 2899	475.3813 5063	140.7742 0118	**244**
245	3.3937 9129	0.2946 5572	478.7582 5739	141.0688 5689	**245**
246	3.4107 6024	0.2931 8977	482.1520 4867	141.3620 4666	**246**
247	3.4278 1404	0.2917 3111	485.5628 0892	141.6537 7777	**247**
248	3.4449 5311	0.2902 7971	488.9906 2296	141.9440 5748	**248**
249	3.4621 7788	0.2888 3553	492.4355 7608	142.2328 9302	**249**
250	3.4794 8877	0.2873 9854	495.8977 5396	142.5202 9156	**250**

$i = \frac{7}{12}\%$

n	$(1+i)^n$	$(1+i)^{-n}$	$s_{\overline{n}\vert i}$	$a_{\overline{n}\vert i}$	n
201	3.2190 7306	0.3106 4843	380.4125 2516	118.1745 5450	201
202	3.2378 5099	0.3088 4683	383.6315 9822	118.4834 0132	202
203	3.2567 3845	0.3070 5567	386.8694 4921	118.7904 5699	203
204	3.2757 3609	0.3052 7490	390.1261 8766	119.0957 3189	204
205	3.2948 4456	0.3035 0445	393.4019 2376	119.3992 3634	205
206	3.3140 6448	0.3017 4428	396.6967 6831	119.7009 8062	206
207	3.3333 9652	0.2999 9431	400.0108 3280	120.0009 7493	207
208	3.3528 4134	0.2982 5450	403.3442 2932	120.2992 2943	208
209	3.3723 9958	0.2965 2477	406.6970 7066	120.5957 5420	209
210	3.3920 7191	0.2948 0507	410.0694 7024	120.8905 5927	210
211	3.4118 5900	0.2930 9535	413.4615 4215	121.1836 5461	211
212	3.4317 6151	0.2913 9554	416.8734 0114	121.4750 5016	212
213	3.4517 8012	0.2897 0559	420.3051 6265	121.7647 5575	213
214	3.4719 1550	0.2880 2544	423.7569 4276	122.0527 8119	214
215	3.4921 6834	0.2863 5504	427.2288 5826	122.3391 3623	215
216	3.5125 3932	0.2846 9432	430.7210 2660	122.6238 3055	216
217	3.5330 2913	0.2830 4324	434.2335 6593	122.9068 7379	217
218	3.5536 3847	0.2814 0173	437.7665 9506	123.1882 7551	218
219	3.5743 6803	0.2797 6974	441.3202 3353	123.4680 4525	219
220	3.5952 1851	0.2781 4721	444.8946 0156	123.7461 9246	220
221	3.6161 9062	0.2765 3410	448.4898 2007	124.0227 2655	221
222	3.6372 8506	0.2749 3033	452.1060 1069	124.2976 5689	222
223	3.6585 0256	0.2733 3588	455.7432 9575	124.5709 9276	223
224	3.6798 4382	0.2717 5066	459.4017 9831	124.8427 4343	224
225	3.7013 0958	0.2701 7464	463.0816 4213	125.1129 1807	225
226	3.7229 0055	0.2686 0777	466.7829 5171	125.3815 2584	226
227	3.7446 1747	0.2670 4997	470.5058 5226	125.6485 7581	227
228	3.7664 6107	0.2655 0122	474.2504 6973	125.9140 7703	228
229	3.7884 3210	0.2639 6144	478.0169 3081	126.1780 3847	229
230	3.8105 3128	0.2624 3060	481.8053 6290	126.4404 6907	230
231	3.8327 5938	0.2609 0863	485.6158 9419	126.7013 7770	231
232	3.8551 1715	0.2593 9549	489.4486 5357	126.9607 7319	232
233	3.8776 0533	0.2578 9112	493.3037 7071	127.2186 6431	233
234	3.9002 2469	0.2563 9548	497.1813 7604	127.4750 5980	234
235	3.9229 7600	0.2549 0852	501.0816 0074	127.7299 6832	235
236	3.9458 6003	0.2534 3018	505.0045 7674	127.9833 9849	236
237	3.9688 7755	0.2519 6041	508.9504 3677	128.2353 5890	237
238	3.9920 2933	0.2504 9916	512.9193 1432	128.4858 5806	238
239	4.0153 1617	0.2490 4639	516.9113 4365	128.7349 0445	239
240	4.0387 3885	0.2476 0205	520.9266 5983	128.9825 0650	240
241	4.0622 9816	0.2461 6608	524.9653 9867	129.2286 7257	241
242	4.0859 9490	0.2447 3844	529.0276 9683	129.4734 1101	242
243	4.1098 2987	0.2433 1907	533.1136 9173	129.7167 3008	243
244	4.1338 0388	0.2419 0794	537.2235 2160	129.9586 3803	244
245	4.1579 1773	0.2405 0500	541.3573 2548	130.1991 4303	245
246	4.1821 7225	0.2391 1019	545.5152 4321	130.4382 5322	246
247	4.2065 6826	0.2377 2347	549.6974 1546	130.6759 7669	247
248	4.2311 0657	0.2363 4479	553.9039 8372	130.9123 2148	248
249	4.2557 8803	0.2349 7411	558.1350 9029	131.1472 9559	249
250	4.2806 1346	0.2336 1138	562.3908 7832	131.3809 0696	250

VII Annuity Whose Present Value at Compound Interest is 1

$$\frac{1}{a_{\overline{n}|}} = \frac{i}{1-v^n} = \frac{1}{s_{\overline{n}|}} + i$$

n	$\frac{1}{4}\%$	$\frac{7}{24}\%$	$\frac{1}{3}\%$	$\frac{5}{12}\%$	n
1	1.0025 0000	1.0029 1667	1.0033 3333	1.0041 6667	1
2	0.5018 7578	0.5021 8856	0.5025 0139	0.5031 2717	2
3	0.3350 0139	0.3352 7967	0.3355 5802	0.3361 1496	3
4	0.2515 6445	0.2518 2557	0.2520 8680	0.2526 0958	4
5	0.2015 0250	0.2017 5340	0.2020 0444	0.2025 0693	5
6	0.1681 2803	0.1683 7219	0.1686 1650	0.1691 0564	6
7	0.1442 8928	0.1445 2866	0.1447 6824	0.1452 4800	7
8	0.1264 1035	0.1266 4620	0.1268 8228	0.1273 5512	8
9	0.1125 0462	0.1127 3777	0.1129 7118	0.1134 3876	9
10	0.1013 8015	0.1016 1117	0.1018 4248	0.1023 0596	10
11	0.0922 7840	0.0925 0772	0.0927 3736	0.0931 9757	11
12	0.0846 9370	0.0849 2163	0.0851 4990	0.0856 0748	12
13	0.0782 7595	0.0785 0274	0.0787 2989	0.0791 8532	13
14	0.0727 7510	0.0730 0093	0.0732 2716	0.0736 8082	14
15	0.0680 0777	0.0682 3279	0.0684 5825	0.0689 1045	15
16	0.0638 3642	0.0640 6076	0.0642 8557	0.0647 3655	16
17	0.0601 5587	0.0603 7964	0.0606 0389	0.0610 5387	17
18	0.0568 8433	0.0571 0761	0.0573 3140	0.0577 8053	18
19	0.0539 5722	0.0541 8008	0.0544 0348	0.0548 5191	19
20	0.0513 2288	0.0515 4537	0.0517 6844	0.0522 1630	20
21	0.0489 3947	0.0491 6166	0.0493 8445	0.0498 3183	21
22	0.0467.7278	0.0469 9471	0.0472 1726	0.0476 6427	22
23	0.0447 9455	0.0450 1625	0.0452 3861	0.0456 8531	23
24	0.0429 8121	0.0432 0272	0.0434 2492	0.0438 7139	24
25	0.0413 1298	0.0415 3433	0.0417 5640	0.0422 0270	25
26	0.0397 7312	0.0399 9434	0.0402 1630	0.0406 6247	26
27	0.0383 4736	0.0385 6847	0.0387 9035	0.0392 3645	27
28	0.0370 2347	0.0372 4450	0.0374 6632	0.0379 1239	28
29	0.0357 9093	0.0360 1188	0.0362 3367	0.0366 7974	29
30	0.0346 4059	0.0348 6149	0.0350 8325	0.0355 2936	30
31	0.0335 6449	0.0337 8536	0.0340 0712	0.0344 5330	31
32	0.0325 5569	0.0327 7653	0.0329 9830	0.0334 4458	32
33	0.0316 0806	0.0318 2889	0.0320 5067	0.0324 9708	33
34	0.0307 1620	0.0309 3703	0.0311 5885	0.0316 0540	34
35	0.0298 7533	0.0300 9618	0.0303 1803	0.0307 6476	35
36	0.0290 8121	0.0293 0208	0.0295 2399	0.0299 7090	36
37	0.0283 3004	0.0285 5094	0.0287 7291	0.0292 2003	37
38	0.0276 1843	0.0278 3938	0.0280 6141	0.0285 0875	38
39	0.0269 4335	0.0271 6434	0.0273 8644	0.0278 3402	39
40	0.0263 0204	0.0265 2308	0.0267 4527	0.0271 9310	40
41	0.0256 9204	0.0259 1315	0.0261 3543	0.0263 8352	41
42	0.0251 1112	0.0253 3229	0.0255 5466	0.0260 0303	42
43	0.0245 5724	0.0247 7848	0.0250 0095	0.0254 4961	43
44	0.0240 2855	0.0242 4987	0.0244 7246	0.0249 2141	44
45	0.0235 2339	0.0237 4479	0.0239 6749	0.0244 1675	45
46	0.0230 4022	0.0232 6170	0.0234 8451	0.0239 3409	46
47	0.0225 7762	0.0227 9920	0.0230 2213	0.0234 7204	47
48	0.0221 3433	0.0223 5600	0.0225 7905	0.0230 2929	48
49	0.0217 0915	0.0219 3092	0.0221 5410	0.0226 0468	49
50	0.0213 0099	0.0215 2287	0.0217 4618	0.0221 9711	50

$$\frac{1}{a_{\overline{n}|}} = \frac{i}{1-v^n} = \frac{1}{s_{\overline{n}|}} + i$$

n	$\frac{1}{4}\%$	$\frac{7}{24}\%$	$\frac{1}{3}\%$	$\frac{5}{12}\%$	n
51	0.0209 0886	0.0211 3085	0.0213 5429	0.0218 0557	51
52	0.0205 3184	0.0207 5393	0.0209 7751	0.0214 2916	52
53	0.0201 6906	0.0203 9126	0.0206 1499	0.0210 6700	53
54	0.0198 1974	0.0200 4205	0.0202 6592	0.0207 1830	54
55	0.0194 8314	0.0197 0557	0.0199 2958	0.0203 8234	55
56	0.0191 5858	0.0193 8113	0.0196 0529	0.0200 5843	56
57	0.0188 4542	0.0190 6810	0.0192 9241	0.0197 4593	57
58	0.0185 4308	0.0187 6588	0.0189 9035	0.0194 4426	58
59	0.0182 5101	0.0184 7394	0.0186 9856	0.0191 5287	59
60	0.0179 6869	0.0181 9175	0.0184 1652	0.0188 7123	60
61	0.0176 9564	0.0179 1883	0.0181 4377	0.0185 9888	61
62	0.0174 3142	0.0176 5474	0.0178 7984	0.0183 3536	62
63	0.0171 7561	0.0173 9906	0.0176 2432	0.0180 8025	63
64	0.0169 2780	0.0171 5139	0.0173 7681	0.0178 3315	64
65	0.0166 8764	0.0169 1136	0.0171 3695	0.0175 9371	65
66	0.0164 5476	0.0166 7863	0.0169 0438	0.0173 6156	66
67	0.0162 2886	0.0164 5286	0.0166 7878	0.0171 3639	67
68	0.0160 0961	0.0162 3376	0.0164 5985	0.0169 1788	68
69	0.0157 9674	0.0160 2102	0.0162 4729	0.0167 0574	69
70	0.0155 8996	0.0158 1439	0.0160 4083	0.0164 9971	70
71	0.0153 8902	0.0156 1359	0.0158 4021	0.0162 9952	71
72	0.0151 9368	0.0154 1840	0.0156 4518	0.0161 0493	72
73	0.0150 0370	0.0152 2857	0.0154 5553	0.0159 1572	73
74	0.0148 1887	0.0150 4389	0.0152 7103	0.0157 3165	74
75	0.0146 3898	0.0148 6415	0.0150 9147	0.0155 5253	75
76	0.0144 6385	0.0146 8916	0.0149 1666	0.0153 7816	76
77	0.0142 9327	0.0145 1974	0.0147 4641	0.0152 0836	77
78	0.0141 2708	0.0143 5270	0.0145 8056	0.0150 4295	78
79	0.0139 6511	0.0141 9089	0.0144 1892	0.0148 8177	79
80	0.0138 0721	0.0140 3313	0.0142 6135	0.0147 2464	80
81	0.0136 5321	0.0138 7929	0.0141 0770	0.0145 7144	81
82	0.0135 0298	0.0137 2922	0.0139 5781	0.0144 2200	82
83	0.0133 5639	0.0135 8278	0.0138 1156	0.0142 7620	83
84	0.0132 1330	0.0134 3985	0.0136 6881	0.0141 3391	84
85	0.0130 7359	0.0133 0030	0.0135 2944	0.0139 9500	85
86	0.0129 3714	0.0131 6400	0.0133 9333	0.0138 5935	86
87	0.0128 0384	0.0130 3086	0.0132 6038	0.0137 2685	87
88	0.0126 7357	0.0129 0076	0.0131 3046	0.0135 9740	88
89	0.0125 4625	0.0127 7360	0.0130 0349	0.0134 7088	89
90	0.0124 2177	0.0126 4928	0.0128 7936	0.0133 4721	90
91	0.0123 0004	0.0125 2770	0.0127 5797	0.0132 2629	91
92	0.0121 8096	0.0124 0879	0.0126 3925	0.0131 0803	92
93	0.0120 6446	0.0122 9245	0.0125 2310	0.0129 9234	93
94	0.0119 5044	0.0121 7860	0.0124 0944	0.0128 7915	94
95	0.0118 3884	0.0120 6716	0.0122 9819	0.0127 6837	95
96	0.0117 2957	0.0119 5805	0.0121 8928	0.0126 5992	96
97	0.0116 2257	0.0118 5121	0.0120 8263	0.0125 5374	97
98	0.0115 1776	0.0117 4657	0.0119 7818	0.0124 4976	98
99	0.0114 1508	0.0116 4405	0.0118 7585	0.0123 4790	99
100	0.0113 1446	0.0115 4360	0.0117 7559	0.0122 4811	100

$$\frac{1}{a_{\overline{n}|}} = \frac{i}{1-v^n} = \frac{1}{s_{\overline{n}|}} + i$$

n	$\frac{1}{4}\%$	$\frac{7}{24}\%$	$\frac{1}{3}\%$	$\frac{5}{12}\%$	n
101	0.0112 1584	0.0114 4515	0.0116 7734	0.0121 5033	101
102	0.0111 1917	0.0113 4864	0.0115 8103	0.0120 5449	102
103	0.0110 2439	0.0112 5403	0.0114 8660	0.0119 6054	103
104	0.0109 3144	0.0111 6124	0.0113 9401	0.0118 6842	104
105	0.0108 4027	0.0110 7024	0.0113 0320	0.0117 7809	105
106	0.0107 5083	0.0109 8096	0.0112 1413	0.0116 8948	106
107	0.0106 6307	0.0108 9337	0.0111 2673	0.0116 0256	107
108	0.0105 7694	0.0108 0741	0.0110 4097	0.0115 1727	108
109	0.0104 9241	0.0107 2305	0.0109 5680	0.0114 3358	109
110	0.0104 0942	0.0106 4023	0.0108 7417	0.0113 5143	110
111	0.0103 2793	0.0105 5891	0.0107 9306	0.0112 7079	111
112	0.0102 4791	0.0104 7906	0.0107 1340	0.0111 9161	112
113	0.0101 6932	0.0104 0064	0.0106 3518	0.0111 1386	113
114	0.0100 9211	0.0103 2360	0.0105 5834	0.0110 3750	114
115	0.0100 1625	0.0102 4792	0.0104 8285	0.0109 6249	115
116	0.0099 4172	0.0101 7355	0.0104 0868	0.0108 8880	116
117	0.0098 6846	0.0101 0046	0.0103 3579	0.0108 1639	117
118	0.0097 9646	0.0100 2863	0.0102 6416	0.0107 4524	118
119	0.0097 2567	0.0099 5801	0.0101 9374	0.0106 7530	119
120	0.0096 5608	0.0098 8859	0.0101 2451	0.0106 0655	120
121	0.0095 8764	0.0098 2032	0.0100 5645	0.0105 3896	121
122	0.0095 2033	0.0097 5318	0.0099 8951	0.0104 7251	122
123	0.0094 5412	0.0096 8715	0.0099 2367	0.0104 0715	123
124	0.0093 8899	0.0096 2219	0.0098 5892	0.0103 4288	124
125	0.0093 2491	0.0095 5828	0.0097 9521	0.0102 7965	125
126	0.0092 6186	0.0094 9540	0.0097 3253	0.0102 1745	126
127	0.0091 9981	0.0094 3352	0.0096 7085	0.0101 5625	127
128	0.0091 3873	0.0093 7262	0.0096 1015	0.0100 9603	128
129	0.0090 7861	0.0093 1267	0.0095 5040	0.0100 3677	129
130	0.0090 1942	0.0092 5366	0.0094 9159	0.0099 7844	130
131	0.0089 6115	0.0091 9556	0.0094 3368	0.0099 2102	131
132	0.0089 0376	0.0091 3834	0.0093 7667	0.0098 6449	132
133	0.0088 4725	0.0090 8200	0.0093 2053	0.0098 0883	133
134	0.0087 9159	0.0090 2651	0.0092 6524	0.0097 5403	134
135	0.0087 3675	0.0089 7186	0.0092 1079	0.0097 0005	135
136	0.0086 8274	0.0089 1801	0.0091 5715	0.0096 4689	136
137	0.0086 2952	0.0088 6497	0.0091 0430	0.0095 9453	137
138	0.0085 7707	0.0088 1270	0.0090 5223	0.0095 4295	138
139	0.0085 2539	0.0087 6119	0.0090 0093	0.0094 9213	139
140	0.0084 7446	0.0087 1043	0.0089 5037	0.0094 4205	140
141	0.0084 2425	0.0086 6040	0.0089 0054	0.0093 9271	141
142	0.0083 7476	0.0086 1109	0.0088 5143	0.0093 4408	142
143	0.0083 2597	0.0085 6247	0.0088 0301	0.0092 9615	143
144	0.0082 7787	0.0085 1454	0.0087 5528	0.0092 4890	144
145	0.0082 3043	0.0084 6728	0.0087 0822	0.0092 0233	145
146	0.0081 8365	0.0084 2067	0.0086 6182	0.0091 5641	146
147	0.0081 3752	0.0083 7471	0.0086 1607	0.0091 1114	147
148	0.0080 9201	0.0083 2938	0.0085 7094	0.0090 6650	148
149	0.0080 4712	0.0082 8467	0.0085 2643	0.0090 2247	149
150	0.0080 0284	0.0082 4056	0.0084 8252	0.0089 7905	150

$$\frac{1}{a_{\overline{n}|}} = \frac{i}{1-v^n} = \frac{1}{s_{\overline{n}|}} + i$$

n	¼%	7/24%	⅓%	5/12%	n
151	0.0079 5915	0.0081 9705	0.0084 3921	0.0089 3623	151
152	0.0079 1605	0.0081 5412	0.0083 9648	0.0088 9398	152
153	0.0078 7351	0.0081 1176	0.0083 5432	0.0088 5231	153
154	0.0078 3153	0.0080 6995	0.0083 1273	0.0088 1119	154
155	0.0077 9010	0.0080 2870	0.0082 7167	0.0087 7063	155
156	0.0077 4921	0.0079 8798	0.0082 3116	0.0087 3060	156
157	0.0077 0885	0.0079 4780	0.0081 9118	0.0086 9110	157
158	0.0076 6900	0.0079 0813	0.0081 5171	0.0086 5211	158
159	0.0076 2966	0.0078 6896	0.0081 1275	0.0086 1364	159
160	0.0075 9082	0.0078 3030	0.0080 7429	0.0085 7566	160
161	0.0075 5246	0.0077 9212	0.0080 3631	0.0085 3817	161
162	0.0075 1459	0.0077 5442	0.0079 9882	0.0085 0116	162
163	0.0074 7719	0.0077 1720	0.0079 6180	0.0084 6462	163
164	0.0074 4025	0.0076 0844	0.0079 2524	0.0084 2855	164
165	0.0074 0377	0.0076 4413	0.0078 8913	0.0083 9293	165
166	0.0073 6773	0.0076 0826	0.0078 5347	0.0083 5775	166
167	0.0073 3213	0.0075 7284	0.0078 1825	0.0083 2302	167
168	0.0072 9695	0.0075 3784	0.0077 8346	0.0082 8871	168
169	0.0072 6220	0.0075 0327	0.0077 4909	0.0082 5482	169
170	0.0072 2787	0.0074 6911	0.0077 1513	0.0082 2135	170
171	0.0071 9394	0.0074 3536	0.0076 8158	0.0081 8829	171
172	0.0071 6042	0.0074 0201	0.0076 4844	0.0081 5563	172
173	0.0071 2728	0.0073 6905	0.0076 1568	0.0081 2336	173
174	0.0070 9454	0.0073 3648	0.0075 8332	0.0080 9147	174
175	0.0070 6217	0.0073 0429	0.0075 5133	0.0080 5997	175
176	0.0070 3018	0.0072 7248	0.0075 1972	0.0080 2884	176
177	0.0069 9855	0.0072 4103	0.0074 8847	0.0079 9808	177
178	0.0069 6729	0.0072 0994	0.0074 5759	0.0079 6768	178
179	0.0069 3638	0.0071 7921	0.0074 2706	0.0079 3763	179
180	0.0069 0582	0.0071 4883	0.0073 9688	0.0079 0794	180
181	0.0068 7560	0.0071 1879	0.0073 6704	0.0078 7858	181
182	0.0068 4572	0.0070 8908	0.0073 3754	0.0078 4957	182
183	0.0068 1617	0.0070 5971	0.0073 0838	0.0078 2088	183
184	0.0067 8695	0.0070 3067	0.0072 7954	0.0077 9253	184
185	0.0067 5805	0.0070 0195	0.0072 5102	0.0077 6449	185
186	0.0067 2947	0.0069 7354	0.0072 2281	0.0077 3677	186
187	0.0067 0120	0.0069 4545	0.0071 9492	0.0077 0936	187
188	0.0066 7323	0.0069 1766	0.0071 6734	0.0076 8226	188
189	0.0066 4557	0.0068 9017	0.0071 4005	0.0076 5546	189
190	0.0066 1820	0.0068 6298	0.0071 1307	0.0076 2895	190
191	0.0065 9112	0.0068 3608	0.0070 8637	0.0076 0274	191
192	0.0065 6434	0.0068 0947	0.0070 5996	0.0075 7681	192
193	0.0065 3783	0.0067 8314	0.0070 3384	0.0075 5117	193
194	0.0065 1160	0.0067 5708	0.0070 0799	0.0075 2580	194
195	0.0064 8565	0.0067 3131	0.0069 8242	0.0075 0071	195
196	0.0064 5997	0.0067 0581	0.0069 5711	0.0074 7589	196
197	0.0064 3455	0.0066 8057	0.0069 3208	0.0074 5133	197
198	0.0064 0939	0.0066 5559	0.0069 0730	0.0074 2704	198
199	0.0063 8450	0.0066 3087	0.0068 8278	0.0074 0300	199
200	0.0063 5985	0.0066 0640	0.0068 5852	0.0073 7922	200

Annuity Whose Present Value at Compound Interest is 1

$$\frac{1}{a_{\overline{n}|}} = \frac{i}{1-v^n} = \frac{1}{s_{\overline{n}|}} + i$$

n	½%	7/12%	5/8%	2/3%	n
1	1.0050 0000	1.0058 3333	1.0062 5000	1.0066 6667	1
2	0.5037 5312	0.5043 7924	0.5046 9237	0.5050 0554	2
3	0.3366 7221	0.3372 2976	0.3375 0865	0.3377 8762	3
4	0.2531 3279	0.2536 5644	0.2539 1842	0.2541 8051	4
5	0.2030 0997	0.2035 1357	0.2037 6558	0.2040 1772	5
6	0.1695 9546	0.1700 8594	0.1703 3143	0.1705 7709	6
7	0.1457 2854	0.1462 0986	0.1464 5082	0.1466 9198	7
8	0.1278 2886	0.1283 0351	0.1285 4118	0.1287 7907	8
9	0.1139 0736	0.1143 7698	0.1146 1218	0.1148 4763	9
10	0.1027 7057	0.1032 3632	0.1034 6963	0.1037 0321	10
11	0.0936 5903	0.0941 2175	0.0943 5358	0.0945 8572	11
12	0.0860 6643	0.0865 2675	0.0867 5742	0.0869 8843	12
13	0.0796 4224	0.0801 0064	0.0803 3039	0.0805 6052	13
14	0.0741 3609	0.0745 9295	0.0748 2198	0.0750 5141	14
15	0.0693 6436	0.0698 1999	0.0700 4845	0.0702 7734	15
16	0.0651 8937	0.0656 4401	0.0658 7202	0.0661 0049	16
17	0.0615 0579	0.0619 5966	0.0621 8732	0.0624 1546	17
18	0.0582 3173	0.0586 8499	0.0589 1239	0.0591 4030	18
19	0.0553 0253	0.0557 5532	0.0559 8252	0.0562 1027	19
20	0.0526 6645	0.0531 1889	0.0533 4597	0.0535 7362	20
21	0.0502 8163	0.0507 3383	0.0509 6083	0.0511 8843	21
22	0.0481 1380	0.0485 6585	0.0487 9281	0.0490 2041	22
23	0.0461 3465	0.0465 8663	0.0468 1360	0.0470 4123	23
24	0.0443 2061	0.0447 7258	0.0449 9959	0.0452 2729	24
25	0.0426 5186	0.0431 0388	0.0433 3096	0.0435 5876	25
26	0.0411 1163	0.0415 6376	0.0417 9094	0.0420 1886	26
27	0.0396 8565	0.0401 3793	0.0403 6523	0.0405 9331	27
28	0.0383 6167	0.0388 1415	0.0390 4159	0.0392 6983	28
29	0.0371 2914	0.0375 8186	0.0378 0946	0.0380 3789	29
30	0.0359 7892	0.0364 3191	0.0366 5969	0.0368 8832	30
31	0.0349 0304	0.0353 5633	0.0355 8430	0.0358 1316	31
32	0.0338 9453	0.0343 4815	0.0345 7633	0.0348 0542	32
33	0.0329 4727	0.0334 0124	0.0336 2964	0.0338 5898	33
34	0.0320 5586	0.0325 1020	0.0327 3883	0.0329 6843	34
35	0.0312 1550	0.0316 7024	0.0318 9911	0.0321 2898	35
36	0.0304 2194	0.0308 7710	0.0311 0622	0.0313 3637	36
37	0.0296 7139	0.0301 2698	0.0303 5636	0.0305 8680	37
38	0.0289 6045	0.0294 1649	0.0296 4614	0.0298 7687	38
39	0.0282 8607	0.0287 4258	0.0289 7250	0.0292 0354	39
40	0.0276 4552	0.0281 0251	0.0283 3271	0.0285 6406	40
41	0.0270 3631	0.0274 9379	0.0277 2429	0.0279 5595	41
42	0.0264 5622	0.0269 1420	0.0271 4499	0.0273 7697	42
43	0.0259 0320	0.0263 6170	0.0265 9278	0.0268 2509	43
44	0.0253 7541	0.0258 3443	0.0260 6583	0.0262 9847	44
45	0.0248 7117	0.0253 3073	0.0255 6243	0.0257 9541	45
46	0.0243 8894	0.0248 4905	0.0250 8106	0.0253 1439	46
47	0.0239 2733	0.0243 8798	0.0246 2032	0.0248 5399	47
48	0.0234 8503	0.0239 4624	0.0241 7890	0.0244 1292	48
49	0.0230 6087	0.0235 2265	0.0237 5563	0.0239 9001	49
50	0.0226 5376	0.0231 1611	0.0233 4943	0.0235 8416	50

$$\frac{1}{a_{\overline{n}|}} = \frac{i}{1-v^n} = \frac{1}{s_{\overline{n}|}} + i$$

n	$\frac{1}{2}\%$	$\frac{7}{12}\%$	$\frac{5}{8}\%$	$\frac{2}{3}\%$	n
51	0.0222 6269	0.0227 2563	0.0229 5928	0.0231 9437	51
52	0.0218 8675	0.0223 5027	0.0225 8425	0.0228 1971	52
53	0.0215 2507	0.0219 8919	0.0222 2350	0.0224 5932	53
54	0.0211 7686	0.0216 4157	0.0218 7623	0.0221 1242	54
55	0.0208 4139	0.0213 0671	0.0215 4171	0.0217 7827	55
56	0.0205 1797	0.0209 8390	0.0212 1925	0.0214 5618	56
57	0.0202 0598	0.0206 7251	0.0209 0821	0.0211 4552	57
58	0.0199 0481	0.0203 7196	0.0206 0801	0.0208 4569	58
59	0.0196 1392	0.0200 8170	0.0203 1809	0.0205 5616	59
60	0.0193 3280	0.0198 0120	0.0200 3795	0.0202 7639	60
61	0.0190 6096	0.0195 2999	0.0197 6709	0.0200 0592	61
62	0.0187 9796	0.0192 6762	0.0195 0508	0.0197 4429	62
63	0.0185 4337	0.0190 1366	0.0192 5148	0.0194 9108	63
64	0.0182 9681	0.0187 6773	0.0190 0591	0.0192 4590	64
65	0.0180 5789	0.0185 2946	0.0187 6800	0.0190 0837	65
66	0.0178 2627	0.0182 9848	0.0185 3739	0.0187 7815	66
67	0.0176 0163	0.0180 7449	0.0183 1376	0.0185 5491	67
68	0.0173 8366	0.0178 5716	0.0180 9680	0.0183 3835	68
69	0.0171 7206	0.0176 4622	0.0178 8622	0.0181 2816	69
70	0.0169 6657	0.0174 4138	0.0176 8175	0.0179 2409	70
71	0.0167 6693	0.0172 4239	0.0174 8313	0.0177 2586	71
72	0.0165 7289	0.0170 4901	0.0172 9011	0.0175 3324	72
73	0.0163 8422	0.0168 6100	0.0171 0247	0.0173 4600	73
74	0.0162 0070	0.0166 7814	0.0169 1999	0.0171 6391	74
75	0.0160 2214	0.0165 0024	0.0167 4246	0.0169 8678	75
76	0.0158 4832	0.0163 2709	0.0165 6968	0.0168 1440	76
77	0.0156 7908	0.0161 5851	0.0164 0147	0.0166 4659	77
78	0.0155 1423	0.0159 9432	0.0162 3766	0.0164 8318	78
79	0.0153 5360	0.0158 3436	0.0160 7808	0.0163 2400	79
80	0.0151 9704	0.0156 7847	0.0159 2256	0.0161 6889	80
81	0.0150 4439	0.0155 2650	0.0157 7096	0.0160 1769	81
82	0.0148 9552	0.0153 7830	0.0156 2314	0.0158 7027	82
83	0.0147 5028	0.0152 3373	0.0154 7895	0.0157 2649	83
84	0.0146 0855	0.0150 9268	0.0153 3828	0.0155 8621	84
85	0.0144 7021	0.0149 5501	0.0152 0098	0.0154 4933	85
86	0.0143 3513	0.0148 2060	0.0150 6696	0.0153 1570	86
87	0.0142 0320	0.0146 8935	0.0149 3608	0.0151 8524	87
88	0.0140 7431	0.0145 6115	0.0148 0826	0.0150 5781	88
89	0.0139 4837	0.0144 3588	0.0146 8337	0.0149 3334	89
90	0.0138 2527	0.0143 1347	0.0145 6134	0.0148 1170	90
91	0.0137 0493	0.0141 9380	0.0144 4205	0.0146 9282	91
92	0.0135 8724	0.0140 7679	0.0143 2542	0.0145 7660	92
93	0.0134 7213	0.0139 6236	0.0142 1137	0.0144 6296	93
94	0.0133 5950	0.0138 5042	0.0140 9982	0.0143 5181	94
95	0.0132 4930	0.0137 4090	0.0139 9067	0.0142 4308	95
96	0.0131 4143	0.0136 3372	0.0138 8387	0.0141 3668	96
97	0.0130 3583	0.0135 2880	0.0137 7933	0.0140 3255	97
98	0.0129 3242	0.0134 2608	0.0136 7700	0.0139 3062	98
99	0.0128 3115	0.0133 2549	0.0135 7679	0.0138 3082	99
100	0.0127 3194	0.0132 2696	0.0134 7865	0.0137 3308	100

$$\frac{1}{a_{\overline{n}|}} = \frac{i}{1-v^n} = \frac{1}{s_{\overline{n}|}} + i$$

n	½%	7/12%	⅝%	⅔%	n
101	0.0126 3473	0.0131 3045	0.0133 8251	0.0136 3735	101
102	0.0125 3947	0.0130 3587	0.0132 8832	0.0135 4357	102
103	0.0124 4611	0.0129 4319	0.0131 9602	0.0134 5168	103
104	0.0123 5457	0.0128 5234	0.0131 0555	0.0133 6162	104
105	0.0122 6481	0.0127 6238	0.0130 1687	0.0132 7334	105
106	0.0121 7679	0.0126 7594	0.0129 2992	0.0131 8680	106
107	0.0120 9045	0.0125 9029	0.0128 4465	0.0131 0194	107
108	0.0120 0575	0.0125 0628	0.0127 6102	0.0130 1871	108
109	0.0119 2264	0.0124 2385	0.0126 7897	0.0129 3708	109
110	0.0118 4107	0.0123 4298	0.0125 9848	0.0128 5700	110
111	0.0117 6102	0.0122 6361	0.0125 1950	0.0127 7842	111
112	0.0116 8242	0.0121 8571	0.0124 4198	0.0127 0131	112
113	0.0116 0526	0.0121 0923	0.0123 6588	0.0126 2562	113
114	0.0115 2948	0.0120 3414	0.0122 9118	0.0125 5132	114
115	0.0114 5506	0.0119 6041	0.0122 1783	0.0124 7838	115
116	0.0113 8195	0.0118 8799	0.0121 4579	0.0124 0675	116
117	0.0113 1013	0.0118 1686	0.0120 7504	0.0123 3641	117
118	0.0112 3956	0.0117 4698	0.0120 0555	0.0122 6732	118
119	0.0111 7021	0.0116 7832	0.0119 3727	0.0121 9944	119
120	0.0111 0205	0.0116 1085	0.0118 7018	0.0121 3276	120
121	0.0110 3505	0.0115 4454	0.0118 0425	0.0120 6724	121
122	0.0109 6918	0.0114 7936	0.0117 3945	0.0120 0284	122
123	0.0109 0441	0.0114 1528	0.0116 7575	0.0119 3955	123
124	0.0108 4072	0.0113 5228	0.0116 1314	0.0118 7734	124
125	0.0107 7808	0.0112 9033	0.0115 5157	0.0118 1618	125
126	0.0107 1647	0.0112 2940	0.0114 9102	0.0117 5604	126
127	0.0106 5586	0.0111 6948	0.0114 3148	0.0116 9690	127
128	0.0105 9623	0.0111 1054	0.0113 7292	0.0116 3875	128
129	0.0105 3755	0.0110 5255	0.0113 1531	0.0115 8154	129
130	0.0104 7981	0.0109 9550	0.0112 5864	0.0115 2527	130
131	0.0104 2298	0.0109 3935	0.0112 0288	0.0114 6992	131
132	0.0103 6704	0.0108 8410	0.0111 4800	0.0114 1545	132
133	0.0103 1197	0.0108 2972	0.0110 9400	0.0113 6185	133
134	0.0102 5775	0.0107 7619	0.0110 4086	0.0113 0910	134
135	0.0102 0436	0.0107 2349	0.0109 8854	0.0112 5719	135
136	0.0101 5179	0.0106 7161	0.0109 3703	0.0112 0609	136
137	0.0101 0002	0.0106 2052	0.0108 8633	0.0111 5578	137
138	0.0100 4902	0.0105 7021	0.0108 3640	0.0111 0625	138
139	0.0099 9879	0.0105 2067	0.0107 8723	0.0110 5749	139
140	0.0099 4930	0.0104 7187	0.0107 3881	0.0110 0947	140
141	0.0099 0055	0.0104 2380	0.0106 9111	0.0109 6218	141
142	0.0098 5250	0.0103 7644	0.0106 4414	0.0109 1560	142
143	0.0098 0516	0.0103 2978	0.0105 9786	0.0108 6972	143
144	0.0097 5850	0.0102 8381	0.0105 5226	0.0108 2453	144
145	0.0097 1252	0.0102 3851	0.0105 0734	0.0107 8000	145
146	0.0096 6719	0.0101 9386	0.0104 6307	0.0107 3613	146
147	0.0096 2250	0.0101 4986	0.0104 1944	0.0106 9291	147
148	0.0095 7844	0.0101 0649	0.0103 7645	0.0106 5031	148
149	0.0095 3500	0.0100 6373	0.0103 3407	0.0106 0833	149
150	0.0094 9217	0.0100 2159	0.0102 9230	0.0105 6695	150

$$\frac{1}{a_{\overline{n}|}} = \frac{i}{1-v^n} = \frac{1}{s_{\overline{n}|}} + i$$

n	½%	7/12%	⅝%	⅔%	n
151	0.0094 4993	0.0099 8003	0.0102 5112	0.0105 2617	151
152	0.0094 0827	0.0099 3905	0.0102 1052	0.0104 8597	152
153	0.0093 6719	0.0098 9865	0.0101 7049	0.0104 4633	153
154	0.0093 2666	0.0098 5880	0.0101 3102	0.0104 0726	154
155	0.0092 8668	0.0098 1950	0.0100 9209	0.0103 6873	155
156	0.0092 4723	0.0097 8074	0.0100 5370	0.0103 3074	156
157	0.0092 0832	0.0097 4251	0.0100 1584	0.0102 9327	157
158	0.0091 6992	0.0097 0479	0.0099 7850	0.0102 5632	158
159	0.0091 3203	0.0096 6758	0.0099 4166	0.0102 1988	159
160	0.0090 9464	0.0096 3087	0.0099 0532	0.0101 8394	160
161	0.0090 5774	0.0095 9464	0.0098 6947	0.0101 4848	161
162	0.0090 2131	0.0095 5890	0.0098 3410	0.0101 1350	162
163	0.0089 8536	0.0095 2362	0.0097 9919	0.0100 7899	163
164	0.0089 4987	0.0094 8881	0.0097 6475	0.0100 4494	164
165	0.0089 1483	0.0094 5445	0.0097 3076	0.0100 1134	165
166	0.0088 8024	0.0094 2053	0.0096 9722	0.0099 7819	166
167	0.0088 4608	0.0093 8705	0.0096 6411	0.0099 4547	167
168	0.0088 1236	0.0093 5400	0.0096 3143	0.0099 1318	168
169	0.0087 7906	0.0093 2138	0.0095 9918	0.0098 8131	169
170	0.0087 4617	0.0092 8917	0.0095 6733	0.0098 4986	170
171	0.0087 1369	0.0092 5736	0.0095 3589	0.0098 1881	171
172	0.0086 8161	0.0092 2595	0.0095 0486	0.0097 8816	172
173	0.0086 4992	0.0091 9494	0.0094 7421	0.0097 5791	173
174	0.0086 1862	0.0091 6431	0.0094 4395	0.0097 2803	174
175	0.0085 8770	0.0091 3406	0.0094 1407	0.0096 9854	175
176	0.0085 5715	0.0091 0418	0.0093 8456	0.0096 6942	176
177	0.0085 2697	0.0090 7468	0.0093 5542	0.0096 4066	177
178	0.0084 9715	0.0090 4553	0.0093 2664	0.0096 1226	178
179	0.0084 6769	0.0090 1673	0.0092 9821	0.0095 8422	179
180	0.0084 3857	0.0089 8828	0.0092 7012	0.0095 5652	180
181	0.0084 0979	0.0089 6018	0.0092 4238	0.0095 2917	181
182	0.0083 8136	0.0089 3241	0.0092 1498	0.0095 0215	182
183	0.0083 5325	0.0089 0497	0.0091 8791	0.0094 7546	183
184	0.0083 2547	0.0088 7786	0.0091 6116	0.0094 4909	184
185	0.0082 9802	0.0088 5107	0.0091 3473	0.0094 2305	185
186	0.0082 7087	0.0088 2459	0.0091 0862	0.0093 9732	186
187	0.0082 4404	0.0087 9843	0.0090 8282	0.0093 7189	187
188	0.0082 1752	0.0087 7257	0.0090 5732	0.0093 4678	188
189	0.0081 9129	0.0087 4701	0.0090 3212	0.0093 2196	189
190	0.0081 6537	0.0087 2174	0.0090 0722	0.0092 9743	190
191	0.0081 3973	0.0086 9677	0.0089 8260	0.0092 7320	191
192	0.0081 1438	0.0086 7208	0.0089 5828	0.0092 4925	192
193	0.0080 8931	0.0086 4767	0.0089 3423	0.0092 2558	193
194	0.0080 6452	0.0086 2355	0.0089 1046	0.0092 0219	194
195	0.0080 4000	0.0085 9969	0.0088 8696	0.0091 7907	195
196	0.0080 1576	0.0085 7610	0.0088 6374	0.0091 5622	196
197	0.0079 9178	0.0085 5278	0.0088 4077	0.0091 3363	197
198	0.0079 6806	0.0085 2972	0.0088 1807	0.0091 1130	198
199	0.0079 4459	0.0085 0691	0.0087 9562	0.0090 8923	199
200	0.0079 2138	0.0084 8436	0.0087 7343	0.0090 6741	200

Annuity Whose Present Value at Compound Interest is 1

$$\frac{1}{a_{\overline{n}|}} = \frac{i}{1-v^n} = \frac{1}{s_{\overline{n}|}} + i$$

n	$\frac{3}{4}\%$	$\frac{7}{8}\%$	1%	$1\frac{1}{8}\%$	n
1	1.0075 0000	1.0087 5000	1.0100 0000	1.0112 5000	1
2	0.5056 3200	0.5065 7203	0.5075 1244	0.5084 5323	2
3	0.3383 4579	0.3391 8361	0.3400 2211	0.3408 6130	3
4	0.2547 0501	0.2554 9257	0.2562 8109	0.2570 7058	4
5	0.2045 2242	0.2052 8049	0.2060 3980	0.2068 0034	5
6	0.1710 6891	0.1718 0789	0.1725 4837	0.1732 9034	6
7	0.1471 7488	0.1479 0070	0.1486 2828	0.1493 5762	7
8	0.1292 5552	0.1299 7190	0.1306 9029	0.1314 1071	8
9	0.1153 1929	0.1160 2868	0.1167 4037	0.1174 5432	9
10	0.1041 7123	0.1048 7538	0.1055 8208	0.1062 9131	10
11	0.0950 5094	0.0957 5111	0.0964 5408	0.0971 5984	11
12	0.0874 5148	0.0881 4860	0.0888 4879	0.0895 5203	12
13	0.0810 2188	0.0817 1669	0.0824 1482	0.0831 1626	13
14	0.0755 1146	0.0762 0453	0.0769 0117	0.0776 0138	14
15	0.0707 3639	0.0714 2817	0.0721 2378	0.0728 2321	15
16	0.0665 5879	0.0672 4965	0.0679 4460	0.0686 4363	16
17	0.0628 7321	0.0635 6346	0.0642 5806	0.0649 5698	17
18	0.0595 9766	0.0602 8756	0.0609 8205	0.0616 8113	18
19	0.0566 6740	0.0573 5715	0.0580 5175	0.0587 5120	19
20	0.0540 3063	0.0547 2042	0.0554 1532	0.0561 1531	20
21	0.0516 4543	0.0523 3541	0.0530 3075	0.0537 3145	21
22	0.0494 7748	0.0501 6779	0.0508 6371	0.0515 6525	22
23	0.0474 9846	0.0481 8921	0.0488 8584	0.0495 8833	23
24	0.0456 8474	0.0463 7604	0.0470 7347	0.0477 7701	24
25	0.0440 1650	0.0447 0843	0.0454 0675	0.0461 1144	25
26	0.0424 7693	0.0431 6959	0.0438 6888	0.0445 7479	26
27	0.0410 5176	0.0417 4520	0.0424 4553	0.0431 5273	27
28	0.0397 2871	0.0404 2300	0.0411 2444	0.0418 3299	28
29	0.0384 9723	0.0391 9243	0.0398 9502	0.0406 0498	29
30	0.0373 4816	0.0380 4431	0.0387 4811	0.0394 5953	30
31	0.0362 7352	0.0369 7068	0.0376 7573	0.0383 8866	31
32	0.0352 6634	0.0359 6454	0.0366 7089	0.0373 8535	32
33	0.0343 2048	0.0350 1976	0.0357 2744	0.0364 4349	33
34	0.0334 3053	0.0341 3092	0.0348 3997	0.0355 5763	34
35	0.0325 9170	0.0332 9324	0.0340 0368	0.0347 2299	35
36	0.0317 9973	0.0325 0244	0.0332 1431	0.0339 3529	36
37	0.0310 5082	0.0317 5473	0.0324 6805	0.0331 9072	37
38	0.0303 4157	0.0310 4671	0.0317 6150	0.0324 8589	38
39	0.0296 6893	0.0303 7531	0.0310 9160	0.0318 1773	39
40	0.0290 3016	0.0297 3780	0.0304 5560	0.0311 8349	40
41	0.0284 2276	0.0291 3169	0.0298 5102	0.0305 8069	41
42	0.0278 4452	0.0285 5475	0.0292 7563	0.0300 0709	42
43	0.0272 9338	0.0280 0493	0.0287 2737	0.0294 6064	43
44	0.0267 6751	0.0274 8039	0.0282 0441	0.0289 3949	44
45	0.0262 6521	0.0269 7943	0.0277 0505	0.0284 4197	45
46	0.0257 8495	0.0265 0053	0.0272 2775	0.0279 6652	46
47	0.0253 2532	0.0260 4228	0.0267 7111	0.0275 1173	47
48	0.0248 8504	0.0256 0338	0.0263 3384	0.0270 7632	48
49	0.0244 6292	0.0251 8265	0.0259 1474	0.0266 5910	49
50	0.0240 5787	0.0247 7900	0.0255 1273	0.0262 5898	50

$$\frac{1}{a_{\overline{n}|}} = \frac{i}{1-v^n} = \frac{1}{s_{\overline{n}|}} + i$$

n	¾%	⅞%	1%	1⅛%	n
51	0.0236 6888	0.0243 9142	0.0251 2680	0.0258 7494	51
52	0.0232 9503	0.0240 1899	0.0247 5603	0.0255 0606	52
53	0.0229 3546	0.0236 6084	0.0243 9956	0.0251 5149	53
54	0.0225 8938	0.0233 1619	0.0240 5658	0.0248 1043	54
55	0.0222 5605	0.0229 8430	0.0237 2637	0.0244 8213	55
56	0.0219 3478	0.0226 6449	0.0234 0823	0.0241 6592	56
57	0.0216 2496	0.0223 5611	0.0231 0156	0.0238 6116	57
58	0.0213 2597	0.0220 5858	0.0228 0573	0.0235 6726	58
59	0.0210 3727	0.0217 7135	0.0225 2020	0.0232 8366	59
60	0.0207 5836	0.0214 9390	0.0222 4445	0.0230 0985	60
61	0.0204 8873	0.0212 2575	0.0219 7800	0.0227 4534	61
62	0.0202 2795	0.0209 6644	0.0217 2041	0.0224 8969	62
63	0.0199 7560	0.0207 1557	0.0214 7125	0.0222 4247	63
64	0.0197 3127	0.0204 7273	0.0212 3013	0.0220 0329	64
65	0.0194 9460	0.0202 3754	0.0209 9667	0.0217 7178	65
66	0.0192 6524	0.0200 0968	0.0207 7052	0.0215 4758	66
67	0.0190 4286	0.0197 8879	0.0205 5136	0.0213 3037	67
68	0.0188 2716	0.0195 7459	0.0203 3888	0.0211 1985	68
69	0.0186 1785	0.0193 6677	0.0201 3280	0.0209 1571	69
70	0.0184 1464	0.0191 6506	0.0199 3282	0.0207 1769	70
71	0.0182 1728	0.0189 6921	0.0197 3870	0.0205 2552	71
72	0.0180 2554	0.0187 7897	0.0195 5019	0.0203 3896	72
73	0.0178 3917	0.0185 9411	0.0193 6706	0.0201 5779	73
74	0.0176 5796	0.0184 1441	0.0191 8910	0.0199 8177	74
75	0.0174 8170	0.0182 3966	0.0190 1609	0.0198 1072	75
76	0.0173 1020	0.0180 6967	0.0188 4784	0.0196 4442	76
77	0.0171 4328	0.0179 0426	0.0186 8416	0.0194 8269	77
78	0.0169 8074	0.0177 4324	0.0185 2488	0.0193 2536	78
79	0.0168 2244	0.0175 8645	0.0183 6984	0.0191 7226	79
80	0.0166 6821	0.0174 3374	0.0182 1885	0.0190 2323	80
81	0.0165 1790	0.0172 8494	0.0180 7180	0.0188 7812	81
82	0.0163 7136	0.0171 3992	0.0179 2851	0.0187 3678	82
83	0.0162 2847	0.0169 9854	0.0177 8886	0.0185 9908	83
84	0.0160 8908	0.0168 6067	0.0176 5273	0.0184 6489	84
85	0.0159 5308	0.0167 2619	0.0175 1998	0.0183 3409	85
86	0.0158 2034	0.0165 9497	0.0173 9050	0.0182 0654	86
87	0.0156 9076	0.0164 6691	0.0172 6417	0.0180 8215	87
88	0.0155 6423	0.0163 4190	0.0171 4089	0.0179 6081	88
89	0.0154 4064	0.0162 1982	0.0170 2056	0.0178 4240	89
90	0.0153 1989	0.0161 0060	0.0169 0306	0.0177 2684	90
91	0.0152 0190	0.0159 8413	0.0167 8832	0.0176 1403	91
92	0.0150 8657	0.0158 7031	0.0166 7624	0.0175 0387	92
93	0.0149 7382	0.0157 5908	0.0165 6673	0.0173 9629	93
94	0.0148 6356	0.0156 5033	0.0164 5971	0.0172 9119	94
95	0.0147 5571	0.0155 4401	0.0163 5511	0.0171 8851	95
96	0.0146 5020	0.0154 4002	0.0162 5284	0.0170 8816	96
97	0.0145 4696	0.0153 3829	0.0161 5284	0.0169 9007	97
98	0.0144 4592	0.0152 3877	0.0160 5503	0.0168 9418	98
99	0.0143 4701	0.0151 4137	0.0159 5936	0.0168 0041	99
100	0.0142 5017	0.0150 4604	0.0158 6574	0.0167 0870	100

$$\frac{1}{a_{\overline{n}|}} = \frac{i}{1-v^n} = \frac{1}{s_{\overline{n}|}} + i$$

n	¾%	⅞%	1%	1⅛%	n
101	0.0141 5533	0.0149 5271	0.0157 7413	0.0166 1899	101
102	0.0140 6243	0.0148 6133	0.0156 8446	0.0165 3122	102
103	0.0139 7143	0.0147 7184	0.0155 9668	0.0164 4534	103
104	0.0138 8226	0.0146 8418	0.0155 1073	0.0163 6128	104
105	0.0137 9487	0.0145 9830	0.0154 2656	0.0162 7900	105
106	0.0137 0922	0.0145 1415	0.0153 4412	0.0161 9844	106
107	0.0136 2524	0.0144 3169	0.0152 6336	0.0161 1956	107
108	0.0135 4291	0.0143 5086	0.0151 8423	0.0160 4231	108
109	0.0134 6217	0.0142 7162	0.0151 0669	0.0159 6665	109
110	0.0133 8296	0.0141 9393	0.0150 3069	0.0158 9252	110
111	0.0133 0527	0.0141 1774	0.0149 5620	0.0158 1990	111
112	0.0132 2905	0.0140 4301	0.0148 8317	0.0157 4873	112
113	0.0131 5425	0.0139 6971	0.0148 1156	0.0156 7898	113
114	0.0130 8084	0.0138 9780	0.0147 4133	0.0156 1061	114
115	0.0130 0878	0.0138 2724	0.0146 7245	0.0155 4358	115
116	0.0129 3803	0.0137 5799	0.0146 0488	0.0154 7786	116
117	0.0128 6857	0.0136 9003	0.0145 3860	0.0154 1342	117
118	0.0128 0037	0.0136 2331	0.0144 7356	0.0153 5022	118
119	0.0127 3338	0.0135 5781	0.0144 0973	0.0152 8824	119
120	0.0126 6758	0.0134 9350	0.0143 4709	0.0152 2743	120
121	0.0126 0294	0.0134 3034	0.0142 8561	0.0151 6777	121
122	0.0125 3942	0.0133 6832	0.0142 2525	0.0151 0924	122
123	0.0124 7702	0.0133 0740	0.0141 6599	0.0150 5179	123
124	0.0124 1568	0.0132 4754	0.0141 0780	0.0149 9542	124
125	0.0123 5540	0.0131 8874	0.0140 5065	0.0149 4008	125
126	0.0122 9614	0.0131 3096	0.0139 9452	0.0148 8576	126
127	0.0122 3788	0.0130 7418	0.0139 3939	0.0148 3244	127
128	0.0121 8060	0.0130 1838	0.0138 8524	0.0147 8008	128
129	0.0121 2428	0.0129 6352	0.0138 3203	0.0147 2866	129
130	0.0120 6888	0.0129 0960	0.0137 7975	0.0146 7817	130
131	0.0120 1440	0.0128 5659	0.0137 2837	0.0146 2858	131
132	0.0119 6080	0.0128 0446	0.0136 7788	0.0145 7987	132
133	0.0119 0808	0.0127 5320	0.0136 2825	0.0145 3202	133
134	0.0118 5621	0.0127 0279	0.0135 7947	0.0144 8501	134
135	0.0118 0516	0.0126 5321	0.0135 3151	0.0144 3882	135
136	0.0117 5493	0.0126 0444	0.0134 8437	0.0143 9343	136
137	0.0117 0550	0.0125 5646	0.0134 3801	0.0143 4883	137
138	0.0116 5684	0.0125 0926	0.0133 9242	0.0143 0499	138
139	0.0116 0894	0.0124 6281	0.0133 4759	0.0142 6190	139
140	0.0115 6179	0.0124 1711	0.0133 0349	0.0142 1955	140
141	0.0115 1536	0.0123 7213	0.0132 6012	0.0141 7792	141
142	0.0114 6965	0.0123 2787	0.0132 1746	0.0141 3699	142
143	0.0114 2464	0.0122 8430	0.0131 7549	0.0140 9674	143
144	0.0113 8031	0.0122 4141	0.0131 3419	0.0140 5717	144
145	0.0113 3664	0.0121 9918	0.0130 9356	0.0140 1826	145
146	0.0112 9364	0.0121 5761	0.0130 5358	0.0139 7999	146
147	0.0112 5127	0.0121 1668	0.0130 1423	0.0139 4235	147
148	0.0112 0953	0.0120 7638	0.0129 7551	0.0139 0533	148
149	0.0111 6841	0.0120 3669	0.0129 3739	0.0138 6891	149
150	0.0111 2790	0.0119 9760	0.0128 9988	0.0138 3309	150

$$\frac{1}{a_{\overline{n}|}} = \frac{i}{1-v^n} = \frac{1}{s_{\overline{n}|}} + i$$

n	¾%	⅞%	1%	1⅛%	n
151	0.0110 8797	0.0119 5910	0.0128 6294	0.0137 9784	151
152	0.0110 4862	0.0119 2117	0.0128 2659	0.0137 6317	152
153	0.0110 0984	0.0118 8381	0.0127 9079	0.0137 2905	153
154	0.0109 7162	0.0118 4701	0.0127 5554	0.0136 9547	154
155	0.0109 3395	0.0118 1075	0.0127 2084	0.0136 6243	155
156	0.0108 9680	0.0117 7502	0.0126 8666	0.0136 2992	156
157	0.0108 6019	0.0117 3981	0.0126 5300	0.0135 9791	157
158	0.0108 2409	0.0117 0512	0.0126 1986	0.0135 6642	158
159	0.0107 8849	0.0116 7093	0.0125 8720	0.0135 3541	159
160	0.0107 5340	0.0116 3724	0.0125 5504	0.0135 0489	160
161	0.0107 1878	0.0116 0402	0.0125 2336	0.0134 7484	161
162	0.0106 8465	0.0115 7128	0.0124 9215	0.0134 4526	162
163	0.0106 5098	0.0115 3901	0.0124 6141	0.0134 1614	163
164	0.0106 1777	0.0115 0720	0.0124 3111	0.0133 8746	164
165	0.0105 8502	0.0114 7583	0.0124 0126	0.0133 5923	165
166	0.0105 5270	0.0114 4490	0.0123 7185	0.0133 3142	166
167	0.0105 2083	0.0114 1441	0.0123 4286	0.0133 0404	167
168	0.0104 8937	0.0113 8434	0.0123 1430	0.0132 7707	168
169	0.0104 5834	0.0113 5469	0.0122 8614	0.0132 5051	169
170	0.0104 2772	0.0113 2544	0.0122 5840	0.0132 2435	170
171	0.0103 9751	0.0112 9660	0.0122 3105	0.0131 9858	171
172	0.0103 6769	0.0112 6816	0.0122 0409	0.0131 7319	172
173	0.0103 3827	0.0112 4010	0.0121 7751	0.0131 4819	173
174	0.0103 0922	0.0112 1242	0.0121 5132	0.0131 2356	174
175	0.0102 8056	0.0111 8512	0.0121 2549	0.0130 9929	175
176	0.0102 5226	0.0111 5818	0.0121 0003	0.0130 7537	176
177	0.0102 2433	0.0111 3161	0.0120 7492	0.0130 5182	177
178	0.0101 9676	0.0111 0539	0.0120 5016	0.0130 2860	178
179	0.0101 6954	0.0110 7952	0.0120 2575	0.0130 0573	179
180	0.0101 4267	0.0110 5399	0.0120 0168	0.0129 8319	180
181	0.0101 1613	0.0110 2880	0.0119 7794	0.0129 6097	181
182	0.0100 8993	0.0110 0394	0.0119 5453	0.0129 3908	182
183	0.0100 6406	0.0109 7941	0.0119 3144	0.0129 1750	183
184	0.0100 3851	0.0109 5520	0.0119 0867	0.0128 9624	184
185	0.0100 1328	0.0109 3130	0.0118 8621	0.0128 7528	185
186	0.0099 8837	0.0109 0771	0.0118 6405	0.0128 5462	186
187	0.0099 6376	0.0108 8443	0.0118 4219	0.0128 3425	187
188	0.0099 3945	0.0108 6145	0.0118 2063	0.0128 1418	188
189	0.0099 1544	0.0108 3876	0.0117 9936	0.0127 9439	189
190	0.0098 9173	0.0108 1637	0.0117 7838	0.0127 7488	190
191	0.0098 6830	0.0107 9425	0.0117 5768	0.0127 5564	191
192	0.0098 4516	0.0107 7242	0.0117 3725	0.0127 3668	192
193	0.0098 2230	0.0107 5087	0.0117 1710	0.0127 1798	193
194	0.0097 9971	0.0107 2959	0.0116 9721	0.0126 9955	194
195	0.0097 7739	0.0107 0857	0.0116 7759	0.0126 8137	195
196	0.0097 5534	0.0106 8782	0.0116 5822	0.0126 6345	196
197	0.0097 3355	0.0106 6733	0.0116 3911	0.0126 4577	197
198	0.0097 1202	0.0106 4709	0.0116 2026	0.0126 2834	198
199	0.0096 9074	0.0106 2711	0.0116 0164	0.0126 1115	199
200	0.0096 6972	0.0106 0737	0.0115 8328	0.0125 9420	200

Annuity Whose Present Value at Compound Interest is 1

$$\frac{1}{a_{\overline{n}|i}} = \frac{i}{1-v^n} = \frac{1}{s_{\overline{n}|i}} + i$$

n	1¼%	1⅜%	1½%	1¾%	n
1	1.0125 0000	1.0137 5000	1.0150 0000	1.0175 0000	1
2	0.5093 9441	0.5103 3597	0.5112 7792	0.5131 6295	2
3	0.3417 0117	0.3425 4173	0.3433 8296	0.3450 6746	3
4	0.2578 6102	0.2586 5243	0.2594 4478	0.2610 3237	4
5	0.2075 6211	0.2083 2510	0.2090 8932	0.2106 2142	5
6	0.1740 3381	0.1747 7877	0.1755 2521	0.1770 2256	6
7	0.1500 8872	0.1508 2157	0.1515 5616	0.1530 3059	7
8	0.1321 3314	0.1328 5758	0.1335 8402	0.1350 4292	8
9	0.1181 7055	0.1188 8906	0.1196 0982	0.1210 5813	9
10	0.1070 0307	0.1077 1737	0.1084 3418	0.1098 7534	10
11	0.0978 6839	0.0985 7973	0.0992 9384	0.1007 3038	11
12	0.0902 5831	0.0909 6764	0.0916 7999	0.0931 1377	12
13	0.0838 2100	0.0845 2903	0.0852 4036	0.0866 7283	13
14	0.0783 0515	0.0790 1246	0.0797 2332	0.0811 5562	14
15	0.0735 2646	0.0742 3351	0.0749 4436	0.0763 7739	15
16	0.0693 4672	0.0700 5388	0.0707 6508	0.0721 9958	16
17	0.0656 6023	0.0663 6780	0.0670 7966	0.0685 1623	17
18	0.0623 8479	0.0630 9301	0.0638 0578	0.0652 4492	18
19	0.0594 5548	0.0601 6457	0.0608 7847	0.0623 2061	19
20	0.0568 2039	0.0575 3054	0.0582 4574	0.0596 9122	20
21	0.0544 3748	0.0551 4884	0.0558 6550	0.0573 1464	21
22	0.0522 7238	0.0529 8507	0.0537 0331	0.0551 5638	22
23	0.0502 9666	0.0510 1080	0.0517 3075	0.0531 8796	23
24	0.0484 8665	0.0492 0235	0.0499 2410	0.0513 8565	24
25	0.0468 2247	0.0475 3981	0.0482 6345	0.0497 2952	25
26	0.0452 8729	0.0460 0635	0.0467 3196	0.0482 0269	26
27	0.0438 6677	0.0445 8763	0.0453 1527	0.0467 9079	27
28	0.0425 4863	0.0432 7134	0.0440 0108	0.0454 8151	28
29	0.0413 2228	0.0420 4689	0.0427 7878	0.0442 6424	29
30	0.0401 7854	0.0409 0511	0.0416 3919	0.0431 2975	30
31	0.0391 0942	0.0398 3798	0.0405 7430	0.0420 7005	31
32	0.0381 0791	0.0388 3850	0.0395 7710	0.0410 7812	32
33	0.0371 6786	0.0379 0053	0.0386 4144	0.0401 4779	33
34	0.0362 8387	0.0370 1864	0.0377 6189	0.0392 7363	34
35	0.0354 5111	0.0361 8801	0.0369 3363	0.0384 5082	35
36	0.0346 6533	0.0354 0438	0.0361 5240	0.0376 7507	36
37	0.0339 2270	0.0346 6394	0.0354 1437	0.0369 4257	37
38	0.0332 1983	0.0339 6327	0.0347 1613	0.0362 4990	38
39	0.0325 5365	0.0332 9931	0.0340 5463	0.0355 9399	39
40	0.0319 2141	0.0326 6931	0.0334 2710	0.0349 7209	40
41	0.0313 2063	0.0320 7078	0.0328 3106	0.0343 8170	41
42	0.0307 4906	0.0315 0148	0.0322 6426	0.0338 2057	42
43	0.0302 0466	0.0309 5936	0.0317 2465	0.0332 8666	43
44	0.0296 8557	9.0304 4257	0.0312 1038	0.0327 7810	44
45	0.0291 9012	0.0299 4941	0.0307 1976	0.0322 9321	45
46	0.0287 1675	0.0294 7836	0.0302 5125	0.0318 3043	46
47	0.0282 6406	0.0290 2799	0.0298 0342	0.0313 8836	47
48	0.0278 3075	0.0285 9701	0.0293 7500	0.0309 6569	48
49	0.0274 1563	0.0281 8424	0.0289 6478	0.0305 6124	49
50	0.0270 1763	0.0277 8857	0.0285 7168	0.0301 7391	50

$$\frac{1}{a_{\overline{n}|}} = \frac{i}{1-v^n} = \frac{1}{s_{\overline{n}|}} + i$$

n	1¼%	1⅜%	1½%	1¾%	n
51	0.0266 3571	0.0274 0900	0.0281 9469	0.0298 0269	51
52	0.0262 6897	0.0270 4461	0.0278 3287	0.0294 4665	52
53	0.0259 1653	0.0266 9453	0.0274 8537	0.0291 0492	53
54	0.0255 7760	0.0263 5797	0.0271 5138	0.0287 7672	54
55	0.0252 5145	0.0260 3418	0.0268 3018	0.0284 6129	55
56	0.0249 3739	0.0257 2249	0.0265 2106	0.0281 5795	56
57	0.0246 3478	0.0254 2225	0.0262 2341	0.0278 6606	57
58	0.0243 4303	0.0251 3287	0.0259 3661	0.0275 8503	58
59	0.0240 6158	0.0248 5380	0.0256 6012	0.0273 1430	59
60	0.0237 8993	0.0245 8452	0.0253 9343	0.0270 5336	60
61	0.0235 2758	0.0243 2455	0.0251 3604	0.0268 0172	61
62	0.0232 7410	0.0240 7344	0.0248 8751	0.0265 5892	62
63	0.0230 2904	0.0238 3076	0.0246 4741	0.0263 2455	63
64	0.0227 9203	0.0235 9612	0.0244 1534	0.0260 9821	64
65	0.0225 6268	0.0233 6914	0.0241 9094	0.0258 7952	65
66	0.0223 4065	0.0231 4949	0.0239 7386	0.0256 6813	66
67	0.0221 2560	0.0229 3682	0.0237 6376	0.0254 6372	67
68	0.0219 1724	0.0227 3082	0.0235 6033	0.0252 6596	68
69	0.0217 1527	0.0225 3122	0.0233 6329	0.0250 7459	69
70	0.0215 1941	0.0223 3773	0.0231 7235	0.0248 8930	70
71	0.0213 2941	0.0221 5009	0.0229 8727	0.0247 0985	71
72	0.0211 4501	0.0219 6806	0.0228 0779	0.0245 3600	72
73	0.0209 6600	0.0217 9140	0.0226 3368	0.0243 6750	73
74	0.0207 9215	0.0216 1991	0.0224 6473	0.0242 0413	74
75	0.0206 2325	0.0214 5336	0.0223 0072	0.0240 4570	75
76	0.0204 5910	0.0212 9157	0.0221 4146	0.0238 9200	76
77	0.0202 9953	0.0211 3435	0.0219 8676	0.0237 4284	77
78	0.0201 4435	0.0209 8151	0.0218 3645	0.0235 9806	78
79	0.0199 9341	0.0208 3290	0.0216 9036	0.0234 5746	79
80	0.0198 4652	0.0206 8836	0.0215 4832	0.0233 2093	80
81	0.0197 0356	0.0205 4772	0.0214 1019	0.0231 8828	81
82	0.0195 6437	0.0204 1086	0.0212 7583	0.0230 5936	82
83	0.0194 2881	0.0202 7762	0.0211 4509	0.0229 3406	83
84	0.0192 9675	0.0201 4789	0.0210 1784	0.0228 1223	84
85	0.0191 6808	0.0200 2153	0.0208 9396	0.0226 9375	85
86	0.0190 4267	0.0198 9843	0.0207 7333	0.0225 7850	86
87	0.0189 2041	0.0197 7847	0.0206 5584	0.0224 6636	87
88	0.0188 0119	0.0196 6155	0.0205 4138	0.0223 5724	88
89	0.0186 8490	0.0195 4756	0.0204 2984	0.0222 5102	89
90	0.0185 7146	0.0194 3641	0.0203 2113	0.0221 4760	90
91	0.0184 6076	0.0193 2799	0.0202 1516	0.0220 4690	91
92	0.0183 5271	0.0192 2222	0.0201 1182	0.0219 4882	92
93	0.0182 4724	0.0191 1902	0.0200 1104	0.0218 5327	93
94	0.0181 4425	0.0190 1829	0.0199 1273	0.0217 6017	94
95	0.0180 4366	0.0189 1997	0.0198 1681	0.0216 6944	95
96	0.0179 4540	0.0188 2397	0.0197 2321	0.0215 8101	96
97	0.0178 4941	0.0187 3022	0.0196 3186	0.0214 9480	97
98	0.0177 5560	0.0186 3866	0.0195 4268	0.0214 1074	98
99	0.0176 6391	0.0185 4921	0.0194 5560	0.0213 2876	99
100	0.0175 7428	0.0184 6181	0.0193 7057	0.0212 4880	100

Annuity Whose Present Value at Compound Interest is 1

$$\frac{1}{a_{\overline{n}|}} = \frac{i}{1-v^n} = \frac{1}{s_{\overline{n}|}} + i$$

n	2%	2¼%	2½%	2¾%	n
1	1.0200 0000	1.0225 0000	1.0250 0000	1.0275 0000	1
2	0.5150 4950	0.5169 3758	0.5188 2716	0.5207 1825	2
3	0.3467 5467	0.3484 4458	0.3501 3717	0.3518 3243	3
4	0.2626 2375	0.2642 1893	0.2658 1788	0.2674 2059	4
5	0.2121 5839	0.2137 0021	0.2152 4686	0.2167 9832	5
6	0.1785 2581	0.1800 3496	0.1815 4997	0.1830 7083	6
7	0.1545 1196	0.1560 0025	0.1574 9543	0.1589 9747	7
8	0.1365 0980	0.1379 8462	0.1394 6735	0.1409 5795	8
9	0.1225 1544	0.1239 8170	0.1254 5689	0.1269 4095	9
10	0.1113 2653	0.1127 8768	0.1142 5876	0.1157 3972	10
11	0.1021 7794	0.1036 3649	0.1051 0596	0.1065 8629	11
12	0.0945 5960	0.0960 1740	0.0974 8713	0.0989 6871	12
13	0.0881 1835	0.0895 7686	0.0910 4827	0.0925 3252	13
14	0.0826 0197	0.0840 6230	0.0855 3653	0.0870 2457	14
15	0.0778 2547	0.0792 8852	0.0807 6646	0.0822 5917	15
16	0.0736 5013	0.0751 1663	0.0765 9899	0.0780 9710	16
17	0.0699 6984	0.0714 4039	0.0729 2777	0.0744 3186	17
18	0.0667 0210	0.0681 7720	0.0696 7008	0.0711 8063	18
19	0.0637 8177	0.0652 6182	0.0667 6062	0.0682 7802	19
20	0.0611 5672	0.0626 4207	0.0641 4713	0.0656 7173	20
21	0.0587 8477	0.0602 7572	0.0617 8733	0.0633 1941	21
22	0.0566 3140	0.0581 2821	0.0596 4661	0.0611 8640	22
23	0.0546 6810	0.0561 7097	0.0576 9638	0.0592 4410	23
24	0.0528 7110	0.0543 8023	0.0559 1282	0.0574 6863	24
25	0.0512 2044	0.0527 3599	0.0542 7592	0.0558 3997	25
26	0.0496 9923	0.0512 2134	0.0527 6875	0.0543 4116	26
27	0.0482 9309	0.0498 2188	0.0513 7687	0.0529 5776	27
28	0.0469 8967	0.0485 2525	0.0500 8793	0.0516 7738	28
29	0.0457 7836	0.0473 2081	0.0488 9127	0.0504 8935	29
30	0.0446 4992	0.0461 9934	0.0477 7764	0.0493 8442	30
31	0.0435 9635	0.0451 5280	0.0467 3900	0.0483 5453	31
32	0.0426 1061	0.0441 7415	0.0457 6831	0.0473 9263	32
33	0.0416 8653	0.0432 5722	0.0448 5938	0.0464 9253	33
34	0.0408 1867	0.0423 9655	0.0440 0675	0.0456 4875	34
35	0.0400 0221	0.0415 8731	0.0432 0558	0.0448 5645	35
36	0.0392 3285	0.0408 2522	0.0424 5158	0.0441 1132	36
37	0.0385 0678	0.0401 0643	0.0417 4090	0.0434 0953	37
38	0.0378 2057	0.0394 2753	0.0410 7012	0.0427 4764	38
39	0.0371 7114	0.0387 8543	0.0404 3615	0.0421 2256	39
40	0.0365 5575	0.0381 7738	0.0398 3623	0.0415 3151	40
41	0.0359 7188	0.0376 0087	0.0392 6786	0.0409 7200	41
42	0.0354 1729	0.0370 5364	0.0387 2876	0.0404 4175	42
43	0.0348 8993	0.0365 3364	0.0382 1688	0.0399 3871	43
44	0.0343 8794	0.0360 3901	0.0377 3037	0.0394 6100	44
45	0.0339 0962	0.0355 6805	0.0372 6752	0.0390 0693	45
46	0.0334 5342	0.0351 1921	0.0368 2676	0.0385 7493	46
47	0.0330 1792	0.0346 9107	0.0364 0669	0.0381 6358	47
48	0.0326 0184	0.0342 8233	0.0360 0599	0.0377 7158	48
49	0.0322 0396	0.0338 9179	0.0356 2348	0.0373 9773	49
50	0.0318 2321	0.0335 1836	0.0352 5806	0.0370 4092	50

$$\frac{1}{a_{\overline{n}|}} = \frac{i}{1-v^n} = \frac{1}{s_{\overline{n}|}} + i$$

n	2%	2¼%	2½%	2¾%	n
51	0.0314 5856	0.0331 6102	0.0349 0870	0.0367 0014	51
52	0.0311 0909	0.0328 1884	0.0345 7446	0.0363 7444	52
53	0.0307 7392	0.0324 9094	0.0342 5449	0.0360 6297	53
54	0.0304 5226	0.0321 7654	0.0339 4799	0.0357 6491	54
55	0.0301 4337	0.0318 7489	0.0336 5419	0.0354 7953	55
56	0.0298 4656	0.0315 8530	0.0333 7243	0.0352 0612	56
57	0.0295 6120	0.0313 0712	0.0331 0204	0.0349 4404	57
58	0.0292 8667	0.0310 3977	0.0328 4244	0.0346 9270	58
59	0.0290 2243	0.0307 8268	0.0325 9307	0.0344 5153	59
60	0.0287 6797	0.0305 3533	0.0323 5340	0.0342 2002	60
61	0.0285 2278	0.0302 9724	0.0321 2294	0.0339 9767	61
62	0.0282 8643	0.0300 6795	0.0319 0126	0.0337 8402	62
63	0.0280 5848	0.0298 4704	0.0316 8790	0.0335 7866	63
64	0.0278 3855	0.0296 3411	0.0314 8249	0.0333 8118	64
65	0.0276 2624	0.0294 2878	0.0312 8463	0.0331 9120	65
66	0.0274 2122	0.0292 3070	0.0310 9398	0.0330 0837	66
67	0.0272 2316	0.0290 3955	0.0309 1021	0.0328 3236	67
68	0.0270 3173	0.0288 5500	0.0307 3300	0.0326 6285	68
69	0.0268 4665	0.0286 7677	0.0305 6206	0.0324 9955	69
70	0.0266 6765	0.0285 0458	0.0303 9712	0.0323 4218	70
71	0.0264 9446	0.0283 3816	0.0302 3790	0.0321 9048	71
72	0.0263 2683	0.0281 7728	0.0300 8417	0.0320 4420	72
73	0.0261 6454	0.0280 2169	0.0299 3568	0.0319 0311	73
74	0.0260 0736	0.0278 7118	0.0297 9222	0.0317 6698	74
75	0.0258 5508	0.0277 2554	0.0296 5358	0.0316 3560	75
76	0.0257 0751	0.0275 8457	0.0295 1956	0.0315 0878	76
77	0.0255 6447	0.0274 4808	0.0293 8997	0.0313 8633	77
78	0.0254 2576	0.0273 1589	0.0292 6463	0.0312 6806	78
79	0.0252 9123	0.0271 8784	0.0291 4338	0.0311 5382	79
80	0.0251 6071	0.0270 6376	0.0290 2605	0.0310 4342	80
81	0.0250 3405	0.0269 4350	0.0289 1248	0.0309 3674	81
82	0.0249 1110	0.0268 2692	0.0288 0254	0.0308 3361	82
83	0.0247 9173	0.0267 1387	0.0286 9608	0.0307 3389	83
84	0.0246 7581	0.0266 0423	0.0285 9298	0.0306 3747	84
85	0.0245 6321	0.0264 9787	0.0284 9310	0.0305 4420	85
86	0.0244 5381	0.0263 9467	0.0283 9633	0.0304 5397	86
87	0.0243 4750	0.0262 9452	0.0283 0255	0.0303 6667	87
88	0.0242 4416	0.0261 9730	0.0282 1165	0.0302 8219	88
89	0.0241 4370	0.0261 0291	0.0281 2353	0.0302 0041	89
90	0.0240 4602	0.0260 1126	0.0280 3809	0.0301 2125	90
91	0.0239 5101	0.0259 2224	0.0279 5523	0.0300 4460	91
92	0.0238 5859	0.0258 3577	0.0278 7486	0.0299 7038	92
93	0.0237 6868	0.0257 5176	0.0277 9690	0.0298 9850	93
94	0.0236 8118	0.0256 7012	0.0277 2126	0.0298 2887	94
95	0.0235 9602	0.0255 9078	0.0276 4786	0.0297 6141	95
96	0.0235 1313	0.0255 1366	0.0275 7662	0.0296 9605	96
97	0.0234 3242	0.0254 3868	0.0275 0747	0.0296 3272	97
98	0.0233 5383	0.0253 6578	0.0274 4034	0.0295 7134	98
99	0.0232 7729	0.0252 9489	0.0273 7517	0.0295 1185	99
100	0.0232 0274	0.0252 2594	0.0273 1188	0.0294 5418	100

Annuity Whose Present Value at Compound Interest is 1

$$\frac{1}{a_{\overline{n}|}} = \frac{i}{1-v^n} = \frac{1}{s_{\overline{n}|}} + i$$

n	3%	3½%	4%	4½%	n
1	1.0300 0000	1.0350 0000	1.0400 0000	1.0450 0000	1
2	0.5226 1084	0.5264 0049	0.5301 9608	0.5339 9756	2
3	0.3535 3036	0.3569 3418	0.3603 4854	0.3637 7336	3
4	0.2690 2705	0.2722 5114	0.2754 9005	0.2787 4365	4
5	0.2183 5457	0.2214 8137	0.2246 2711	0.2277 9164	5
6	0.1845 9750	0.1876 6821	0.1907 6190	0.1938 7839	6
7	0.1605 0635	0.1635 4449	0.1666 0961	0.1697 0147	7
8	0.1424 5639	0.1454 7665	0.1485 2783	0.1516 0965	8
9	0.1284 3386	0.1314 4601	0.1344 9299	0.1375 7447	9
10	0.1172 3051	0.1202 4137	0.1232 9094	0.1263 7882	10
11	0.1080 7745	0.1110 9197	0.1141 4904	0.1172 4818	11
12	0.1004 6209	0.1034 8395	0.1065 5217	0.1096 6619	12
13	0.0940 2954	0.0970 6157	0.1001 4373	0.1032 7535	13
14	0.0885 2634	0.0915 7073	0.0946 6897	0.0978 2032	14
15	0.0837 6658	0.0868 2507	0.0899 4110	0.0931 1381	15
16	0.0796 1085	0.0826 8483	0.0858 2000	0.0890 1537	16
17	0.0759 5253	0.0790 4313	0.0821 9852	0.0854 1758	17
18	0.0727 0870	0.0758 1684	0.0789 9333	0.0822 3690	18
19	0.0698 1388	0.0729 4033	0.0761 3862	0.0794 0734	19
20	0.0672 1571	0.0703 6108	0.0735 8175	0.0768 7614	20
21	0.0648 7178	0.0680 3659	0.0712 8011	0.0746 0057	21
22	0.0627 4739	0.0659 3207	0.0691 9881	0.0725 4565	22
23	0.0608 1390	0.0640 1880	0.0673 0906	0.0706 8249	23
24	0.0590 4742	0.0622 7283	0.0655 8683	0.0689 8703	24
25	0.0574 2787	0.0606 7404	0.0640 1196	0.0674 3903	25
26	0.0559 3829	0.0592 0540	0.0625 6738	0.0660 2137	26
27	0.0545 6421	0.0578 5241	0.0612 3854	0.0647 1946	27
28	0.0532 9323	0.0566 0265	0.0600 1298	0.0635 2081	28
29	0.0521 1467	0.0554 4538	0.0588 7993	0.0624 1461	29
30	0.0510 1926	0.0543 7133	0.0578 3010	0.0613 9154	30
31	0.0499 9893	0.0533 7240	0.0568 5535	0.0604 4345	31
32	0.0490 4662	0.0524 4150	0.0559 4859	0.0595 6320	32
33	0.0481 5612	0.0515 7242	0.0551 0357	0.0587 4453	33
34	0.0473 2196	0.0507 5966	0.0543 1477	0.0579 8191	34
35	0.0465 3929	0.0499 9835	0.0535 7732	0.0572 7045	35
36	0.0458 0379	0.0492 8416	0.0528 8688	0.0566 0578	36
37	0.0451 1162	0.0486 1325	0.0522 3957	0.0559 8402	37
38	0.0444 5934	0.0479 8214	0.0516 3192	0.0554 0169	38
39	0.0438 4385	0.0473 8775	0.0510 6083	0.0548 5567	39
40	0.0432 6238	0.0468 2728	0.0505 2349	0.0543 4315	40
41	0.0427 1241	0.0462 9822	0.0500 1738	0.0538 6158	41
42	0.0421 9167	0.0457 9828	0.0495 4020	0.0534 0868	42
43	0.0416 9811	0.0453 2539	0.0490 8989	0.0529 8235	43
44	0.0412 2985	0.0448 7768	0.0486 6454	0.0525 8071	44
45	0.0407 8518	0.0444 5343	0.0482 6246	0.0522 0202	45
46	0.0403 6254	0.0440 5108	0.0478 8205	0.0518 4471	46
47	0.0399 6051	0.0436 6919	0.0475 2189	0.0515 0734	47
48	0.0395 7777	0.0433 0646	0.0471 8065	0.0511 8858	48
49	0.0392 1314	0.0429 6167	0.0468 5712	0.0508 8722	49
50	0.0388 6550	0.0426 3371	0.0465 5020	0.0506 0215	50

$$\frac{1}{a_{\overline{n}|}} = \frac{i}{1-v^n} = \frac{1}{s_{\overline{n}|}} + i$$

n	3%	3½%	4%	4½%	n
51	0.0385 3382	0.0423 2156	0.0462 5885	0.0503 3232	51
52	0.0382 1718	0.0420 2429	0.0459 8212	0.0500 7679	52
53	0.0379 1471	0.0417 4100	0.0457 1915	0.0498 3469	53
54	0.0376 2558	0.0414 7090	0.0454 6910	0.0496 0519	54
55	0.0373 4907	0.0412 1323	0.0452 3124	0.0493 8754	55
56	0.0370 8447	0.0409 6730	0.0450 0487	0.0491 8105	56
57	0.0368 3114	0.0407 3245	0.0447 8932	0.0489 8506	57
58	0.0365 8848	0.0405 0810	0.0445 8401	0.0487 9897	58
59	0.0363 5593	0.0402 9366	0.0443 8836	0.0486 2221	59
60	0.0361 3296	0.0400 8862	0.0442 0185	0.0484 5426	60
61	0.0359 1908	0.0398 9249	0.0440 2398	0.0482 9462	61
62	0.0357 1385	0.0397 0480	0.0438 5430	0.0481 4284	62
63	0.0355 1682	0.0395 2513	0.0436 9237	0.0479 9848	63
64	0.0353 2760	0.0393 5308	0.0435 3780	0.0478 6115	64
65	0.0351 4581	0.0391 8826	0.0433 9019	0.0477 3047	65
66	0.0349 7110	0.0390 3031	0.0432 4921	0.0476 0608	66
67	0.0348 0313	0.0388 7892	0.0431 1451	0.0474 8765	67
68	0.0346 4159	0.0387 3375	0.0429 8578	0.0473 7487	68
69	0.0344 8618	0.0385 9453	0.0428 6272	0.0472 6745	69
70	0.0343 3663	0.0384 6095	0.0427 4506	0.0471 6511	70
71	0.0341 9266	0.0383 3277	0.0426 3253	0.0470 6759	71
72	0.0340 5404	0.0382 0973	0.0425 2489	0.0469 7465	72
73	0.0339 2053	0.0380 9160	0.0424 2190	0.0468 8606	73
74	0.0337 9191	0.0379 7816	0.0423 2334	0.0468 0159	74
75	0.0336 6796	0.0378 6919	0.0422 2900	0.0467 2104	75
76	0.0335 4849	0.0377 6450	0.0421 3869	0.0466 4422	76
77	0.0334 3331	0.0376 6390	0.0420 5221	0.0465 7094	77
78	0.0333 2224	0.0375 6721	0.0419 6939	0.0465 0104	78
79	0.0332 1510	0.0374 7426	0.0418 9007	0.0464 3434	79
80	0.0331 1175	0.0373 8489	0.0418 1408	0.0463 7069	80
81	0.0330 1201	0.0372 9894	0.0417 4127	0.0463 0995	81
82	0.0329 1576	0.0372 1628	0.0416 7150	0.0462 5197	82
83	0.0328 2284	0.0371 3676	0.0416 0463	0.0461 9663	83
84	0.0327 3313	0.0370 6025	0.0415 4054	0.0461 4379	84
85	0.0326 4650	0.0369 8662	0.0414 7909	0.0460 9334	85
86	0.0325 6284	0.0369 1576	0.0414 2018	0.0460 4516	86
87	0.0324 8202	0.0368 4756	0.0413 6370	0.0459 9915	87
88	0.0324 0393	0.0367 8190	0.0413 0953	0.0459 5522	88
89	0.0323 2848	0.0367 1868	0.0412 5758	0.0459 1325	89
90	0.0322 5556	0.0366 5781	0.0412 0775	0.0458 7316	90
91	0.0321 8508	0.0365 9919	0.0411 5995	0.0458 3486	91
92	0.0321 1694	0.0365 4273	0.0411 1410	0.0457 9827	92
93	0.0320 5107	0.0364 8834	0.0410 7010	0.0457 6331	93
94	0.0319 8737	0.0364 3594	0.0410 2789	0.0457 2991	94
95	0.0319 2577	0.0363 8546	0.0409 8738	0.0456 9799	95
96	0.0318 6619	0.0363 3682	0.0409 4850	0.0456 6749	96
97	0.0318 0856	0.0362 8995	0.0409 1119	0.0456 3834	97
98	0.0317 5281	0.0362 4478	0.0408 7538	0.0456 1048	98
99	0.0316 9886	0.0362 0124	0.0408 4100	0.0455 8385	99
100	0.0316 4667	0.0361 5927	0.0408 0800	0.0455 5839	100

VII Annuity Whose Present Value at Compound Interest is 1

$$\frac{1}{a_{\overline{n}|}} = \frac{i}{1-v^n} = \frac{1}{s_{\overline{n}|}} + i$$

n	5%	5½%	6%	6½%	n
1	1.0500 0000	1.0550 0000	1.0600 0000	1.0650 0000	1
2	0.5378 0488	0.5416 1800	0.5454 3689	0.5492 6150	2
3	0.3672 0856	0.3706 5407	0.3741 0981	0.3775 7570	3
4	0.2820 1183	0.2852 9449	0.2885 9149	0.2919 0274	4
5	0.2309 7480	0.2341 7644	0.2373 9640	0.2406 3454	5
6	0.1970 1747	0.2001 7895	0.2033 6263	0.2065 6831	6
7	0.1728 1982	0.1759 6442	0.1791 3502	0.1823 3137	7
8	0.1547 2181	0.1578 6401	0.1610 3594	0.1642 3730	8
9	0.1406 9008	0.1438 3946	0.1470 2224	0.1502 3803	9
10	0.1295 0458	0.1326 6777	0.1358 6796	0.1391 0469	10
11	0.1203 8889	0.1235 7065	0.1267 9294	0.1300 5521	11
12	0.1128 2541	0.1160 2923	0.1192 7703	0.1225 6817	12
13	0.1064 5577	0.1096 8426	0.1129 6011	0.1162 8256	13
14	0.1010 2397	0.1042 7912	0.1075 8491	0.1109 4048	14
15	0.0963 4229	0.0996 2560	0.1029 6276	0.1063 5278	15
16	0.0922 6991	0.0955 8254	0.0989 5214	0.1023 7757	16
17	0.0886 9914	0.0920 4197	0.0954 4480	0.0989 0633	17
18	0.0855 4622	0.0889 1992	0.0923 5654	0.0958 5461	18
19	0.0827 4501	0.0861 5006	0.0896 2086	0.0931 5575	19
20	0.0802 4259	0.0836 7933	0.0871 8456	0.0907 5640	20
21	0.0779 9611	0.0814 6478	0.0850 0455	0.0886 1333	21
22	0.0759 7051	0.0794 7123	0.0830 4557	0.0866 9120	22
23	0.0741 3682	0.0776 6965	0.0812 7848	0.0849 6078	23
24	0.0724 7090	0.0760 3580	0.0796 7900	0.0833 9770	24
25	0.0709 5246	0.0745 4935	0.0782 2672	0.0819 8148	25
26	0.0695 6432	0.0731 9307	0.0769 0435	0.0806 9480	26
27	0.0682 9186	0.0719 5228	0.0756 9717	0.0795 2288	27
28	0.0671 2253	0.0708 1440	0.0745 9255	0.0784 5305	28
29	0.0660 4551	0.0697 6857	0.0735 7961	0.0774 7440	29
30	0.0650 5144	0.0688 0539	0.0726 4891	0.0765 7744	30
31	0.0641 3212	0.0679 1665	0.0717 9222	0.0757 5393	31
32	0.0632 8042	0.0670 9519	0.0710 0234	0.0749 9665	32
33	0.0624 9004	0.0663 3469	0.0702 7293	0.0742 9924	33
34	0.0617 5545	0.0656 2958	0.0695 9843	0.0736 5610	34
35	0.0610 7171	0.0649 7493	0.0689 7386	0.0730 6226	35
36	0.0604 3446	0.0643 6635	0.0683 9483	0.0725 1332	36
37	0.0598 3979	0.0637 9993	0.0678 5743	0.0720 0534	37
38	0.0592 8423	0.0632 7217	0.0673 5812	0.0715 3480	38
39	0.0587 6462	0.0627 7991	0.0668 9377	0.0710 9854	39
40	0.0582 7816	0.0623 2034	0.0664 6154	0.0706 9373	40
41	0.0578 2229	0.0618 9090	0.0660 5886	0.0703 1779	41
42	0.0573 9471	0.0614 8927	0.0656 8342	0.0699 6842	42
43	0.0569 9333	0.0611 1337	0.0653 3312	0.0696 4352	43
44	0.0566 1625	0.0607 6128	0.0650 0606	0.0693 4119	44
45	0.0562 6173	0.0604 3127	0.0647 0050	0.0690 5968	45
46	0.0559 2820	0.0601 2175	0.0644 1485	0.0687 9743	46
47	0.0556 1421	0.0598 3129	0.0641 4768	0.0685 5300	47
48	0.0553 1843	0.0595 5854	0.0638 9766	0.0683 2506	48
49	0.0550 3965	0.0593 0230	0.0636 6356	0.0681 1240	49
50	0.0547 7674	0.0590 6145	0.0634 4429	0.0679 1393	50

$$\frac{1}{a_{\overline{n}|}} = \frac{i}{1-v^n} = \frac{1}{s_{\overline{n}|}} + i$$

n	5%	5½%	6%	6½%	n
51	0.0545 2867	0.0588 3495	0.0632 3880	0.0677 2861	51
52	0.0542 9450	0.0586 2186	0.0630 4617	0.0675 5553	52
53	0.0540 7334	0.0584 2130	0.0628 6551	0.0673 9382	53
54	0.0538 6438	0.0582 3245	0.0626 9602	0.0672 4267	54
55	0.0536 6686	0.0580 5458	0.0625 3696	0.0671 0137	55
56	0.0534 8010	0.0578 8698	0.0623 8765	0.0669 6923	56
57	0.0533 0343	0.0577 2900	0.0622 4744	0.0668 4563	57
58	0.0531 3626	0.0575 8006	0.0621 1574	0.0667 2999	58
59	0.0529 7802	0.0574 3959	0.0619 9200	0.0666 2177	59
60	0.0528 2818	0.0573 0707	0.0618 7572	0.0665 2047	60
61	0.0526 8627	0.0571 8202	0.0617 6642	0.0664 2564	61
62	0.0525 5183	0.0570 6400	0.0616 6366	0.0663 3684	62
63	0.0524 2442	0.0569 5258	0.0615 6704	0.0662 5367	63
64	0.0523 0365	0.0568 4737	0.0614 7615	0.0661 7577	64
65	0.0521 8915	0.0567 4800	0.0613 9066	0.0661 0280	65
66	0.0520 8057	0.0566 5413	0.0613 1022	0.0660 3442	66
67	0.0519 7757	0.0565 6544	0.0612 3454	0.0659 7034	67
68	0.0518 7986	0.0564 8163	0.0611 6330	0.0659 1029	68
69	0.0517 8715	0.0564 0242	0.0610 9625	0.0658 5400	69
70	0.0516 9915	0.0563 2754	0.0610 3313	0.0658 0124	70
71	0.0516 1563	0.0562 5675	0.0609 7370	0.0657 5177	71
72	0.0515 3633	0.0561 8982	0.0609 1774	0.0657 0539	72
73	0.0514 6103	0.0561 2652	0.0608 6505	0.0656 6190	73
74	0.0513 8953	0.0560 6665	0.0608 1542	0.0656 2112	74
75	0.0513 2161	0.0560 1002	0.0607 6867	0.0655 8287	75
76	0.0512 5709	0.0559 5645	0.0607 2463	0.0655 4699	76
77	0.0511 9580	0.0559 0577	0.0606 8315	0.0655 1335	77
78	0.0511 3756	0.0558 5781	0.0606 4407	0.0654 8178	78
79	0.0510 8222	0.0558 1243	0.0606 0724	0.0654 5217	79
80	0.0510 2962	0.0557 6948	0.0605 7254	0.0654 2440	80
81	0.0509 7963	0.0557 2884	0.0605 3984	0.0653 9834	81
82	0.0509 3211	0.0556 9036	0.0605 0903	0.0653 7388	82
83	0.0508 8694	0.0556 5395	0.0604 7998	0.0653 5094	83
84	0.0508 4399	0.0556 1947	0.0604 5261	0.0653 2941	84
85	0.0508 0316	0.0555 8683	0.0604 2681	0.0653 0921	85
86	0.0507 6433	0.0555 5593	0.0604 0249	0.0652 9026	86
87	0.0507 2740	0.0555 2667	0.0603 7956	0.0652 7247	87
88	0.0506 9228	0.0554 9896	0.0603 5795	0.0652 5577	88
89	0.0506 5888	0.0554 7273	0.0603 3757	0.0652 4010	89
90	0.0506 2711	0.0554 4788	0.0603 1836	0.0652 2540	90
91	0.0505 9689	0.0554 2435	0.0603 0025	0.0652 1160	91
92	0.0505 6815	0.0554 0207	0.0602 8318	0.0651 9864	92
93	0.0505 4080	0.0553 8096	0.0602 6708	0.0651 8649	93
94	0.0505 1478	0.0553 6097	0.0602 5190	0.0651 7507	94
95	0.0504 9003	0.0553 4204	0.0602 3758	0.0651 6436	95
96	0.0504 6648	0.0553 2410	0.0602 2408	0.0651 5431	96
97	0.0504 4407	0.0553 0711	0.0602 1135	0.0651 4487	97
98	0.0504 2274	0.0552 9101	0.0601 9935	0.0651 3601	98
99	0.0504 0245	0.0552 7577	0.0601 8803	0.0651 2769	99
100	0.0503 8314	0.0552 6132	0.0601 7736	0.0651 1988	100

Annuity Whose Present Value at Compound Interest is 1

$$\frac{1}{a_{\overline{n}|}} = \frac{i}{1-v^n} = \frac{1}{s_{\overline{n}|}} + i$$

n	7%	7½%	8%	8½%	n
1	1.0700 0000	1.0750 0000	1.0800 0000	1.0850 0000	1
2	0.5530 9179	0.5569 2771	0.5607 6923	0.5646 1631	2
3	0.3810 5166	0.3845 3763	0.3880 3351	0.3915 3925	3
4	0.2952 2812	0.2985 6751	0.3019 2080	0.3052 8789	4
5	0.2438 9069	0.2471 6472	0.2504 5645	0.2537 6575	5
6	0.2097 9580	0.2130 4489	0.2163 1539	0.2196 0708	6
7	0.1855 5322	0.1888 0032	0.1920 7240	0.1953 6922	7
8	0.1674 6776	0.1707 2702	0.1740 1476	0.1773 3065	8
9	0.1534 8647	0.1567 6716	0.1600 7971	0.1634 2372	9
10	0.1423 7750	0.1456 8593	0.1490 2949	0.1524 0771	10
11	0.1333 5690	0.1366 9747	0.1400 7634	0.1434 9293	11
12	0.1259 0199	0.1292 7783	0.1326 9502	0.1361 5286	12
13	0.1196 5085	0.1230 6420	0.1265 2181	0.1300 2287	13
14	0.1143 4494	0.1177 9737	0.1212 9685	0.1248 4244	14
15	0.1097 9462	0.1132 8724	0.1168 2954	0.1204 2046	15
16	0.1058 5765	0.1093 9116	0.1129 7687	0.1166 1354	16
17	0.1024 2519	0.1060 0003	0.1096 2943	0.1133 1198	17
18	0.0994 1260	0.1030 2896	0.1067 0210	0.1104 3041	18
19	0.0967 5301	0.1004 1090	0.1041 2763	0.1079 0140	19
20	0.0943 9293	0.0980 9219	0.1018 5221	0.1056 7097	20
21	0.0922 8900	0.0960 2937	0.0998 3225	0.1036 9541	21
22	0.0904 0577	0.0941 8687	0.0980 3207	0.1019 3892	22
23	0.0887 1393	0.0925 3528	0.0964 2217	0.1003 7193	23
24	0.0871 8902	0.0910 5008	0.0949 7796	0.0989 6975	24
25	0.0858 1052	0.0897 1067	0.0936 7878	0.0977 1168	25
26	0.0845 6103	0.0884 9961	0.0925 0713	0.0965 8016	26
27	0.0834 2573	0.0874 0204	0.0914 4809	0.0955 6025	27
28	0.0823 9193	0.0864 0520	0.0904 8891	0.0946 3914	28
29	0.0814 4865	0.0854 9811	0.0896 1854	0.0938 0577	29
30	0.0805 8640	0.0846 7124	0.0888 2743	0.0930 5058	30
31	0.0797 9691	0.0839 1628	0.0881 0728	0.0923 6524	31
32	0.0790 7292	0.0832 2599	0.0874 5081	0.0917 4247	32
33	0.0784 0807	0.0825 9397	0.0868 5163	0.0911 7588	33
34	0.0777 9674	0.0820 1461	0.0863 0411	0.0906 5984	34
35	0.0772 3396	0.0814 8291	0.0858 0326	0.0901 8937	35
36	0.0767 1531	0.0809 9447	0.0853 4467	0.0897 6006	36
37	0.0762 3685	0.0805 4533	0.0849 2440	0.0893 6799	37
38	0.0757 9505	0.0801 3197	0.0845 3894	0.0890 0966	38
39	0.0753 8676	0.0797 5124	0.0841 8513	0.0886 8193	39
40	0.0750 0914	0.0794 0031	0.0838 6016	0.0883 8201	40
41	0.0746 5962	0.0790 7663	0.0835 6149	0.0881 0737	41
42	0.0743 3591	0.0787 7789	0.0832 8684	0.0878 5576	42
43	0.0740 3590	0.0785 0201	0.0830 3414	0.0876 2512	43
44	0.0737 5769	0.0782 4710	0.0828 0152	0 0874 1363	44
45	0.0734 9957	0.0780 1146	0.0825 8728	0.0872 1961	45
46	0.0732 5996	0.0777 9353	0.0823 8991	0.0870 4154	46
47	0.0730 3744	0.0775 9190	0.0822 0799	0.0868 7807	47
48	0.0728 3070	0.0774 0527	0.0820 4027	0.0867 2795	48
49	0.0726 3853	0.0772 3247	0.0818 8557	0.0865 9005	49
50	0.0724 5985	0.0770 7241	0.0817 4286	0.0864 6334	50

$$\frac{1}{a_{\overline{n}|}} = \frac{i}{1-v^n} = \frac{1}{s_{\overline{n}|}} + i$$

n	7%	7½%	8%	8½%	n
51	0.0722 9365	0.0769 2411	0.0816 1116	0.0863 4688	51
52	0.0721 3901	0.0767 8668	0.0814 8959	0.0862 3983	52
53	0.0719 9509	0.0766 5927	0.0813 7735	0.0861 4139	53
54	0.0718 6110	0.0765 4112	0.0812 7370	0.0860 5087	54
55	0.0717 3633	0.0764 3155	0.0811 7796	0.0859 6761	55
56	0.0716 2011	0.0763 2991	0.0810 8952	0.0858 9101	56
57	0.0715 1183	0.0762 3559	0.0810 0780	0.0858 2053	57
58	0.0714 1093	0.0761 4807	0.0809 3227	0.0857 5568	58
59	0.0713 1689	0.0760 6683	0.0808 6247	0.0856 9599	59
60	0.0712 2923	0.0759 9142	0.0807 9795	0.0856 4106	60
61	0.0711 4749	0.0759 2140	0.0807 3830	0.0855 9049	61
62	0.0710 7127	0.0758 5638	0.0806 8314	0.0855 4393	62
63	0.0710 0019	0.0757 9600	0.0806 3214	0.0855 0107	63
64	0.0709 3388	0.0757 3992	0.0805 8497	0.0854 6160	64
65	0.0708 7203	0.0756 8782	0.0805 4135	0.0854 2526	65
66	0.0708 1431	0.0756 3942	0.0805 0100	0.0853 9179	66
67	0.0707 6046	0.0755 9446	0.0804 6367	0.0853 6097	67
68	0.0707 1021	0.0755 5268	0.0804 2914	0.0853 3258	68
69	0.0706 6331	0.0755 1386	0.0803 9719	0.0853 0643	69
70	0.0706 1953	0.0754 7778	0.0803 6764	0.0852 8234	70
71	0.0705 7866	0.0754 4425	0.0803 4029	0.0852 6016	71
72	0.0705 4051	0.0754 1308	0.0803 1498	0.0852 3972	72
73	0.0705 0490	0.0753 8412	0.0802 9157	0.0852 2089	73
74	0.0704 7164	0.0753 5719	0.0802 6989	0.0852 0354	74
75	0.0704 4060	0.0753 3216	0.0802 4984	0.0851 8756	75
76	0.0704 1160	0.0753 0889	0.0802 3128	0.0851 7284	76
77	0.0703 8453	0.0752 8726	0.0802 1410	0.0851 5927	77
78	0.0703 5924	0.0752 6714	0.0801 9820	0.0851 4677	78
79	0.0703 3563	0.0752 4844	0.0801 8349	0.0851 3526	79
80	0.0703 1357	0.0752 3106	0.0801 6987	0.0851 2465	80
81	0.0702 9297	0.0752 1489	0.0801 5726	0.0851 1487	81
82	0.0702 7373	0.0751 9986	0.0801 4559	0.0851 0586	82
83	0.0702 5576	0.0751 8588	0.0801 3479	0.0850 9756	83
84	0.0702 3897	0.0751 7288	0.0801 2479	0.0850 8990	84
85	0.0702 2329	0.0751 6079	0.0801 1553	0.0850 8285	85
86	0.0702 0863	0.0751 4955	0.0801 0696	0.0850 7636	86
87	0.0701 9495	0.0751 3910	0.0800 9903	0.0850 7037	87
88	0.0701 8216	0.0751 2938	0.0800 9168	0.0850 6485	88
89	0.0701 7021	0.0751 2034	0.0800 8489	0.0850 5977	89
90	0.0701 5905	0.0751 1193	0.0800 7859	0.0850 5508	90
91	0.0701 4863	0.0751 0411	0.0800 7277	0.0850 5077	91
92	0.0701 3888	0.0750 9684	0.0800 6737	0.0850 4679	92
93	0.0701 2978	0.0750 9007	0.0800 6238	0.0850 4312	93
94	0.0701 2128	0.0750 8378	0.0800 5775	0.0850 3974	94
95	0.0701 1333	0.0750 7793	0.0800 5347	0.0850 3663	95
96	0.0701 0590	0.0750 7249	0.0800 4951	0.0850 3375	96
97	0.0700 9897	0.0750 6743	0.0800 4584	0.0850 3111	97
98	0.0700 9248	0.0750 6272	0.0800 4244	0.0850 2867	98
99	0.0700 8643	0.0750 5834	0.0800 3930	0.0850 2642	99
100	0.0700 8076	0.0750 5427	0.0800 3638	0.0850 2435	100

$$(1+i)^{1/p}$$

p	$\frac{1}{4}\%$	$\frac{7}{24}\%$	$\frac{1}{3}\%$	$\frac{5}{12}\%$	p
2	1.0012 4922	1.0014 5727	1.0016 6528	1.0020 8117	2
3	1.0008 3264	1.0009 7128	1.0011 0988	1.0013 8696	3
4	1.0006 2441	1.0007 2837	1.0008 3229	1.0010 4004	4
6	1.0004 1623	1.0004 8552	1.0005 5479	1.0006 9324	6
12	1.0002 0890	1.0002 4273	1.0002 7735	1.0003 4656	12
13	1.0001 9209	1.0002 2406	1.0002 5602	1.0003 1990	13
26	1.0000 9604	1.0001 1202	1.0001 2800	1.0001 5994	26
52	1.0000 4802	1.0000 5601	1.0000 6400	1.0000 7996	52
365	1.0000 0684	1.0000 0798	1.0000 0912	1.0000 1139	365
∞	1.0000 0000	1.0000 0000	1.0000 0000	1.0000 0000	∞

p	$\frac{1}{2}\%$	$\frac{7}{12}\%$	$\frac{5}{8}\%$	$\frac{2}{3}\%$	p
2	1.0024 9688	1.0029 1243	1.0031 2013	1.0033 2780	2
3	1.0016 6390	1.0019 4068	1.0020 7901	1.0022 1730	3
4	1.0012 4766	1.0014 5515	1.0015 5885	1.0016 6252	4
6	1.0008 3160	1.0009 6987	1.0010 3896	1.0011 0804	6
12	1.0004 1571	1.0004 8482	1.0005 1935	1.0005 5387	12
13	1.0003 8373	1.0004 4751	1.0004 7939	1.0005 1125	13
26	1.0001 9185	1.0002 2373	1.0002 3967	1.0002 5559	26
52	1.0000 9592	1.0001 1186	1.0001 1983	1.0001 2779	52
365	1.0000 1366	1.0000 1594	1.0000 1707	1.0000 1820	365
∞	1.0000 0000	1.0000 0000	1.0000 0000	1.0000 0000	∞

p	$\frac{3}{4}\%$	$\frac{7}{8}\%$	1%	$1\frac{1}{8}\%$	p
2	1.0037 4299	1.0043 6547	1.0049 8756	1.0056 0927	2
3	1.0024 9378	1.0029 0820	1.0033 2228	1.0037 3602	3
4	1.0018 6975	1.0021 8036	1.0024 9068	1.0028 0081	4
6	1.0012 4611	1.0014 5304	1.0016 5977	1.0018 6627	6
12	1.0006 2286	1.0007 2626	1.0008 2954	1.0009 3270	12
13	1.0005 7494	1.0006 7037	1.0007 6570	1.0008 6092	13
26	1.0002 8743	1.0003 3513	1.0003 8276	1.0004 3037	26
52	1.0001 4370	1.0001 6755	1.0001 9137	1.0002 1516	52
365	1.0000 2047	1.0000 2387	1.0000 2726	1.0000 3065	365
∞	1.0000 0000	1.0000 0000	1.0000 0000	1.0000 0000	∞

p	$1\frac{1}{4}\%$	$1\frac{3}{8}\%$	$1\frac{1}{2}\%$	$1\frac{3}{4}\%$	p
2	1.0062 3059	1.0068 5153	1.0074 7208	1.0087 1205	2
3	1.0041 4943	1.0045 6249	1.0049 7521	1.0057 9963	3
4	1.0031 1046	1.0034 1992	1.0037 2909	1.0043 4658	4
6	1.0020 7257	1.0022 7865	1.0024 8452	1.0028 9562	6
12	1.0010 3575	1.0011 3868	1.0012 4149	1.0014 4677	12
13	1.0009 5604	1.0010 5104	1.0011 4594	1.0013 3540	13
26	1.0004 7790	1.0005 2538	1.0005 7280	1.0006 6748	26
52	1.0002 3892	1.0002 6266	1.0002 8636	1.0003 3368	52
365	1.0000 3403	1.0000 3742	1.0000 4079	1.0000 4753	365
∞	1.0000 0000	1.0000 0000	1.0000 0000	1.0000 0000	∞

$(1+i)^{1/p}$

p	2%	2¼%	2½%	2¾%	p
2	1.0099 5050	1.0111 8742	1.0124 2284	1.0136 5675	2
3	1.0066 2271	1.0074 4444	1.0082 6484	1.0090 8390	3
4	1.0049 6293	1.0055 7815	1.0061 9225	1.0068 0522	4
6	1.0033 0589	1.0037 1532	1.0041 2392	1.0045 3168	6
12	1.0016 5158	1.0018 5594	1.0020 5984	1.0022 6328	12
13	1.0015 2444	1.0017 1305	1.0019 0124	1.0020 8900	13
26	1.0007 6193	1.0008 5616	1.0009 5017	1.0010 4396	26
52	1.0003 8089	1.0004 2799	1.0004 7497	1.0005 2184	52
365	1.0000 5426	1.0000 6096	1.0000 6765	1.0000 7433	365
∞	1.0000 0000	1.0000 0000	1.0000 0000	1.0000 0000	∞

p	3%	3½%	4%	4½%	p
2	1.0148 8916	1.0173 4950	1.0198 0390	1.0222 5242	2
3	1.0099 0163	1.0115 3314	1.0131 5941	1.0147 8046	3
4	1.0074 1707	1.0086 3745	1.0098 5341	1.0110 6499	4
6	1.0049 3862	1.0057 5004	1.0065 5820	1.0073 6312	6
12	1.0024 6627	1.0028 7090	1.0032 7374	1.0036 7481	12
13	1.0022 7634	1.0026 4977	1.0030 2153	1.0033 9165	13
26	1.0011 3752	1.0013 2401	1.0015 0963	1.0016 9439	26
52	1.0005 6860	1.0006 6179	1.0007 5453	1.0008 4684	52
365	1.0000 8099	1.0000 9425	1.0001 0746	1.0001 2060	365
∞	1.0000 0000	1.0000 0000	1.0000 0000	1.0000 0000	∞

p	5%	5½%	6%	6½%	p
2	1.0246 9508	1.0271 3193	1.0295 6302	1.0319 8837	2
3	1.0163 9636	1.0180 0713	1.0196 1282	1.0212 1347	3
4	1.0122 7224	1.0134 7518	1.0146 7385	1.0158 6828	4
6	1.0081 6485	1.0089 6340	1.0097 5880	1.0105 5107	6
12	1.0040 7412	1.0044 7170	1.0048 6755	1.0052 6169	12
13	1.0037 6014	1.0041 2701	1.0044 9228	1.0048 5597	13
26	1.0018 7831	1.0020 6138	1.0022 4363	1.0024 2504	26
52	1.0009 3871	1.0010 3016	1.0011 2118	1.0012 1179	52
365	1.0001 3368	1.0001 4670	1.0001 5965	1.0001 7255	365
∞	1.0000 0000	1.0000 0000	1.0000 0000	1.0000 0000	∞

p	7%	7½%	8%	8½%	p
2	1.0344 0804	1.0368 2207	1.0392 3048	1.0416 3333	2
3	1.0228 0912	1.0243 9981	1.0259 8557	1.0275 6644	3
4	1.0170 5853	1.0182 4460	1.0194 2655	1.0206 0440	4
6	1.0113 4026	1.0121 2638	1.0129 0946	1.0136 8952	6
12	1.0056 5415	1.0060 4492	1.0064 3403	1.0068 2149	12
13	1.0052 1808	1.0055 7863	1.0059 3764	1.0062 9511	13
26	1.0026 0564	1.0027 8544	1.0029 6443	1.0031 4262	26
52	1.0013 0197	1.0013 9175	1.0014 8112	1.0015 7008	52
365	1.0001 8538	1.0001 9816	1.0002 1087	1.0002 2353	365
∞	1.0000 0000	1.0000 0000	1.0000 0000	1.0000 0000	∞

Amount at End of Period at Compound Interest of p Installments Each of $1/p$ Deposited at End of Each pth Part of the Period

$$s_{\overline{1}|i}^{(p)} = i/j_{(p)}$$

p	$\frac{1}{4}\%$	$\frac{7}{24}\%$	$\frac{1}{3}\%$	$\frac{5}{12}\%$	p
2	1.0006 2461	1.0007 2864	1.0008 3264	1.0010 4058	2
3	1.0008 3267	1.0009 7159	1.0011 1029	1.0013 8761	3
4	1.0009 3701	1.0010 9309	1.0012 4913	1.0015 6115	4
6	1.0010 4116	1.0012 1459	1.0013 8799	1.0017 3471	6
12	1.0011 4532	1.0013 3610	1.0015 2686	1.0019 0829	12
13	1.0011 5333	1.0013 4545	1.0015 3754	1.0019 2164	13
26	1.0012 0140	1.0014 0154	1.0016 0164	1.0020 0176	26
52	1.0012 2544	1.0014 2958	1.0016 3369	1.0020 4183	52
365	1.0012 4606	1.0014 5363	1.0016 6118	1.0020 7618	365
∞	1.0012 4948	1.0014 5763	1.0016 6574	1.0020 8189	∞

p	$\frac{1}{2}\%$	$\frac{7}{12}\%$	$\frac{5}{8}\%$	$\frac{2}{3}\%$	p
2	1.0012 4844	1.0014 5621	1.0015 6007	1.0016 6390	2
3	1.0016 6482	1.0019 4193	1.0020 8045	1.0022 1894	3
4	1.0018 7305	1.0021 8485	1.0023 4071	1.0024 9654	4
6	1.0020 8131	1.0024 2781	1.0026 0101	1.0027 7419	6
12	1.0022 8960	1.0026 7080	1.0028 6136	1.0030 5189	12
13	1.0023 0563	1.0026 8950	1.0028 8139	1.0030 7325	13
26	1.0024 2182	1.0028 0166	1.0030 1762	1.0032 0144	26
52	1.0024 4985	1.0028 5775	1.0030 6166	1.0032 6554	52
365	1.0024 9107	1.0029 0585	1.0031 1319	1.0033 2051	365
∞	1.0024 9792	1.0029 1384	1.0031 2175	1.0033 2964	∞

p	$\frac{3}{4}\%$	$\frac{7}{8}\%$	1%	$1\frac{1}{8}\%$	p
2	1.0018 7150	1.0021 8274	1.0024 9378	1.0028 0463	2
3	1.0024 9585	1.0029 1102	1.0033 2596	1.0037 4068	3
4	1.0028 0812	1.0032 7529	1.0037 4223	1.0042 0892	4
6	1.0031 2046	1.0036 3967	1.0041 5861	1.0046 7730	6
12	1.0034 3286	1.0040 0411	1.0045 7510	1.0051 4583	12
13	1.0034 5690	1.0040 3215	1.0046 0714	1.0051 8188	13
26	1.0036 0111	1.0042 0039	1.0047 9941	1.0053 9818	26
52	1.0036 7322	1.0042 8452	1.0048 9556	1.0055 0634	52
365	1.0037 3506	1.0043 5666	1.0049 7801	1.0055 9910	365
∞	1.0037 4533	1.0043 6865	1.0049 9171	1.0056 1451	∞

p	$1\frac{1}{4}\%$	$1\frac{3}{8}\%$	$1\frac{1}{2}\%$	$1\frac{3}{4}\%$	p
2	1.0031 1529	1.0034 2576	1.0037 3604	1.0043 6176	2
3	1.0041 5516	1.0045 6942	1.0049 8346	1.0058 1064	3
4	1.0046 7537	1.0051 4158	1.0056 0755	1.0065 3878	4
6	1.0051 9575	1.0057 1395	1.0062 3191	1.0072 6707	6
12	1.0057 1632	1.0062 8654	1.0068 5652	1.0079 9571	12
13	1.0057 5637	1.0063 3060	1.0069 0458	1.0080 5177	13
26	1.0059 9669	1.0065 9495	1.0071 9296	1.0083 8820	26
52	1.0061 1687	1.0067 2715	1.0073 3717	1.0085 5644	52
365	1.0062 1994	1.0068 4052	1.0074 6084	1.0087 0073	365
∞	1.0062 3706	1.0068 5935	1.0074 8139	1.0087 2470	∞

Amount at End of Period at Compound Interest of p Installments Each of $1/p$ Deposited at End of Each pth Part of the Period

$$s_{\overline{1}|i}^{(p)} = i/j_{(p)}$$

p	2%	2¼%	2½%	2¾%	p
2	1.0049 7525	1.0055 9371	1.0062 1142	1.0068 2837	2
3	1.0066 3733	1.0074 6292	1.0082 8761	1.0091 1141	3
4	1.0074 6856	1.0083 9839	1.0093 2677	1.0102 5422	4
6	1.0083 0125	1.0093 3444	1.0103 6665	1.0113 9789	6
12	1.0091 3389	1.0102 7107	1.0114 0725	1.0125 4243	12
13	1.0091 9796	1.0103 4314	1.0114 8732	1.0126 3051	13
26	1.0095 8243	1.0107 7565	1.0119 6786	1.0131 5908	26
52	1.0097 7470	1.0109 9195	1.0122 0819	1.0134 2343	52
365	1.0099 3960	1.0111 7746	1.0124 1431	1.0136 5016	365
∞	1.0099 6700	1.0112 0828	1.0124 4856	1.0136 8783	∞
p	3%	3½%	4%	4½%	p
2	1.0074 4458	1.0086 7475	1.0099 0195	1.0111 2621	2
3	1.0099 3431	1.0115 7748	1.0132 1713	1.0148 5328	3
4	1.0111 8072	1.0130 3094	1.0148 7744	1.0167 2026	4
6	1.0124 2816	1.0144 8578	1.0165 3957	1.0185 8953	6
12	1.0136 7662	1.0159 4203	1.0182 0351	1.0204 6109	12
13	1.0137 7270	1.0160 5410	1.0183 3158	1.0206 0515	13
26	1.0143 4929	1.0167 2674	1.0191 0023	1.0214 6980	26
52	1.0146 3757	1.0170 6316	1.0194 8470	1.0219 6231	52
365	1.0148 8501	1.0173 5172	1.0198 1447	1.0222 7330	365
∞	1.0149 2610	1.0173 9966	1.0198 6927	1.0223 3494	∞
p	5%	5½%	6%	6½%	p
2	1.0123 4754	1.0135 6596	1.0147 8151	1.0159 9419	2
3	1.0164 8597	1.0181 1522	1.0197 4104	1.0213 6348	3
4	1.0185 5942	1.0203 9495	1.0222 2688	1.0240 5523	4
6	1.0206 3570	1.0226 7810	1.0247 1676	1.0267 5172	6
12	1.0227 1479	1.0249 6465	1.0272 1070	1.0294 5294	12
13	1.0228 7484	1.0251 4068	1.0274 0270	1.0296 6093	13
26	1.0238 3548	1.0261 9729	1.0285 5526	1.0309 0941	26
52	1.0243 1602	1.0267 2586	1.0291 3186	1.0315 3404	52
365	1.0247 2822	1.0271 7928	1.0296 2648	1.0320 6987	365
∞	1.0247 9672	1.0272 5462	1.0297 0867	1.0321 5891	∞
p	7%	7½%	8%	8½%	p
2	1.0172 0402	1.0184 1103	1.0196 1524	1.0208 1667	2
3	1.0229 8254	1.0245 9826	1.0262 1065	1.0278 1974	3
4	1.0258 8002	1.0277 0129	1.0295 1904	1.0313 3332	4
6	1.0287 8298	1.0308 1059	1.0328 3456	1.0348 5492	6
12	1.0316 9143	1.0339 2617	1.0361 5721	1.0383 8455	12
13	1.0319 1538	1.0341 6609	1.0364 1309	1.0386 5642	13
26	1.0332 5978	1.0356 0640	1.0379 4927	1.0402 8845	26
52	1.0339 3242	1.0363 2705	1.0387 1794	1.0411 0511	52
365	1.0345 0947	1.0369 4530	1.0393 7739	1.0418 0577	365
∞	1.0346 0535	1.0370 4804	1.0394 8698	1.0419 2221	∞

Age x	l_x	d_x	$1000\ q_x$	D_x	Age x
0	1023102	23102	22 58	1023102 00	0
1	1000000	5770	5 77	975609 76	1
2	994230	4116	4 14	946322 43	2
3	990114	3347	3 38	919419 28	3
4	986767	2950	2 99	893962 20	4
5	983817	2715	2 76	869550 88	5
6	981102	2561	2 61	846001 18	6
7	978541	2417	2 47	823212 53	7
8	976124	2255	2 31	801150 42	8
9	973869	2065	2 12	779804 53	9
10	971804	1914	1 97	759171 73	10
11	969890	1852	1 91	739196 60	11
12	968038	1859	1 92	719790 36	12
13	966179	1913	1 98	700885 94	13
14	964266	1996	2 07	682437 28	14
15	962270	2069	2 15	664414 29	15
16	960201	2103	2 19	646815 33	16
17	958098	2156	2 25	629657 27	17
18	955942	2199	2 30	612917 42	18
19	953743	2260	2 37	596592 68	19
20	951483	2312	2 43	580662 42	20
21	949171	2382	2 51	565123 40	21
22	946789	2452	2 59	549956 28	22
23	944337	2531	2 68	535153 17	23
24	941806	2609	2 77	520701 32	24
25	939197	2705	2 88	506594 02	25
26	936492	2800	2 99	492814 61	26
27	933692	2904	3 11	479357 22	27
28	930788	3025	3 25	466211 03	28
29	927763	3154	3 40	453361 83	29
30	924609	3292	3 56	440800 58	30
31	921317	3437	3 73	428518 18	31
32	917880	3598	3 92	416506 91	32
33	914282	3767	4 12	404755 37	33
34	910515	3961	4 35	393256 29	34
35	906554	4161	4 59	381995 63	35
36	902393	4386	4 86	370968 10	36
37	898007	4625	5 15	360161 02	37
38	893382	4878	5 46	349566 90	38
39	888504	5162	5 81	339178 75	39
40	883342	5459	6 18	328983 61	40
41	877883	5785	6 59	318976 11	41
42	872098	6131	7 03	309145 51	42
43	865967	6503	7 51	299485 04	43
44	859464	6910	8 04	289986 39	44
45	852554	7340	8 61	280638 95	45
46	845214	7801	9 23	271436 89	46
47	837413	8299	9 91	262372 33	47
48	829114	8822	10 64	253436 24	48
49	820292	9392	11 45	244624 00	49

Age x	l_x	d_x	$1000\,q_x$	D_x	Age x
50	810900	9990	12 32	235925 04	50
51	800910	10628	13 27	227335 15	51
52	790282	11301	14 30	218847 25	52
53	778981	12020	15 43	210456 33	53
54	766961	12770	16 65	202155 03	54
55	754191	13560	17 98	193940 61	55
56	740631	14390	19 43	185808 43	56
57	726241	15251	21 00	177754 43	57
58	710990	16147	22 71	169777 17	58
59	694843	17072	24 57	161874 57	59
60	677771	18022	26 59	154046 23	60
61	659749	18988	28 78	146292 80	61
62	640761	19979	31 18	138616 97	62
63	620782	20958	33 76	131019 40	63
64	599824	21942	36 58	123508 39	64
65	577882	22907	39 64	116088 15	65
66	554975	23842	42 96	108767 29	66
67	531133	24730	46 56	101555 70	67
68	506403	25553	50 46	94465 545	68
69	480850	26302	54 70	87511 050	69
70	454548	26955	59 30	80706 625	70
71	427593	27481	64 27	74068 942	71
72	400112	27872	69 66	67618 148	72
73	372240	28104	75 50	61373 498	73
74	344136	28154	81 81	55355 921	74
75	315982	28009	88 64	49587 526	75
76	287973	27651	96 02	44089 787	76
77	260322	27071	103 99	38884 206	77
78	233251	26262	112 59	33990 850	78
79	206989	25224	121 86	29428 077	79
80	181765	23966	131 85	25211 636	80
81	157799	22502	142 60	21353 602	81
82	135297	20857	154 16	17862 047	82
83	114440	19062	166 57	14739 984	83
84	95378	17157	179 88	11985 151	84
85	78221	15185	194 13	9589 4746	85
86	63036	13198	209 37	7539 3905	86
87	49838	11245	225 63	5815 4632	87
88	38593	9378	243 00	4393 4773	88
89	29215	7638	261 44	3244 7546	89
90	21577	6063	280 99	2337 9929	90
91	15514	4681	301 73	1640 0309	91
92	10833	3506	323 64	1117 2571	92
93	7327	2540	346 66	737 2363	93
94	4787	1776	371 00	469 9158	94
95	3011	1193	396 21	288 3657	95
96	1818	813	447 19	169 8646	96
97	1005	551	548 26	91 6117	97
98	454	329	724 67	40 3755	98
99	125	125	1000 00	10 8454	99

Age x	N_x	C_x	M_x	Age x
0	31374229 80	22538 5366	257876 8839	0
1	30351127 80	5491 9691	235338 3473	1
2	29375518 04	3822 1152	229846 3782	2
3	28429195 61	3032 2168	226024 2630	3
4	27509776 33	2607 3702	222992 0462	4
5	26615814 13	2341 1360	220384 6760	5
6	25746263 25	2154 4803	218043 5400	6
7	24900262 07	1983 7445	215889 0597	7
8	24077049 54	1805 6425	213905 3152	8
9	23275899 12	1613 1747	212099 6727	9
10	22496094 59	1458 7451	210486 4980	10
11	21736922 86	1377 0655	209027 7529	11
12	20997726 26	1348 5565	207650 6874	12
13	20277935 90	1353 8821	206302 1309	13
14	19577049 96	1378 1693	204948 2488	14
15	18894612 68	1393 7300	203570 0795	15
16	18230198 39	1382 0812	202176 3495	16
17	17583383 06	1382 3537	200794 2683	17
18	16953725 79	1375 5355	199411 9146	18
19	16340808 37	1379 2123	198036 3791	19
20	15744215 69	1376 5331	196657 1668	20
21	15163553 27	1383 6196	195280 6337	21
22	14598429 87	1389 5416	193897 0141	22
23	14048473 59	1399 3275	192507 4725	23
24	13513320 42	1407 2700	191108 1450	24
25	12992619 10	1423 4649	189700 8750	25
26	12486025 08	1437 5192	188277 4101	26
27	11993210 47	1454 5491	186839 8909	27
28	11513853 25	1478 2003	185385 3418	28
29	11047642 22	1503 6464	183907 1415	29
30	10594280 39	1531 1580	182403 4951	30
31	10153479 81	1559 6094	180872 3371	31
32	9724961 63	1592 8453	179312 7277	32
33	9308454 72	1626 9874	177719 8824	33
34	8903699 35	1669 0508	176092 8950	34
35	8510443 06	1710 5610	174423 8442	35
36	8128447 43	1759 0801	172713 2832	36
37	7757479 33	1809 6928	170954 2031	37
38	7397318 31	1862 1345	169144 5103	38
39	7047751 41	1922 4869	167282 3758	39
40	6708572 66	1983 5110	165359 8889	40
41	6379589 05	2050 6947	163376 3779	41
42	6060612 94	2120 3381	161325 6832	42
43	5751467 43	2194 1367	159205 3451	43
44	5451982 39	2274 5951	157011 2084	44
45	5161996 00	2357 2099	154736 6133	45
46	4881357 05	2444 1542	152379 4034	46
47	4609920 16	2536 7650	149935 2492	47
48	4347547 83	2630 8594	147398 4842	48
49	4094111 59	2732 5292	144767 6248	49

Age x	N_x	C_x	M_x	Age x
50	3849487 59	2835 6221	142035 0956	50
51	3613562 55	2943 1374	139199 4735	51
52	3386227 40	3053 1772	136256 3361	52
53	3167380 15	3168 2229	133203 1589	53
54	2956923 82	3283 8121	130034 9360	54
55	2754768 79	3401 9131	126751 1239	55
56	2560828 18	3522 0901	123349 2108	56
57	2375019 75	3641 7835	119827 1207	57
58	2197265 32	3761 6968	116185 3372	58
59	2027488 15	3880 1854	112423 6404	59
60	1865613 58	3996 1999	108543 4550	60
61	1711567 35	4107 7080	104547 2551	61
62	1565274 55	4216 6760	100439 5471	62
63	1426657 58	4315 4138	96222 8711	63
64	1295638 18	4407 8312	91907 4573	64
65	1172129 79	4489 4497	87499 6261	65
66	1056041 64	4558 7282	83010 1764	66
67	947274 35	4613 1893	78451 4482	67
68	845718 651	4650 4521	73838 2589	68
69	751253 106	4670 0143	69187 8068	69
70	663742 056	4669 2260	64517 7925	70
71	583035 431	4644 2354	59848 5665	71
72	508966 489	4595 4281	55204 3311	72
73	441348 341	4520 6627	50608 9030	73
74	379974 843	4418 2492	46088 2403	74
75	324618 922	4288 2869	41669 9911	75
76	275031 396	4130 2202	37381 7042	76
77	230941 609	3944 9618	33251 4840	77
78	192057 403	3733 7258	29306 5222	78
79	158066 553	3498 6841	25572 7964	79
80	128638 476	3243 1158	22074 1123	80
81	103426 840	2970 7368	18830 9965	81
82	82073 238	2686 4020	15860 2597	82
83	64211 191	2395 3212	13173 8577	83
84	49471 207	2103 3561	10778 5365	84
85	37486 0561	1816 1946	8675 1804	85
86	27896 5815	1540 0394	6858 9858	86
87	20357 1910	1280 1454	5318 9464	87
88	14541 7278	1041 5646	4038 8010	88
89	10148 2505	827 6215	2997 2364	89
90	6903 4959	640 9377	2169 6149	90
91	4565 5030	482 7730	1528 6772	91
92	2925 4721	352 7707	1045 9042	92
93	1808 2150	249 3391	693 1335	93
94	1070 9787	170 0888	443 7944	94
95	601 0629	111 4678	273 7056	95
96	312 6972	74 1098	162 2378	96
97	142 8326	49 0019	88 1280	97
98	51 2209	28 5451	39 1261	98
99	10 8454	10 5810	10 5810	99

Age x	u_x	1000 k_x	1000 c_x	Age x
5	1 0278365	2 76730	2 69236	5
6	1 0276826	2 61716	2 54667	6
7	1 0275380	2 47612	2 40976	7
8	1 0273734	2 31551	2 25382	8
9	1 0271780	2 12491	2 06870	9
10	1 0270228	1 97342	1 92150	10
11	1 0269610	1 91315	1 86293	11
12	1 0269722	1 92407	1 87354	12
13	1 0270335	1 98389	1 93168	13
14	1 0271261	2 07426	2 01949	14
15	1 0272086	2 15476	2 09769	15
16	1 0272498	2 19497	2 13675	16
17	1 0273118	2 25537	2 19541	17
18	1 0273633	2 30565	2 24425	18
19	1 0274346	2 37524	2 31182	19
20	1 0274967	2 43581	2 37063	20
21	1 0275788	2 51587	2 44835	21
22	1 0276614	2 59653	2 52664	22
23	1 0277546	2 68739	2 61482	23
24	1 0278473	2 77790	2 70265	24
25	1 0279606	2 88844	2 80988	25
26	1 0280738	2 99885	2 91696	26
27	1 0281979	3 11994	3 03438	27
28	1 0283420	3 26053	3 17067	28
29	1 0284964	3 41117	3 31666	29
30	1 0286625	3 57315	3 47359	30
31	1 0288381	3 74450	3 63954	31
32	1 0290337	3 93533	3 82430	32
33	1 0292407	4 13722	4 01968	33
34	1 0294785	4 36929	4 24418	34
35	1 0297264	4 61107	4 47796	35
36	1 0300062	4 88415	4 74187	36
37	1 0303064	5 17696	5 02468	37
38	1 0306274	5 49013	5 32698	38
39	1 0309898	5 84372	5 66807	39
40	1 0313738	6 21837	6 02921	40
41	1 0317993	6 63343	6 42900	41
42	1 0322569	7 07995	6 85871	42
43	1 0327555	7 56634	7 32637	43
44	1 0333077	8 10506	7 84380	44
45	1 0339013	8 68419	8 39944	45
46	1 0345485	9 31559	9 00451	46
47	1 0352597	10 00948	9 66857	47
48	1 0360236	10 75471	10 38076	48
49	1 0368717	11 58219	11 17033	49
50	1 0377851	12 47331	12 01917	50
51	1 0387846	13 44836	12 94625	51
52	1 0398701	14 50741	13 95118	52
53	1 0410640	15 67224	15 05407	53
54	1 0423553	16 93205	16 24403	54

Age x	u_x	$1000\ k_x$	$1000\ c_x$	Age x
55	1 0437665	18 30871	17 54101	55
56	1 0453097	19 81436	18 95550	56
57	1 0469866	21 45037	20 48773	57
58	1 0488193	23 23834	22 15667	58
59	1 0508181	25 18845	23 97032	59
60	1 0529994	27 31645	25 94157	60
61	1 0553744	29 63351	28 07868	61
62	1 0579881	32 18360	30 41963	62
63	1 0608138	34 94025	32 93722	63
64	1 0639190	37 96969	35 68852	64
65	1 0673076	41 27573	38 67277	65
66	1 0710112	44 88895	41 91268	66
67	1 0750555	48 83462	45 42522	67
68	1 0794699	53 14131	49 22909	68
69	1 0843106	57 86408	53 36485	69
70	1 0896149	63 03892	57 85431	70
71	1 0954003	68 68327	62 70153	71
72	1 1017483	74 87642	67 96146	72
73	1 1087070	81 66539	73 65823	73
74	1 1163275	89 10001	79 81530	74
75	1 1246942	97 26259	86 47915	75
76	1 1338739	106 21845	93 67749	76
77	1 1439610	116 05952	101 45409	77
78	1 1550483	126 87631	109 84504	78
79	1 1672419	138 77259	118 88932	79
80	1 1806737	151 87675	128 63568	80
81	1 1954734	166 31558	139 12111	81
82	1 2118091	182 25271	150 39722	82
83	1 2298538	199 85741	162 50501	83
84	1 2498235	219 34008	175 49683	84
85	1 2719164	240 89409	189 39459	85
86	1 2964385	264 81800	204 26578	86
87	1 3236584	291 37409	220 12785	87
88	1 3540245	320 99948	237 07067	88
89	1 3878377	353 98805	255 06443	89
90	1 4255786	390 80832	274 14015	90
91	1 4679082	432 10561	294 36827	91
92	1 5154668	478 50417	315 74706	92
93	1 5688687	530 60372	338 20784	93
94	1 6295832	589 83728	361 95591	94
95	1 6976210	656 21561	386 55013	95
96	1 8541792	808 95523	436 28754	96
97	2 2689979	1213 65638	534 88655	97
98	3 7228000	2631 99997	706 99474	98
99			975 60976	99

ANSWERS

Exercise 1-1, page 5

1. 6, 2, 8, 2 **2.** 3, 7, -10, -2.5 **3.** -3, -3, -18, -2 **5.** $5x + 4y + 7w$
6. $5a + 5b + 5c$ **7.** $5b + c - 4w$ **9.** $3x + 3y + w$ **10.** $5a - 5b + (4 - c)w$
11. $2c - d + (2 + a)x$ **13.** $-2y + 3w$ **14.** $2a + b + c$ **15.** $x + 8w$
17. $9a + 5b$ **18.** $x + y$ **19.** $a - b$ **21.** $-6a + 22b$ **22.** $12x + 12y$
23. $3x - 6y$ **25.** $-4a - 36b$ **26.** $-24a + 12b$ **27.** $2x + 2y$

Exercise 1-2, page 8

1. a^7 **2.** a^7 **3.** $6x^5$ **5.** $-21x^7$ **6.** $-8y^9$ **7.** $14b^5$ **9.** $6a^3b^4$
10. $-24x^5y^3$ **11.** $-15c^5d^3$ **13.** $2x^3 + 7x^2 - 9$ **14.** $6x^3 - 7x^2 + 14x - 8$
15. $15x^3 + 4x^2 - 9x + 2$ **17.** $6x^4 - 10x^3 - 13x^2 + 12x + 5$
18. $6x^4 - 19x^3 + 18x^2 - 16x - 10$ **19.** $10x^4 - 21x^3 + 4x^2 + 18x - 9$
21. $2x^5 + x^4 - 3x^3 + 12x^2 - 10x + 4$ **22.** $12x^5 - 17x^4 - 13x^3 + 22x^2 - x - 3$
23. $2x^5 - 7x^4 + 16x^3 - 9x^2 + 16$ **25.** $12x - 8y$ **26.** $-4a + 10b$
27. $-6x + 15y$ **29.** $4a^2 + 4ab + b^2$ **30.** $9a^2 - 12ab + 4b^2$
31. $9x^2 + 24xy + 16y^2$ **33.** $4a^2 - 9b^2$ **34.** $9a^2 - 25b^2$ **35.** $x^4 - 4y^2$
37. $6x^2 - 5xy - 6y^2$ **38.** $10a^2 - 19ab - 15b^2$ **39.** $6x^2 - 11xy - 35y^2$

Exercise 1-3, page 12

1. a^4 **2.** b^7 **3.** c^5 **5.** $3x^3$ **6.** $5x^5$ **7.** $6y^2$ **9.** $2x - 3$ **10.** $x - 5$
11. $2x^2 - 5x - 7$ **13.** $3(x - 4)$ **14.** $5(x + 3)$ **15.** $7(x + 4y)$
17. $(x - 2)(x + 4)$ **18.** $(x - 1)(x + 2)$ **19.** $(x + 3)(x + 5)$
21. $(2x - 1)(x + 1)$ **22.** $(3x - 2)(x + 1)$ **23.** $(2x - 3)(3x - 4)$
25. $(3x - 4)^2$ **26.** $(5x + 3)^2$ **27.** $(4x + 5)^2$ **29.** $(x + 4)(x - 4)$
30. $(2x + 7)(2x - 7)$ **31.** $(5a + 2b)(5a - 2b)$ **33.** $x(3x + 1)(2x + 1)$
34. $x(2x + 1)(x + 3)$ **35.** $2(3x + 2)(2x - 1)$

Exercise 2-1, page 15

1. 3 **2.** -4 **3.** -5 **5.** 2 **6.** 3 **7.** -1 **9.** 5 **10.** -3 **11.** -2
13. 2 **14.** -3 **15.** 0 **17.** 2 **18.** -1 **19.** 0 **21.** $\dfrac{3}{5 - 2m}$
22. $\dfrac{3a + 5c}{5}$ **23.** $\dfrac{b(a - x)}{a}$ **25.** I/rt **26.** $S/(1 + rt)$ **27.** $5/(1 + i)^n$

Exercise 2-2, page 17

1. 13, 6 **2.** 7, 8 **3.** 7, 16 **5.** \$70, \$90 **6.** \$2200, \$2700
7. 35 years, 5 years **9.** 11 **10.** 10 **11.** 3, 9 **13.** County \$24.40, city \$37.20, state \$12.20 **14.** Food \$80, rent \$70, clothes \$40
15. \$2300, \$2700, \$3400 **17.** 25 **18.** 60 **19.** 21

Exercise 3-1, page 22

1. 10/27 **2.** 15/14 **3.** 30/77 **5.** 15/8 **6.** 30/77 **7.** 8/45
9. $\dfrac{2x^2 + 5x}{3x^2 + x - 2}$ **10.** $\dfrac{10x^2 + x - 3}{3x^2 - 11x + 10}$ **11.** $\dfrac{6x^2 + 5x - 4}{4x^2 - 11x + 6}$ **13.** $\dfrac{x^2 + 2x - 3}{10x^2 - 21x - 10}$
14. $\dfrac{2x^2 - 3x - 14}{5x^2 + 17x - 12}$ **15.** $\dfrac{2x^2 - x - 1}{2x^2 + x - 1}$ **17.** $\frac{2}{5}$ **18.** $\frac{4}{5}$ **19.** $\frac{3}{5}$ **21.** $\dfrac{x - 1}{x + 2}$
22. $\dfrac{2x + 1}{2x + 3}$ **23.** $\dfrac{2x + 3}{2x - 3}$ **25.** $\frac{1}{5}$ **26.** $\frac{1}{28}$ **27.** $\frac{4}{21}$ **29.** $\frac{3}{2}$ **30.** $\frac{3}{2}$ **31.** $\frac{10}{3}$ **33.** 1
34. $\dfrac{3x + 2}{x + 3}$ **35.** $\dfrac{2x - 1}{2x + 1}$ **37.** $\dfrac{(2x - 1)^2}{x^2 - 4}$ **38.** $\dfrac{3x + 2}{2x + 3}$ **39.** $\dfrac{2x - 1}{2x + 1}$

Exercise 3-2, page 24

1. 24 **2.** 30 **3.** 156 **5.** $6x^2y^2$ **6.** $15a^3b^3$ **7.** $6a^2b^2c^3$
9. $(2x - 1)(2x + 1)(3x - 4)^2$ **10.** $(x + 2)^2(x - 2)^2$
11. $(2x + 1)^3(x - 3)^2(2x + 3)^2$ **13.** $x(2x - 1)^2(x - 2)$
14. $x(3x + 2)^2(3x - 2)$ **15.** $(x - 1)^2(x + 3)^2$ **17.** 11/18 **18.** 8/15
19. 4/21 **21.** $\dfrac{3a + 2b - 1}{ab}$ **22.** $\dfrac{3y - 2x + 5}{xy}$ **23.** $\dfrac{4xy - 3x^2 - 2}{x^2y}$
25. $\dfrac{5x}{(x - 1)(x - 2)}$ **26.** $\dfrac{7}{(2x + 1)(x + 3)}$ **27.** 0 **29.** $\dfrac{2}{2x - 1}$ **30.** $\dfrac{1}{3x + 2}$
31. $\dfrac{-2(x + 2)}{5x + 3}$ **33.** $\dfrac{45x}{(5x + 1)^2(5x - 2)}$ **34.** $\dfrac{2x^2 + 4x + 3}{(2x + 1)^2(2x + 2)}$
35. $\dfrac{4x - 1}{(x - 1)^2(x + 2)}$

Exercise 3-3, page 27

1. 2 **2.** 3 **3.** 4 **5.** 1 **6.** 2 **7.** 3 **9.** 2 **10.** 3 **11.** 1 **13.** 3
14. 2 **15.** -2 **17.** $d = \dfrac{l - a}{n - 1}$, $n = \dfrac{l - a + d}{d}$

18. $l = \dfrac{a + rS - S}{r}$, $r = \dfrac{S - a}{S - l}$ **19.** $t = \dfrac{S - P}{Pr}$, $r = \dfrac{S - P}{Pt}$

21. Wife got \$6000; younger son got \$4000; daughter got \$4500; older son got \$3500

22. Robert had 20; Tom had 15 **23.** 1/2 **25.** $3\frac{3}{7}$ days **26.** 96 **27.** 2 mi.

Exercise 4-1, page 31

1. 23.5% **2.** 30.25% **3.** 57.1% **5.** 2.5% **6.** 305.2% **7.** 3271.4% **9.** .023 **10.** .0583 **11.** .623 **13.** .386 **14.** .001 **15.** .0063 **17.** 85.02 **18.** 20.748 **19.** .11026 **21.** .492 **22.** .618 **23.** 1.049 **25.** 8.55 **26.** .907 **27.** 100 **29.** 12 **30.** \$6072.21 **31.** \$52 **33.** \$3200 **34.** \$61,441.44 **35.** \$220

Exercise 4-2, page 33

1. \$801.66 **2.** \$563.28 **3.** \$1153.35 **5.** \$1763.39 **6.** \$1563.36 **7.** \$2932.27 **9.** \$1342.95, \$137.74 **10.** \$1899.78, \$212.38 **11.** \$2516.24, \$291.91 **13.** \$17.15, \$64.14, \$14.41 **14.** \$17.84, \$75.14, \$13.28 **15.** \$58.97, \$397.66, \$69.55 **17.** \$11.73, \$94.18, \$14.14 **18.** \$9.24, \$90.52, \$10.20 **19.** \$29.92, \$176.08, \$40.17 **21.** 31 mils **22.** 27 mils **23.** 31.4 mils **25.** \$7200 **26.** \$13,500 **27.** \$3300 **29.** \$1896.26 **30.** \$558.54 **31.** \$1526.10

Exercise 4-3, page 38

1. \$93.84 **2.** \$189.54 **3.** \$527.31 **5.** 37% **6.** $83\frac{1}{3}$% **7.** 53.125% **9.** \$126.90 **10.** \$186 **11.** \$283.92 **13.** \$200 **14.** \$175 **15.** \$724 **17.** (a) \$612.88; (b) \$625.92; (c) \$652 **18.** (a) \$272.65; (b) \$278.39; (c) \$287 **19.** \$233.97 **21.** \$541.26 **22.** \$536.37 **23.** \$1411.78 **25.** \$1197.72 **26.** \$975.58 **27.** \$192.63

Exercise 4-4, page 40

1. \$160 **2.** \$255 **3.** \$600 **5.** \$151.47 **6.** \$627 **7.** \$700 **9.** \$576 **10.** \$862.50 **11.** \$2828.57

Exercise 5-1, page 45

1. 8 **2.** 9 **3.** 625 **5.** 32 **6.** 243 **7.** 125 **9.** 16 **10.** 81 **11.** 343 **13.** 216 **14.** 225 **15.** 10,000 **17.** 81/16 **18.** 125/343 **19.** 36/49 **21.** 64 **22.** 729 **23.** 512 **25.** m^5 **26.** a^7 **27.** b^{m+p} **29.** $4a^3b$ **30.** $3a^3b^6$ **31.** $4r^3s$ **33.** $(1+i)^4$ **34.** $(1+i)^2$ **35.** 1.02^2 **37.** x^{5n+1} **38.** a^7x^8 **39.** b^{2n}

Exercise 5-2, page 47

1. $\frac{1}{8}$ **2.** $\frac{1}{9}$ **3.** $\frac{1}{16}$ **5.** 2 **6.** 2 **7.** 2 **9.** 8 **10.** 9 **11.** 8 **13.** $1/x^8$ **14.** $1/y^5$ **15.** $1/a$ **17.** $\dfrac{1}{x + y}$ **18.** $\dfrac{1}{(1+i)^3}$ **19.** $\dfrac{1}{(1+i)^5}$ **21.** $\dfrac{x^2y}{y - x^2}$ **22.** $\dfrac{xy^3}{y^3 - x^2}$ **23.** $\dfrac{xy + x^3}{y}$ **25.** $1.03^2 - 1$ **26.** $1.04^3 - 1$

27. $1.02^7 - 1$ **29.** $\sqrt[3]{(a + x)^2}$ **30.** $\sqrt{1.02}$ **31.** $\sqrt[4]{1.02^3}$ **33.** $1.03^{1/2}$
34. $1.02^{1/3}$ **35.** $1.01^{1/4}$ **37.** $1/(1 + i)^{2/3}$ **38.** $(1 + i)^{3/4}$ **39.** $1/(1 + i)^{5/6}$

Exercise 5-3, page 50

1. $\sqrt{15}$ **2.** $\sqrt{14}$ **3.** $\sqrt{55}$ **5.** $\sqrt{2}$ **6.** $\sqrt{3}$ **7.** $\sqrt{5}$ **9.** 9 **10.** 28
11. 4 **13.** $a^3b^2 \sqrt{ab}$ **14.** $4xy^2 \sqrt{xy}$ **15.** $3xy^2 \sqrt{10xy}$ **17.** $ab \sqrt{5}$
18. $2xy \sqrt{x}$ **19.** $3ax^2 \sqrt{a}$ **21.** 4 **22.** 3 **23.** 2 **25.** $\dfrac{\sqrt{2zx}}{2z}$
26. $\dfrac{\sqrt{6ac}}{3c}$ **27.** $\dfrac{\sqrt{15bd}}{5d}$ **29.** $\dfrac{y\sqrt{70x}}{10x}$ **30.** $\dfrac{\sqrt[3]{294y}}{7x}$ **31.** $\dfrac{\sqrt[3]{36a^2b}}{2b^2}$
33. $6(2 + \sqrt{3})$ **34.** $7(3 - \sqrt{2})$ **35.** $3(\sqrt{7} + \sqrt{2})$

Exercise 6-1, page 53

1. $4^2 = 16$ **2.** $2^3 = 8$ **3.** $2^7 = 128$ **5.** $b^2 = 17$ **6.** $3^{2.7} = N$ **7.** $3^l = 72$
9. $\log_3 81 = 4$ **10.** $\log_2 128 = 7$ **11.** $\log_5 125 = 3$ **13.** $\log_b 183 = 5$
14. $\log_3 407 = a$ **15.** $\log_{13} N = 1.9$ **17.** 3 **18.** 4 **19.** $\frac{1}{2}$ **21.** $\frac{3}{2}$
22. $\frac{2}{3}$ **23.** -2 **25.** 9 **26.** 64 **27.** $\frac{1}{4}$ **29.** 3 **30.** 8 **31.** $\frac{1}{4}$
33. 4 **34.** 4 **35.** 8

Exercise 6-2, page 56

1. 0 **2.** 1 **3.** -1 **5.** 3 **6.** -2 **7.** 3 **9.** -3 **10.** 4 **11.** -4
13. 0.7686 **14.** 0.4713 **15.** 1.6010 **17.** 2.9943 **18.** 2.6053 **19.** 0.8591
21. 9.7050 − 10 **22.** 8.5465 − 10 **23.** 7.7143 − 10 **25.** 25.3
26. 1.96 **27.** 247 **29.** 38.6 **30.** 4.97 **31.** 999 **33.** .852 **34.** .00306
35. .0243

Exercise 6-3, page 59

1. 2.4113 **2.** 1.5153 **3.** 0.2751 **5.** 3.5901 **6.** 8.7325 − 10
7. 9.4741 − 10 **9.** 17.24 **10.** 608.6 **11.** 1.939 **13.** .1535 **14.** .02654
15. .4412 **17.** 34280 **18.** 50.97 **19.** 73.28 **21.** 67.36 **22.** 67.36
23. 491.9 **25.** 1.0913 **26.** 8.1329 − 10 **27.** 0.9896 **29.** 1.5675
30. 2.9408 **31.** 8.4140 − 10

Exercise 6-4, page 63

1. $\log a + \log b$ **2.** $\log 2 + \log c + \log d$ **3.** $2 \log c + \log d$
5. $\log a + \log b - \log c$ **6.** $\log a - \log b - \log c$ **7.** $\frac{1}{2} \log a + \frac{1}{2} \log b$
9. 82.9 **10.** 46.5 **11.** 739 **13.** 1.66 **14.** .795 **15.** 1.14 **17.** 1.28
18. 1.63 **19.** .722 **21.** 9.68 **22.** .897 **23.** .985 **25.** 96.02 **26.** 21.64
27. 2970 **29.** 43.20 **30.** .07967 **31.** 5.632

Exercise 7-1, page 67

1. $-1, 3, -2$ **2.** $2, -1, 0$ **3.** $1, -2, 13$ **5.** $4, 2, 1.75$ **6.** $1, 10, -\frac{2}{3}$
7. $43, 3, 1$ **9.** $3, -1$ **10.** $0, -3$ **11.** $5, 3$ **13.** $x^2 + x + 1, 4x^2 - 6x + 3$

14. $6x^2 + 14x + 7,\ 6x^2 - 22x + 19$ **15.** $x^2 - 3x,\ x^2 + 5x + 4$ **17.** 32, 27
18. 9, -3 **19.** 12, 44 **21.** \$224.97, \$416.16 **22.** \$272.11, \$457.57
23. \$101.50, \$218.51 **25.** 0, 3, -2 **26.** 0, 5, 13/8 **27.** 0, 0, 10

Exercise 7-2, page 71

1. First, fourth **2.** Second, third **3.** Fourth, second

Exercise 8-1, page 75

1. 1, 3, 5, 7, 9, 11, 13 **2.** 8, 6, 4, 2, 0, -2 **3.** $-9, -6, -3, 0, 3, 6, 9, 12$
5. 3, 5, 7, 9, 11, 13 **6.** 2, 3, 4, 5, 6, 7, 8, 9 **7.** $-1, 1, 3, 5, 7, 9, 11$
9. $l = 12,\ s = 77$ **10.** $l = -3,\ s = 0$ **11.** $l = -9,\ s = -9$
13. $l = 11,\ s = 32$ **14.** $l = 12,\ s = -21$ **15.** $l = -8,\ s = 18$
17. 625 **18.** 650 **19.** 1755 **21.** \$2.55 **22.** Gains \$3.25 by choosing \$9.25 per day **23.** Gets an extra \$8 at the variable rate

Exercise 8-2, page 78

1. 1, 2, 4, 8, 16, 32 **2.** 2, 4, 8, 16, 32, 64, 128 **3.** 128, 64, 32, 16, 8, 4, 2, 1
5. $\frac{1}{4}, \frac{1}{12}, \frac{1}{36}, \frac{1}{108}, \frac{1}{324}, \frac{1}{972}$ **6.** $\frac{1}{32}, \frac{1}{16}, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4$
7. $81,\ \pm 27,\ 9,\ \pm 3,\ 1,\ \pm\frac{1}{3},\ \frac{1}{9}$ **9.** $l = 4,\ s = 7\frac{7}{8}$ **10.** $l = 5,\ s = 6\frac{81}{125}$
11. $l = \frac{1}{3},\ s = 364\frac{4}{27}$ **13.** $l = \frac{1}{3},\ s = 20\frac{1}{3}$ **14.** $l = 1,\ s = 13\frac{3}{16}$
15. $l = 1,\ s = 127, 43$ **17.** 2046 **18.** 88, 572 **19.** $\frac{63}{64}$ **21.** $\dfrac{1.03^6 - 1}{.03}$
22. $\dfrac{1.04^5 - 1}{.04}$ **23.** $\dfrac{1.01^{18} - 1}{.01}$ **25.** $\dfrac{1 - 1.02^{-5}}{1 - 1.02^{-1}}$ **26.** $\dfrac{1 - 1.04^{-6}}{1 - 1.04^{-1}}$
27. $\dfrac{1 - 1.01^{-20}}{1 - 1.01^{-1}}$ **29.** \$655.35 **30.** \$1.15 **31.** Two

Exercise 8-3, page 81

1. $a^7 + 7a^6 + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + b^7$
2. $a^4 + 12a^3 + 54a^2 + 108a + 81$ **3.** $1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5$
5. $x^6 - 6x^5y + 15x^4y^2 - 20x^3y^3 + 15x^2y^4 - 6xy^5 + y^6$
6. $x^4 - 12x^3 + 54x^2 - 108x + 81$ **7.** $1 - 5a + 10a^2 - 10a^3 + 5a^4 - a^5$
9. $a^5 - 10a^4b + 40a^3b^2 - 80a^2b^3 + 80ab^4 - 32b^5$
10. $81x^4 - 108x^3y + 54x^2y^2 - 12xy^3 + y^4$
11. $64x^6 - 192x^5y + 240x^4y^2 - 160x^3y^3 + 60x^2y^4 - 12xy^5 + y^6$
13. $x^6 - 6x^4 + 12x^2 - 8$ **14.** $x^4 + 4x^3y^3 + 6x^2y^6 + 4xy^9 + y^{12}$
15. $a^{10} + 5a^8b^3 + 10a^6b^6 + 10a^4b^9 + 5a^2b^{12} + b^{15}$
17. $a^{1/2} + \frac{1}{2}a^{-1/2}b - \frac{1}{8}a^{-3/2}b^2 + \frac{1}{16}a^{-5/2}b^3$ **18.** $1^{2/3} - \frac{2}{3}a - \frac{1}{9}a^2 - \frac{4}{81}a^3$
19. $2^{-1/4}[2 - \frac{3}{4}x - \frac{3}{64}x^2 - \frac{5}{512}x^3]$ **21.** $1 - 2x + 3x^2 - 4x^3$
22. $1 + 3x + 6x^2 + 10x^3$ **23.** $\frac{1}{2} - \frac{1}{4}x + \frac{1}{8}x^2 - \frac{1}{16}x^3$ **25.** 1.061208
26. 1.15927 **27.** 1.072135 **29.** .94232 **30.** .862555 **31.** .86375

33. 1.019804 **34.** 1.0049629 **35.** 1.0097592 **37.** .9950616
38. .9758984 **39.** .990195

Exercise 9-1, page 86

1. Mean, 76; median, 78; mode, 78 **2.** Mean, 27; median, 27; mode, 28
3. Mean, \$2.14; median, \$2.11; mode, \$2.13
5. Mean, \$1.04; median, \$1.08; mode, \$1.18; mode too near highest rate to be representative average.
6. Mean, \$5500; median, \$5300; mode, \$5300; arithmetic mean not a desirable average since one item is much larger than any other
7. Mean, 27; median, 14; mode, 10; arithmetic mean not a representative average because one fisherman far excelled others; mode not a desirable average since it is one of the smaller numbers
9. Mean, 36.04; median, 37; mode, 47 **10.** Mean, 144.3; median, 147; mode, 156
11. Mean, \$327.83; median, \$299.50; mode, \$299.50

Exercise 9-2, page 90

1. 11.7 **2.** 19.3 **3.** .54 **5.** 8.2 **6.** \$180.06 **7.** 4 in.
9. About 100 between 38 and 54; about 142 between 30 and 62; about 148 between 22 and 70
10. About 600 between 73 and 85; about 855 between 67 and 91; about 891 between 61 and 97
11. About 1200 between 110 and 124; about 1710 between 103 and 131; about 1782 between 96 and 138.

Exercise 9-3, page 95

1. .978 **2.** .799 **3.** .957 **5.** .855 **6.** $-.466$ **7.** .732 **9.** .796
10. $-.429$ **11.** .653

Exercise 10-1, page 102

1. \$30.79, \$1057.21 **2.** \$1.68, \$123.98 **3.** \$22.27, \$857.41
5. 120 days, 120 days **6.** 130 days, 127 days **7.** 117 days, 116 days
9. I_o (approx.) \$21.67, I_o (exact) \$21.67, I_e (approx.) \$21.37, I_e (exact) \$21.37
10. I_o (approx.) \$27.34, I_o (exact) \$27.99, I_e (approx.) \$26.97, I_e (exact) \$27.60
11. I_o (approx.) \$6.32, I_o (exact) \$6.32, I_e (approx.) \$6.23, I_e (exact) \$6.23
13. \$17.52 **14.** \$164.44 **15.** \$326.78 **17.** \$17.04 **18.** \$159.97
19. \$317.88 **21.** \$12.82, \$1038.24 **22.** \$16.88, \$766.88
23. \$13.17, \$5281.36

Exercise 10-2, page 104

1. (a) $P = \frac{I}{rt}$; (b) $r = \frac{I}{Pt}$; (c) $t = \frac{I}{Pr}$ **2.** 7% **3.** 2.97% **5.** 6.5%
6. \$414 **7.** \$683.20 **9.** \$1555.56 **10.** 6 mo. **11.** 1 yr. **13.** 2 mo.
14. 2.83% **15.** 3.11% **17.** 3.40% **18.** 3.49% **19.** 3.57% **21.** 3.74%

Exercise 10-3, page 105

1. $418, 3% **2.** $660, 4% **3.** $660, $2\frac{1}{2}$ yrs. **5.** $900, 4% **6.** $225, 4% **7.** $900, $126 **9.** $7.12, 6% **10.** $5.27, 4% **11.** $22.13, 1 mo.

Exercise 10-4, page 111

1. 6.185% **2.** 8.219% **3.** 4.040% **5.** 4.938% **6.** 5.660% **7.** 3.480% **9.** $980 **10.** $505.81 **11.** $250.39 **13.** $989.90, 6.061% **14.** 8% **15.** 6%

Exercise 10-5, page 117

1. $20,000 **2.** $55,555.56 **3.** $25,000 **5.** $120,000 **6.** $4,000,000 **7.** $350,000 **9.** 9.986%, 10.511% **10.** 10.450%, 11.669% **11.** 10.465%, 12.040% **13.** 22.308%, 28.713% **14.** 22.857%, 29.629% **15.** 22.564%, 29.139% **17.** 19.919%, 26.523% **18.** 19.792%, 26.298% **19.** 19.792%, 26.298% **21.** 19.792%, 26.298% **22.** 19.758%, 26.238% **23.** 19.756%, 26.235% **25.** 56.76% **26.** 34.44% **27.** 24.81% **29.** 21.29% **30.** 19.43% **31.** 17.17% **33.** 14.46% **34.** 14.76% **35.** 13.58%

Exercise 10-6, page 120

1. (a) $41.17, $2057.45; (b) $4.33, $328.83; (c) $135, $885; (d) $16.44, $1002.74 **2.** I_o (approx.) $2.63, I_e (exact) $2.68, I_o (approx.) $2.60, I_e (exact) $2.65 **3.** $1494.93 **5.** 19.251%, 19.715% **6.** 323.08% **7.** 9.468% **9.** 14.932% **10.** 24.740% **11.** 14.526% **13.** (a) First $10.36 better; (b) first $10.73 better; (c) first $10.40 better **14.** 3.015% **15.** 5.063% **17.** 8.333% **18.** 6.122% **19.** 2.985% **21.** 6.103% **22.** 7.692% **23.** 5.882% **25.** 14.159% **26.** 13.609% **27.** 13.980%

Exercise 11-1, page 127

1. $1275.08 **2.** $1283.77 **3.** $100.79 **5.** $1144.90, $1402.55, $1967.15, $29,457.03 **6.** $1149.81, $1417.63, $2009.66 **7.** $868.22, $1055.33, $1989.97, $4137.01 **9.** $2927.15 **10.** $2906.25 **11.** $1282.21 **13.** $1097.97 **14.** $1084.17 **15.** $2584.19 **17.** Time payment by $464.96 **18.** Time payment by $471.44 **19.** $1429.70

Exercise 11-2, page 133

In Problems 1, 2, 3 the accumulated value at simple interest is given in parentheses:
1. ($106), $106; ($112), $112.36; ($130), $133.82; ($160), $179.08; ($190), $239.66; ($220), $320.71; ($250), $429.19 **2.** ($105), $105; ($120), $121.55; ($140), $147.75; ($160), $179.59; ($180), $218.29; ($200), $265.33; ($220), $322.51
3. ($104), $104; ($112), $112.49; ($124), $126.53; ($140), $148.02; ($160), $180.09; ($184), $227.88

In Problems 5, 6, 7 the first figure was obtained by interpolation, the second by using logarithms:

5. (a) 23.45 yrs., 23.44 yrs.; (b) 17.67 yrs., 17.68 yrs.; (c) 14.20 yrs., 14.21 yrs.; (d) 11.89 yrs., 11.89 yrs. **6.** (a) 23.28 yrs., 23.26 yrs.; (b) 17.5 yrs., 17.5 yrs.; (c) 14.04 yrs., 14.04 yrs.; (d) 23.45 yrs., 23.44 yrs. **7.** (a) 26.76 yrs., 26.77 yrs.; (b) 23.85 yrs., 23.85 yrs.; (c) 19.60 yrs., 19.61 yrs.; (d) 13.63 yrs., 13.64 yrs.
9. 19.09 yrs. **10.** 18.59 yrs. **11.** 34.56 yrs. **13.** 25 yrs. **14.** 24.67 yrs. **15.** 6.63 yrs.

Exercise 11-3, page 136

1. 3.18%, 4% **2.** .77% **3.** 3.80% **5.** 2.92%, 3.33% **6.** 1.52% **7.** 1.26% **9.** 1.79%, 1.88% **10.** .15% **11.** 3.77% **13.** 2.45% **14.** 4.35% **15.** Yes

Exercise 11-4, page 142

1. 5.84% **2.** 4.94% **3.** 3.94% **5.** 6.17% **6.** 5.06% **7.** 4.06% **9.** \$548.28 **10.** \$9.68 **11.** \$2000.17 **13.** \$551.30 **14.** \$5637.72 **15.** \$741.10 **17.** \$944.12 **18.** \$7646.87 **19.** \$7195.90 **21.** 3.65% **22.** 5.76% **23.** 3.97%

Exercise 11-5, page 147

The first figure was obtained by the approximate method, the second by the exact method:

1. \$681.67, \$681.57 **2.** \$1085.88, \$1085.78 **3.** \$5821.29, \$5820.45 **5.** \$7327.11, \$7326.93 **6.** \$1021.39, \$1021.39 **7.** \$756.85, 756.84 **9.** \$571.13, \$571.23 **10.** \$858.32, \$858.37 **11.** \$2565.18, \$2565.21 **13.** \$1061.42, \$1061.61 **14.** \$620.96, \$621.00 **15.** \$683.20, 683.21

Exercise 11-6, page 149

1. \$933.88 **2.** \$1398.72 **3.** \$22,830.66 **5.** \$719.29 **6.** \$5085.20 **7.** \$6976.29 **9.** \$4181.79 **10.** \$7207.81 **11.** \$7111.49

Exercise 11-7, page 151

1. \$2020.28 **2.** \$5071.50 **3.** \$1820.29 **5.** \$1654.46 **6.** \$1102.27 **7.** \$8024.51 **9.** 6.09% **10.** 4.06% **11.** 4.89% **13.** (a) 2.77%; (b) 2.75%; (c) 2.74% **14.** (a) 4.08%; (b) 4.01% **15.** (a) 2.81%; (b) 2.77% **17.** (a) \$3923.40; (b) \$3924.57 **18.** (a) \$850.74; (b) \$850.75 **19.** (a) \$12,273.45; (b) \$14,674.36; (c) \$15,574.83

Exercise 12-1, page 158

1. \$6231.11 **2.** \$9501.86 **3.** \$17,807.29 **5.** \$9942.36 **6.** \$8673.20 **7.** \$2586.28 **9.** \$11,503.97 **10.** \$28,596.72 **11.** \$59,509.90 **13.** \$13,137.76 **14.** (b) by \$10.58 **15.** Offer by \$134.36

Exercise 12-2, page 160

1. \$9411.43 **2.** \$2293.32 **3.** \$5100.46 **5.** \$83,367.90 **6.** \$100,562.19
7. \$107,061.56 **9.** \$896,239.46 **10.** (a) \$23,414.44; (b) \$36,459.26
11. (a) \$23,637.98; (b) \$36,841.41 **13.** \$17,456.98 **14.** \$15,450.42
15. \$2401.22 **17.** \$4168.21 **18.** \$21,634.28 **19.** \$5238.86

Exercise 12-3, page 164

1. .07365013, .05365013 **2.** .06227283, .02727283 **3.** .16746776, .09746776
5. .08024259, \$80.24 **6.** .03024259, \$30.24 **7.** .10635278, \$1063.53
9. (a) \$641.94; (b) \$588.65 **10.** (a) \$241.94; (b) \$268.65
11. (a) \$537.11; (b) \$511.03 **13.** (a) \$224.85; (b) \$209.42
14. (a) \$124.85; (b) \$134.42 **15.** (a) \$745.99; (b) \$542.76
17. (a) \$313.01; (b) \$292.63 **18.** \$117,230.51 **19.** \$1862.28
21. (a) \$691.89; (b) \$784.66 **22.** (a) \$2145.26; (b) \$1495.08 **23.** \$83,158.63

Exercise 12-4, page 168

1. 20 yrs. **2.** 12 yrs. **3.** 10 yrs. **5.** 15 yrs., \$689.61, \$730.99
6. 8, \$473.63 **7.** 13, \$602.05 **9.** 8, \$125,338.55 **10.** 10, \$5.46
11. 44, \$426.05

Exercise 12-5, page 170

1. 6.64% **2.** 2.63% **3.** 5.07% **5.** 4.49% **6.** 4.08% **7.** 5.70%
9. 2.57%, 10.29% **10.** 2.79%, 5.58% **11.** 9.70% **13.** 5.59% **14.** 3.32%
15. 5.53% **17.** 3.27% **18.** 6.10% **19.** 4.02% **21.** 9.29%
22. 9.33% **23.** 9.30%

Exercise 12-6, page 175

1. \$1560.34 **2.** \$4811.39 **3.** \$4291.39 **5.** \$4847.35 **6.** \$18,697.02
7. \$7293.53 **9.** \$13,395.01 **10.** \$121.37 **11.** \$446.50 **13.** \$93.40
14. \$11,308.26 **15.** \$11,080.55

Exercise 12-7, page 179

1. \$1000.55 **2.** \$1876.52 **3.** \$13,595.58 **5.** \$9138.50 **6.** \$1620.36
7. \$2331.08 **9.** \$278.92 **10.** \$1087.11 **11.** \$1050.84 **13.** \$11,766.15
14. \$34,273.61 **15.** \$6524.51 **17.** \$27,710.61 **18.** \$14,603.25
19. Deferred payment plan, \$3621.32

Exercise 12-8, page 181

1. \$9118.49 **2.** \$573.33 **3.** \$11,684.96 **5.** \$46,925.37 **6.** \$120,287.28
7. \$13,617.02 **9.** \$5365.43 **10.** \$12,787.49 **11.** \$11,403.73 **13.** \$5132.00
14. \$13,085.32 **15.** \$2112.80 **17.** \$20,519.31 **18.** \$49,155.29
19. \$11,207.22 **21.** \$17,077.44 **22.** 3.21% **23.** 2.28% **25.** 6.79%
26. No **27.** 3.40% **29.** 4.22% **30.** \$3,176.35 **31.** \$9956.17

33. 5, \$23.10 **34.** \$950.28 **35.** 4.02% **37.** 3.05% **38.** 3.22%
39. 4.02% **41.** 3.53% **42.** 32 **43.** \$222.01

Exercise 13-1, page 187

1. \$368.25 **2.** \$342.78 **3.** \$180.52 **5.** \$97.37 **6.** \$71.91 **7.** \$84.39
9. \$2062.73 **10.** \$1289.53 **11.** \$700.07 **13.** \$4146.05 **14.** \$4586.54
15. \$2345.54 **17.** \$8664.69 **18.** \$4126.45 **19.** \$8913.43

Exercise 13-2, page 193

Only the periodic rent is given:

1. \$1092.08 **2.** \$1661.38 **3.** \$1901.02 **5.** \$1329.10 **6.** \$2156.42
7. \$1829.23 **9.** \$672.80 **10.** \$278.47 **11.** \$219.70

Exercise 13-3, page 196

1. \$220.97 **2.** \$734.10 **3.** \$257.08 **5.** \$642.92 **6.** \$120.50 **7.** \$622.32
9. \$429.58 **10.** \$1386.46 **11.** \$1275.50 **13.** \$3194.43 **14.** \$5818.70
15. \$4601.83 **17.** \$7075.49 **18.** \$6723.37 **19.** \$5454.28

Exercise 13-4, page 199

1. Amortization **2.** Sinking fund, both cost the same **3.** Amortize, \$293.78
5. Pay within \$500 of \$7426.39 each period **6.** Pay within \$250 of \$7880.70 each period **7.** Pay within \$500 of \$19,093.30 each period
9. Pay within \$500 of \$29,438.01 each year **10.** Pay within \$250 of \$23,845.24 each period **11.** Pay within \$500 of \$92,389.92 each period

Exercise 13-5, page 200

Only the periodic expense is given for Problems 1 to 8:
1. \$2073.84 **2.** \$7931.14 **3.** \$10,684.23 **5.** \$5788.87 **6.** \$13,245.72
7. \$8977.99 **9.** \$26,794.59 **10.** \$12,015.12 **11.** Pay within \$250 of \$7630.05 each period

Exercise 14-1, page 209

In Problems 1 to 3, only the annual depreciation charge is given:
1. \$440 **2.** \$100 **3.** \$24
In Problems 5 to 7, only the fixed percentage is given:
5. .26155 **6.** .19245 **7.** .13582
In Problems 9 to 15, only the first annual depreciation charge is given:
9. \$1120 **10.** \$216.17 **11.** \$55.14 **13.** \$733.33 **14.** \$184.62 **15.** \$44.80

Exercise 14-2, page 213

1. $R = \$642.55, B_5 = \$4519.74$ **2.** $R = \$60.03, B_2 = \$378.14$
3. $R = \$205.09, B_5 = \$3805.08$ **5.** $R = \$705.43, B_3 = \$4997.93$
6. $R = \$1520.06, B_7 = \$10,994.12$ **7.** $R = \$721.45, B_{14} = \$5,173.07$

9. $1487.80 **10.** $13,764.96 **11.** $3623.70 **13.** 9.329 **14.** 11.910
15. 8.904

Exercise 14-3, page 215

1. R = $4595.25, N = $9595.25 **2.** R = $40,159.52, N = $65,659.52
3. V = $36,061.93, N = $3842.48 **5.** V = $26,077.23, i = 4.728%
6. V = $38,514.27, i = 13.482% **7.** R = $7517.52, i = 13.82%
9. $409,310.08 **10.** $850,118.40 **11.** $19,010.26

Exercise 14-4, page 219

1. $99.19 **2.** $4676.52 **3.** Second **5.** $919.84 **6.** Second **7.** $7792

Exercise 14-5, page 222

1. $8549.04 **2.** $85,309.89 **3.** $16,084.30 **5.** $17,339.53 **6.** $14,138.12
7. $11,226.20 **9.** $977.60 **10.** $856.23 **11.** $13,708.04 **13.** Second
14. First **15.** $2456.74 **17.** $2.46 **18.** $919.11 **19.** $.79 or less

Exercise 14-6, page 224

1. Annual charge is $500 **2.** Fixed percentage is .30117 **3.** $28,948.15
5. $4687.50, $4375, $625, $312.50 **6.** B_8 = $22,070.86, B_{12} = $11,385.36
7. $3,333.33 **9.** 14.748% **10.** $82,537.72 **11.** 18.57% **13.** $2789.65
14. 8.475 **15.** $1126.62

Exercise 15-1, page 228

1. $877.79 **2.** $1802.03 **3.** $471.91 **5.** $1000 **6.** $922.05 **7.** $451.00
9. $571.76 **10.** $924.01 **11.** $865.15 **13.** $902.68 **14.** $971.02
15. $1809.28 **17.** $1816.73 **18.** $2000 **19.** $2237.06

Exercise 15-2, page 231

1. $76.34, $2136.34 **2.** $4.26, $104.26 **3.** −$505.93, $4544.07
5. $62.72, $1062.72 **6.** −$137.08, $882.92 **7.** −$701.18, $1898.82
9. −$71.77, $928.23 **10.** $0, $1000 **11.** $47, $1047

Exercise 15-3, page 234

The answers given are the purchase prices:
1. $956.71 **2.** $2225.18 **3.** $5000 **5.** $1418.39 **6.** $507.88
7. $710.34 **9.** $771.10 **10.** $953.83 **11.** $2015.83

Exercise 15-4, page 236

1. −$17.33 **2.** −$17.32 **3.** $22.39 **5.** −$.18 **6.** $8.80 **7.** −$15.98
9. −$9.16 **10.** −$.28 **11.** −$4.95

Exercise 15-5, page 238

1. $1008.75, $959.57 **2.** $5041.67, $4749.24 **3.** $502.50, $501.79
5. $2007.50, $1821.03 **6.** $3018.75, $3335.82 **7.** $505.83, $563.38
9. $505.21, $473.59 **10.** $501.67, $424.51 **11.** $707, $681.85

Exercise 15-6, page 243

1. $425.60 **2.** $511.40 **3.** $482.05 **5.** $509.65 **6.** $362.05
7. $467.25 **9.** 3% **10.** 3% **11.** 3.5% **13.** 4.5% **14.** 3.5% **15.** 6%
17. 6.91% **18.** 5.46% **19.** 3.16% **21.** 6.91% **22.** 5.46%
23. 3.16% **25.** 3.37% **26.** 4.41% **27.** 5.60%

Exercise 15-7, page 245

1. $48,198.77 **2.** $42,188.52 **3.** $40,804.21 **5.** $88,427.55
6. $93,974.39 **7.** $102,418.90 **9.** $19,713.03 **10.** $19,432.47
11. $12,002.27 **13.** $6745.07 **14.** $6920.45 **15.** $6764.80
17. $20,628.19 **18.** $15,074.44 **19.** $13,853.89

Exercise 15-8, page 247

1. $1026.85 **3.** $1043.09 **5.** $3,109,611.48 **6.** 4.19% **7.** 3.88%
9. $528.85 **10.** $516.38 **11.** 2.69% **13.** $3,247,222.72 **14.** $259,033.64
15. $245,479.49

Exercise 16-1, page 255

1. $122.73 **2.** $619.01 **3.** $602.99 **5.** (a) $9910.25; (b) $14,669.59
6. (a) $3957.95; (b) $5847.70 **7.** (a) $7713.17; (b) $15,520.42
9. (a) $10,365.38; (b) $13,960.66 **10.** (a) $15,804.02; (b) $38,485.79
11. (a) $52,487.41; (b) $172,212.81 **13.** (a) $1661.81; (b) $2689.96
14. (a) $8616.39; (b) $11,605.03 **15.** (a) $5715.04; (b) $9213.94
17. $13,424.74 **18.** $135.82 **19.** $30,819.95

Exercise 16-2, page 260

1. $195.56 **2.** $618.05 **3.** $826.42 **5.** (a) $22,121.92; (b) $40,268.56
6. (a) $22,143.15; (b) $40,227.39 **7.** (a) $22,174.71; (b) $40,166.41
9. (a) $22,365.69; (b) $40,631.68 **10.** (a) $22,457.08; (b) $40,878.67
11. (a) $19,744.55; (b) $53,559.87 **13.** (a) $19,834.29; (b) $53,256.34
14. (a) $19,993.94; (b) $54,236.38 **15.** (a) $20,029.48; (b) $54,109.34
17. (a) $36,695.10; (b) $73,744.73 **18.** (a) $37,346.70; (b) $75,054.23
19. (a) $37,416.99; (b) $74,893.74 **21.** (a) $565.19; (b) $5788.40
22. $6309.56 **23.** $836.54

Exercise 16-3, page 262

1. $22,647.73 **2.** $27,932.69 **3.** $22,710.18 **5.** $116.30 **6.** $74.57
7. $706.01 **9.** $994.59 **10.** (a) $4540.43; (b) $57,592.03

11. (a) $8475.53; (b) $2940.90 **13.** Lose $334.34
14. (a) $10,186.41; (b) $2718.43 **15.** $2121.84 **17.** $10,944.71
18. $526.22 **19.** First by $502.92

Exercise 17-1, page 270

1. (a) .91799; (b) .08201; (c) .02040 **2.** (a) .99757; (b) .47773; (c) .28767
6. (a) .94053; (b) .05957; (c) .07488; (d) .00459; (e) .99541

Exercise 17-2, page 280

1. $1999.24 **2.** $11,642.71 **3.** $349.47 **5.** $7308.11
6. (a) $57,585.45; (b) $60,085.45 **7.** (a) $10,916.27; (b) $12,116.27
9. $12,791.86 **10.** $42,072.33 **11.** $423.04 **13.** $4568.66 **14.** $1895.20
15. $1253.22

Exercise 17-3, page 285

6. (a) $1123.39; (b) $43.80 **7.** $126.97 **9.** (a) $2908.00; (b) $169.52
10. $252.32 **11.** $188.67

Exercise 17-4, page 291

1. (a) $328.57; (b) $24.33 **2.** $39.07 **3.** (a) $231.71; (b) $13.48
5. $342.09 **6.** $322.32 **7.** $670.65 **9.** $554.04 **10.** $30.30

Exercise 17-5, page 299

5. $143.19 **6.** $27.55, $55.63, etc. **7.** $674.12 **9.** $426.01
10. $38.82, $78.38, $118.70, etc. **11.** $1000

Exercise 17-6, page 300

2. (a) .98369; (b) .06392; (c) .02159
3. $7853.52; $7815.40; endowment by $38.12
6. (a) $.11736; (b) $.70687; (c) $.50264 **7.** $6771.58 **9.** $1764.69
10. $875.87, $1775.72, $2700.57, etc. **11.** $4630.48, $10,000

INDEX

DATE DUE

MAY 19 1971			
MAY 18 REC'D			
MAR 7 '78			
MAR 22 '78			
MAR 24 1978 REC'D			
DEC 16 '81			
DEC 16 1981 REC'D			
GAYLORD			PRINTED IN U.S.A.